

Life Insurance and the Agency Conflict: An Analysis of Prudential Regulation to Guard Policyholders from Excessive Risk Taking*

Christian Wiehenkamp
House of Finance
Goethe University Frankfurt[†]

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Abstract

The paper builds on the current discussion on reforming insurance regulation in light of the EU's move towards the Solvency II regime and studies the agency problem in a life insurance environment. It compares different regulatory regimes in their effectiveness to control the owner's incentive for excessive risk taking, both in an analytically tractable life insurance model as well as in the realistic market consistent valuation framework. As such it is the first paper to investigate the Solvency I & II regimes in an analytical framework going back to ??, as well as extend the analysis to the industry standard market-consistent embedded value (MCEV) methodology to address valuation, solvency, and agency questions in a life insurance context. The results suggest that the new Solvency regime eliminates an unfair subsidy of equity holders at the expense of policyholders in bad states of the world. By imposing an implicit restriction on asset performance through the link of capital requirements to asset performance, Solvency II makes policyholder protection compatible with the shareholder incentive of equity value maximization with positive impact on welfare. In the MCEV setup, it is further shown that the value of future operations reduces the owner's incentive for excessive risk taking. Lastly, low market yields can threaten solvency and, despite the optionality of the liabilities, the default put option can dominate equity payoffs.

KEYWORDS: Solvency II; life insurance valuation; limited liability;
market-consistent embedded value; value-based management

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[†]Christian Wiehenkamp, House of Finance, Goethe University Frankfurt, Campus Westend - Grüneburgplatz 1, Uni-Pf 13, 60323 Frankfurt am Main, Germany. Email: wiehenkamp@finance.uni-frankfurt.de.

1. Introduction

While banking regulation has been a hotly debated topic in the aftermath of the crisis, little concern has been voiced about (life) insurers in the public. The recession and its low interest rate environment has also hit the life insurance industry, though, whose business model can typically be characterized as an asset manager who guarantees a minimum return to policyholders. This optionality of the provided floors can become costly for the owner of the insurance company at low market yields and possibly lead to the classical problem of gambling for resurrection as the value of the default option dominates equity value. The paper sets out to study the agency problem in the life insurance environment and compares different regulatory regimes in their effectiveness to control the owner's incentive for excessive risk taking, both in an analytically tractable life insurance model as well as in a realistic market consistent valuation framework.

The long-term nature of the life insurance business makes it easy to conceal difficulties by diluting current problems at the expense of possibly huge future losses. Furthermore, the insurer's remuneration scheme of receiving a fraction of returns generated on behalf of the policyholders adds an incentive to maximize returns. These agency problems are magnified by the absence of sophisticated claim holders. While banks usually commit to an important part of the monitoring of non-financial firms, they are absent in the insurance context, and individual policyholders have no incentive to take over. Consequently, prudential regulation, known from the banking sector has found its way into the insurance landscape to deal with the underrepresentation of claim holders and to control the risk taking behavior of the firm.

The paper builds on the current discussion on the reform of insurance regulation, and the EU's move towards the Solvency II regime. In light of the rationale for prudential insurance regulation, the results suggest that the Solvency II regime, indeed, incentivizes management to reduce risk. Given the link of capital requirements and asset portfolio riskiness, management actually optimizes equity value, even in bad states of the world, when avoiding asset substitution or a gamble for resurrection. In the multiperiod market consistent valuation framework the value of future operations already reduces the owner's incentive for excessive risk taking. The currently low yields do threaten solvency, on the other hand, and Solvency II promises to reward protection of the existing asset portfolio against low interest rate environments. As part of the analysis, it is shown that it can be optimal for the firm owner to contribute additional capital to restore solvency, even in absence of sophisticated regulation.

The remainder of the paper is organized as follows: The following Section introduces the relevant literature, before the optionality of life insurance and solvency regimes are formalized in Section ???. The Section also comprises an analysis of the interplay between regulatory regime and optimal response of the insurance company. Section ??? extends the analysis to a realistic multiperiod setup and discusses risk management and solvency implications of different market environments. Systemic life insurance risk is touched upon in Section ??? before Section ??? concludes.

2. Related Literature

The determination of debt value for firms in general goes back to [?](#) , and [?](#) in an optimal firm value context which sets the stage for the analysis here. Similarly to [?](#) bankruptcy is endogenous in the model, but is based on investment returns and solvency requirements in the insurance setting. With the life insurer essentially being an asset manager who guarantees a minimum return, it is, furthermore, easier to make an assumption about the firm value process, as the asset side of the balance sheet primarily consists of financial assets whose behavior is observable and has been extensively studied in the literature.

The performance based fee structure for managing investments further invokes the issue of asset substitution as first introduced by [?](#) and investigated in the context of firm value by [?](#) . The latter work also introduces the notion of risk management in order to increase value which is highly relevant for insurers.

From a technical point of view of how to value insurance liabilities, [??](#) are the first ones to consider the optionality of minimum guarantees and analyze investment strategies based on hedging arguments following [?](#) . [??](#) further consider the profit sharing component inherent in life insurance policies and take a company perspective by applying the Merton Model to life insurance. One of their key insights is that equity holders do not need the insurance company to bet on interest rate risk and that the equity performance should be immunized against interest rate movements.

Solvency concerns and regulatory action upon breaching solvency rules are introduced by [?](#) in a barrier option framework. Although the paper recognizes that regulatory modeling can effectively limit the value of the limited liability put option, the focus is purely option pricing driven and without a link to a prevailing solvency regime or a managerial view of how to deal with breaching the solvency barrier. The technical analysis of life insurance default risk is extended by [?](#) who allow for a grace period after default, in line with Chapter 11 bankruptcy law, using Parisian barrier options.

The afore mentioned papers on pricing the optionality in life insurance share the common feature of assuming a geometric Brownian motion as determinant of the insurer's asset value dynamics for analytical tractability. For applications in the industry, this constitutes an unrealistic setup for practical purposes, though, and [?](#) provide a detailed description of how to value and hedge insurance liabilities when closed form solutions are not generally available.

The hedging behavior of life insurers has also been analyzed empirically by [?](#) who find insurance size, leverage, and the degree of asset-liability mismatch to be positively related to hedging likelihood, lending support to informational asymmetries and the bankruptcy costs hypothesis.

From a solvency perspective, [?](#) elaborate on the economic rationale for prudential regulation of insurers and argue that the sole purpose of capital regulation should be seen in light of the representation hypothesis of [?](#) . [?](#) highlight the need for common economic principles in risk-based capital management. Recently, [?](#) propose to replace minimum capital requirements by restrictions on a company's asset risk and return profile as part of a standard model under Solvency II. The

rationale for the approach is that the asset structure is much easier to adjust in the short term than the distribution of liabilities. As is shown in the following, Solvency II actually implicitly prescribes such standards, if one is willing to assume payoff maximizing behavior of the equity holder.

All previously cited papers on insurance valuation recognize that insurance payouts in the future are a function of the insurer's investment returns and the minimum guarantee and that they are, hence, not known with certainty. This optionality needs to be recognized for valuation purposes. Motivated by the no arbitrage argument, the liabilities can be valued by finding a replicating portfolio of assets whose cash flows match the insurance cash outflows. With this theoretical backing, it has become standard to compute the value as the discounted, risk-neutral expectation of future cash flows using insights from option pricing theory. Exactly this is done in the following two sections.

3. Optionality & Solvency in Analytical Model

3.1. Valuing the Optionality

For the sake of simplicity, consider the balance sheet of a generic life insurer as depicted in Table ??, where time 0 assets have been normalized to 100 without loss of generality. The setup parallels the one of ?, but differs in the important dimension of profit participation being based on the entire investment return and not only on the amount exceeding the guarantee. This is motivated by regulation as will become clear later. Furthermore, no assumption on the asset pool dynamics is needed in order to illustrate the optionality of life insurance contracts.

Table ?? about here

Consider first the policyholder's guarantee. This is modeled as a minimum required payout at maturity $L_T^* = \alpha A_0 e^{r^* T}$, for α representing the fraction of assets at time 0 that are attributable to policyholders. On top of the minimum guaranteed return r^* , further optionality stems from the regulatory requirement of profit participation. Letting δ denote the portion of profits allocated to policyholders¹, the call option payoff is given by

$$C_T = [\delta \alpha A_T - L_T^*]^+. \quad (1)$$

Adding the two asset pool scenarios where the call option is not in the money, the payoff $L_T(A_T)$ to policyholders as a function of asset value at maturity can be summarized as follows

$$L_T(A_T) = \begin{cases} A_T & \text{if } A_T < L_T^* \\ L_T^* & \text{if } L_T^* \leq A_T < \frac{L_T^*}{\delta \alpha} \\ L_T^* + [\delta \alpha A_T - L_T^*] & \text{if } \frac{L_T^*}{\delta \alpha} \leq A_T \end{cases} \quad (2)$$

¹Policyholders are entitled to at least 90% of investment returns generated by the assets covering the actuarial reserve (i.e. $\delta \in [0.9, 1]$) in Germany (§4 Abs. 3 MindZV).

In the first scenario, the policyholder simply receives all assets and the life insurer is bankrupt. The second scenario is slightly more interesting. It marks the outcomes where the life insurer is able to pay the minimum return, but the payoff of the call option, given by Equation (??), is non-positive. Only for asset pool realizations beyond this cut off, the policyholder receives the minimum guarantee as well as the profit participation component. Equation (??) can be rewritten as

$$L_T(A_T) = [\delta\alpha A_T - L_T^*]^+ + L_T^* - [L_T^* - A_T]^+. \quad (3)$$

The first term of Equation (??) represents the profit participation component and is a call option that the policyholder is long. L_T^* is the minimum guarantee and the third component represents the payoff of the default put option that the policyholders have written. Similarly, the equity payoff can be decomposed into the standard limited liability call option and the bonus participation call of the policyholder which the equity holder is short

$$E_T(A_T) = [A_T - L_T^*]^+ - [\delta\alpha A_T - L_T^*]^+. \quad (4)$$

Given an appropriate assumption for the asset pool dynamics, ?? shows that the fair equity and liability values can be computed in closed form.

In the standard corporate finance setting, equity holders' residual claim on the firm's assets pays off as soon as the asset value exceeds the required payout to debt holders (or policyholders in the insurance setting). Without profit participation or minimum guarantees, payoff diagrams would equal the typical concave and convex shapes. It is well known that equity holders embrace risky projects and have an incentive to gamble for resurrection when firm value deteriorates. In the insurance context, the additional optionality alters the payoff profiles (Figure ??), and it is not clear *ex ante* whether the benefit of excessive risk taking still dominates the equity holder payoff in Equation (??).

Figure ?? about here

As can be seen from Figure ??, profit participation and guarantees alter risk preferences. In the region $A_T > L_T^*$ the equity holder's payoff function is actually concave, decreasing the owner's overall risk appetite. On the other hand, in the region close to insolvency, the typical convex shape prevails, suggesting an incentive for asset substitution and gambling for resurrection.

The non-linear effect of risk on value is illustrated in Figure ?? which plots, both the fair equity and liability values as well as their sensitivities to volatility changes as function of the rates and volatility scenario. With the limited liability call dominating total equity value for low interest rate scenarios, the notion of gambling for resurrection is confirmed when looking at the positive impact of volatility on the fair equity value (Subplot (b)). Subplot (d), which shows the Vega² of the

²Vega summarizes the change in the option value (of the equity holder) as a result of a change in volatility.

equity value, confirms that for rate scenarios of up to about 4%, increasing riskiness (i.e. volatility) actually increases value. Looking at the fair liability value (Subplot (a)), it is little surprising that the opposite picture compared to the equity value emerges. For low rates, increasing volatility decreases the liabilities. At the same time, the risk free rate affects the discounting of policyholder cash flows, such that the higher the rate, the lower the total liability value.

Figure ?? about here

3.2. Solvency

The stylized setup of this section not only illustrates that life insurance contracts can be valued using insights from option pricing, but the role of the regulator can also be analyzed. In the traditional, theoretical approach to insurance regulation, the regulator is to ensure that the probability of insolvency (i.e. $L_T > A_T$) is below a given threshold β_{\max} , say 1%. Denoting by $g(\tilde{r})$ the random function of the asset pool dynamics which determines the change in the liabilities, the insolvency condition can be written as

$$L_0(1 + g(\tilde{r})) > A_0(1 + \tilde{r}), \tag{5}$$

where \tilde{r} is the return on the asset portfolio.³ Using the accounting identity $A = L + E$, Equation (??) can be rewritten in terms of the period's net earnings as

$$A_0\tilde{r} - L_0g(\tilde{r}) < -E_0, \tag{6}$$

such that the probability of failure, after defining the random net earning's variable $X = A_0\tilde{r} - L_0g(\tilde{r})$, is given by

$$\text{Prob}(X < -E_0) = \beta. \tag{7}$$

The larger E_0 , the lower β ; and the equity level that ensures a survival probability of $1 - \beta_{\max}$ marks the minimum capital requirement. While postponing a more detailed discussion of current and proposed EU regulation to a later section, the model, nonetheless, allows to highlight the distinct features of the Solvency I and II regimes. Before looking at the implications of the different regimes, though, Figure ?? shows the necessary inputs for the computation of the default probability in Equation (??). Subplot (a) gives the distributions of the time T values of assets and liabilities which correspond to the market risk components on the left hand side of Equation (??). The empirical distribution of X is shown in Subplot (b).

Figure ?? about here

³Note that, based on the liability payoff given by (??), the random function $g(\cdot)$ can in principle be obtained as the combination of the censored distributions determining the embedded option payoffs.

Solvency I. Given the size of the insurance reserves L_0 , $\lambda = \frac{E_0}{\vartheta L_0} > 1$ is the solvency ratio determined by the regulatory constant $\vartheta \in (0, 1]$. The regulator chooses the latter in the hope of it implying a sufficient level of capital in order to ensure $\text{Prob}(X < -E_0) \leq \beta_{\max}$. Furthermore, there is no real consideration of the distributions of different risks⁴ and their joint impact on the insurer’s performance, as summarized above by the distribution of X .

Solvency II. Even in terms of the simple model presented here, the Solvency II framework materially changes the picture of assessing the default probability. For determination of the capital requirement, various risk sources are recognized and modeled. When combining them, the distribution of X is then obtained based on the normal distribution and prespecified correlation assumptions.⁵ In terms of λ , the required capital is no longer a constant fraction of the liabilities, but a function of the value-at-risk (VaR), such that $\lambda = \frac{E_0}{E_{\text{SCR,stan}}}$, where $E_{\text{SCR,stan}} = F(X|\beta_{\max})$ is the capital requirement given the joint distribution of risks and the maximally acceptable failure probability β_{\max} . This approach for determining the capital requirement is much in line with the standard formula of Solvency II, which will be discussed in more detail in Section ??.

Economic Value-at-Risk. Under Solvency II, the regulator also allows using the empirical distribution of X , as shown in Subplot (b) of Figure ??, in order to determine the one period ahead default probability. This approach, referred to as internal model, neither any longer hinges on the constant correlations assumption between risks, nor on the normal distribution summarizing the joint behavior of these. Similarly to the Solvency II regime with standard formula, required capital is then a function of the VaR which is based on the empirical distribution (i.e. true distribution) of X in this case. The solvency ratio λ is the ratio of available capital E_0 and required capital $E_{\text{SCR,int}} = F_{\text{econ}}(X|\beta_{\max})$ as implied by the maximally allowed failure probability.

Table ?? about here

The capital requirements implied by the outlined solvency regimes are analyzed in a numerical application of the section’s model and are shown in Table ?. For different market environments the table reports minimum capital requirements and the corresponding solvency ratios λ for the three regimes.

Following the definition of λ under Solvency I, capital requirements do not reflect the risk position of the insurer’s asset portfolio. As such the implied probability of default can be significantly underestimated compared to the true one (of the economic VaR), given the risk on the asset side. From the insurance owner’s perspective, Solvency I regulatory capital may result in the authorities taking over the insurance company upon the latter breaching the prespecified solvency level λL_T^* and the owner losing up to $(\lambda - 1)L_T^*$, as can be seen in panel (b) of Figure ?. Since λ is independent of the

⁴Inclusion of additional risk sources would add further random variables on the left hand side of Equation (?).

⁵See CEIOPS, Solvency II Final L2 Advice (available at <http://www.ceiops.eu/content/view/706/329/>) for details.

insurer's asset risk, though, the Solvency I regime is insufficient to overcome the asset substitution problem.

The Solvency II framework overcomes this problem, and the required capital reflects the insurer's risk position. Notably the approach requires more capital in scenarios of little asset pool growth or larger volatility. Consider the case where the portfolio drift equals the minimum return r^* of 2.25%. The minimum capital requirement of 4.61 exceeds that of Solvency I by more than 20%. Consequently the implied default probability in the latter regime significantly exceeds the desired 1% level. On the other hand, the table also shows that, in a less risky asset pool environment, the insurer can reduce his equity capital share. With management of (especially) financial institutions typically regarding equity capital as expensive, this result shows that Solvency II incentivizes insurers to avoid excessive risk taking and/or to reduce their risk positions. Interestingly, the regulatory framework is tailored to management perception of capital being scarce and, hence, expensive. This promises to make the Solvency II initiative effective. In the following, it will actually be shown that the regulatory regime is actually incentive compatible, as policyholder protection and value maximization goals are aligned.

Comparing the internal model (i.e. economic VaR) with the standard model of Solvency II, note that the assumption of X being normally distributed in the former setup is little troubling in the simple model of this section; the standard Solvency II framework actually produces the more conservative capital requirements. This, in turn, is not surprising when recalling that the normal distribution theoretically allows for losses of infinite magnitude, whereas they are practically limited by the difference between asset returns and minimum guarantees for any given scenario.

Lastly, the scenario-dependent correlation between terminal asset and liability values (Table ??) highlights the importance of considering market risk of assets and liabilities jointly. Due to the non-linear dependence of reserves on asset growth, policyholder claims co-move more with asset value, the less desirable (i.e. low drift or high volatility) the capital market scenario.

3.3. *Regulation in a Strategic Game between State and Insurance Company*

The analysis of the equity value in terms of a limited liability call and the short bonus participation option has shown that the effect of increased asset side risk is market scenario dependent and not always dominated by profit participation and guarantees. This naturally gives rise to an incentive for gambling for resurrection. At the same time, the discussion of the different solvency regimes has highlighted that Solvency II is likely to discourage excessive risk taking behavior.

To analyze the effectiveness of the regulatory regime in reducing the agency conflict more formally, consider a strategic game where the regulator⁶ chooses a solvency regime and the equity holder, subsequently, maximizes his payoff by altering his risk position on the asset side. In the setup of this section σ is considered as the choice variable determining asset side risk, and the insurer's problem

⁶Regulator, State, or government are used interchangeably.

is given by

$$\begin{aligned} \max_{\sigma} E_0^{\text{fair}} &= A_0 [\Phi(d_3) - \delta\alpha\Phi(d_1)] + L_T^* e^{-r(T)} [\Phi(d_2) - \Phi(d_4)] \\ \text{subject to } E_0 &\geq E_{\text{SCR}} \\ \sigma &\in [0, 100\%] \end{aligned} \quad (8)$$

where E_0^{fair} is the fair equity value, $\Phi(\cdot)$ is the Normal cumulative distribution function, and d_1, \dots, d_4 are defined in ??; furthermore E_0 is the available capital of the insurance company as in Table ??, and E_{SCR} is the minimum capital requirement determined by the prevailing solvency regime and a function of σ for Solvency II. For the Solvency I & II regime options *played* by the regulator, the value maximizing choice σ^* , in accordance with (??), is shown in Panel (a) of Figure ?? for risk free rate r_f and (real world) drift⁷ of the asset portfolio μ between 0 and 10% (i.e. $r_f \in [0, 0.1]$ and $\mu \in [0, 0.1]$). The corresponding equity values are depicted in Panel (b). The results of Figure ?? confirm that for a variety of (low) interest rate environments the value of the limited liability option cannot be ignored. For sufficiently low risk free rates, it is optimal for the equity holder to try to gamble for resurrection by increasing his risk position. With Solvency I not incorporating asset side risk, the owner optimally increases σ^* to 100%. While the incentive to increase equity value by increasing risk remains present under Solvency II, the asset risk dependent capital requirements effectively limit the possibility to do so, and, instead of choosing an asset with $\sigma^* = 100\%$, the insurer optimally chooses $\sigma^* \approx 4\%$. As such, capital requirements within Solvency II actually, implicitly impose minimum standards on asset performance without requiring an explicit model for it as proposed by ?.

Figure ?? about here

While Figure ?? shows that lowering the risk position within the Solvency II framework compared to Solvency I is actually incentive compatible, even in cases where the value of the limited liability option dominates equity value, the effect on welfare can only be assessed after also having considered the second player in the game; the regulator. Given the insurer's chosen risk position, the government payoff is given by the shortfall associated with the insurer not earning the guarantee. In other words, the State is assumed to be paying the guarantee to policyholders, if the insurer becomes insolvent. This choice of payoff function can be motivated by governments having been shown to step in for nearly insolvent financial institutions when serious economic consequences loom. In the case of life insurance, these consequences would be characterized by retirees losing their private pensions in an environment where governments have been encouraging people to rely more and more on non-public retirement plans. Formally, the State's payoff is given by

$$\text{Loss} = E [\min(0, A_T - L_T^*) | \sigma^*, \text{Solvency Regime}]. \quad (9)$$

⁷Note that although the real world drift μ does not enter the objective function of the insurer due to the risk neutral pricing, its effect on σ^* and the fair equity value is not necessarily zero because of its effect on the VaR (i.e. constraint) in the Solvency II setup.

For the two risk free rate scenarios $r_f = 2.5\%$ (Panel (a)) and $r_f = 5\%$ (Panel (b)), Figure ?? shows the losses incurred by the government in a chosen solvency regime for the various drift scenarios underlying Figure ?. If there is no solvency regime in place or if the insurer is regulated within the Solvency I framework, the excessive risk taking incentive in a low interest rate environment leads to frequent insolvencies and to high associated costs for the government (i.e. society). It is interesting to note that the improvement offered by Solvency I compared to the ‘no regulation’ case is actually only marginal, as can be seen from the slightly less negative mean State loss in Panel (a) of Figure ?. The picture, however, changes completely when introducing the risk based capital requirements of Solvency II; with their implicit limits on the insurer’s optimal risk position, the loss due to insolvencies can be significantly reduced and is actually zero in the given setup. Consequently, Solvency II regulation achieves policyholder protection while the owner pursues an equity value maximizing strategy, and as such creditor protection becomes incentive compatible for firm owners. In welfare terms, equity value (Panel (b) of Figure ?) may be lower in some states of the world due to Solvency II, but the (private) gains from less stringent capital regulation are more than offset by the (public) losses due to life insurance failure. Therefore, the Solvency II regime not only improves policyholder protection, but is actually welfare enhancing by doing so.

Figure ?? about here

4. Market-Consistent Embedded Value

With Solvency II having been shown in the simple model of the previous Section to achieve policyholder protection while retaining the profit maximization incentive of owners, insurance companies typically also write new policies at the end of the period and benefit from future profit sharing of existing contracts, though. Consequently the continuation value of existing operations should already induce owners to avoid the risk of bankruptcy in a given period, if the expected future profits exceed the attainable payoff of the current period. In terms of the Bellman Principle, fair equity value E^{fair} is given by

$$E_0^{\text{fair}} = G(\text{Profits}_t) + E \left[\sum_{k=t+1}^{\infty} G(\text{Profits}_k) \right], \quad (10)$$

which shows that foregoing future profits by means of excessive risk taking in the current period may not constitute the optimal policy; $G(\cdot)$ assigns the eligible portion of the period’s profits to equity holders. As such a multiperiod setup already reduces the risk shifting incentive even without regulation. In the following, this Section, therefore, applies the solvency and value maximization insights from the simple model to a realistic, multiperiod setting.

Apart from model horizon, payoffs to policyholders from investment income are generally based on more complex sharing rules. Consequently, one cannot hope for a closed form solution for the fair equity and liability values, in general. Nonetheless, the principal of discounting cash flows and taking

risk neutral expectations remains applicable, and the market-consistent embedded value (MCEV) definition of the \mathbb{V} is based on adjusting the net-asset value by the present value of future profits (PVFP) including the time value of options and guarantees. The decomposition of the MCEV into net asset value, future profits, time value of embedded optionality, and capital cost is shown in Figure ???. The cost of capital captures frictional cost such as the ones related to taxation and investment as well as cost of non-hedgeable risk (CFO Principles 8 & 9); they are mentioned here for the sake of completeness, but the paper abstracts from their inclusion in the following.

Figure ??? about here

In order to determine the (stochastic) present value of future profits, cash flows that can be discounted for pricing purposes are needed. These can either be obtained based on actuarial predictions of insurance cash outflows or by generating asset return scenarios which are mapped into the insurer's balance sheet and subsequently into cash flows based on accounting and regulatory principles. Figure ??? illustrates the idea of the time t balance sheet, together with the capital market scenario between t and $t + 1$, determining the profit and loss account (P&L), which is then shared among policyholders and firm owners based on legally and contractually binding sharing rules.

Figure ??? about here

By simulating the balance sheet forward, one obtains a time series of local GAAP (generally accepted accounting principles) compliant asset, liability, and equity book values until the end of the investigated period (typically 30 years). Based on existing insurance policies (i.e. liability structure) at time $t = 0$, subsequent cash flows to policyholders are determined by the evolution of the actuarial reserve. When returns are simulated using risk-neutral dynamics, in order to allow discounting at the risk free rate, averaging discounted cash flows over many scenarios gives the fair value of insurance assets and liabilities. The PVFP is then the net asset value of the market-consistent balance sheet, and the MCEV is obtained in accordance with Figure ???. Note that the market-consistent balance sheet is not the same as the one implied by local GAAP, even if the latter requires measurement of assets at market values. The reason for this is that even measurement at market values implies a balance sheet that does not account for future cash flows.

Although the simulated balance sheet approach for valuing life insurance (contracts) is the more complex cash flow model compared to relying on actuarial predictions of cash flows, it allows for a more in depth and more realistic analysis of the interplay between asset structure and fair (equity) value as well as the incorporation of solvency rules to investigate the associated costs of breaching these. Before doing so, the previously mentioned complex sharing rules as well as Solvency regulation and other details for simulating the balance sheet need to be elaborated on, though. The German life insurance environment is used as an example of a major European market where global insurance players, such as Allianz, are headquartered.

4.1. Model Building Blocks⁸

4.1.1 Determination of Payout to Policyholders

It is the regulator who determines the minimum guarantee level – currently at 2.25% in Germany – as well as the minimal profit participation. More concretely, policyholders are entitled to at least 90% of the insurer’s investment income (§4 Abs. 3 *MindZV*), where the relevant attributable investment income equals the income generated by the assets covering the actuarial reserve (§3 *MindZV*⁹). Note that this sharing agreement implies that the insurer is remunerated like an asset manager with up to 10% of the returns on the assets invested on behalf of the policyholders, given that the hurdle rate of the minimum guarantee is exceeded.

Similar income participation minimums exist for risk income (i.e. income from mortality events deviating from actuarial predictions) and other income. In the former case, 75% are credited to policyholders and 50% of all other income is shared with contract holders. Since investment income constitutes by far the largest portion on the insurer’s income statement and given that the paper at hand focuses on the interaction between capital market developments and equity value, risk and other income are ignored in the following. Figure ?? summarizes the attribution of income.

Figure ?? about here

Figure ?? also shows that the amount attributable to policyholders is further divided into a profit participation account for policyholders in deferment (*Schlussüberschussanteilsfond (SÜAF)*), a provision for premium refund (*Rückstellung für Beitragsrückerstattung (RfB)*), and a remainder that is added to the actuarial reserve and available for payout in the next period. The remainder equals the excess of the *RfB* level over the sum of the last three periods’ contributions, since the *RfB* level is effectively limited by the tax code (§21 Abs. 2 *KStG*).

On top of the income participation, the regulator requires the insurer to share 50% of the valuation reserve¹⁰ with policyholders at the end of their deferment period (§153 Abs. 3 *VVG*). Regulation does, however, allow to use the *RfB* for this (§56a Abs. 3 *VAG*), which reduces the remainder described in the previous paragraph. Only if the available amount is insufficient will the rest be charged against the P&L. On the contrary, if the available amount exceeds the required valuation reserve participation, the surplus is attributed as shown in Figure ?. The periodic surplus attribution takes, both, the contract’s level of actuarial reserve as well as the difference between its minimum guaranteed return and the realized investment return into account (§28 *VVG*). Given a sufficient level of investment returns, this ensures that different policy holders receive the same return, irrespective of the individually guaranteed minimum rates.

⁸For the sake of completeness, the following description includes references to German regulation, but no knowledge of the latter is required for a proper understanding.

⁹The actual computation of the attributable investment income, as prescribed by §3 *MindZV*, is slightly more involved, but the paper abstracts from this complication here.

¹⁰The valuation reserve arises from assets’ market values exceeding their book values.

Figure ?? about here

4.1.2 Lapses

A small percentage of policyholders surrenders its insurance contract typically every period. The insurer is obliged to return the redemption value (§169 VVG), determined by the accumulated individual actuarial reserve and the surplus investment income in excess of the cancellation fee¹¹, to the policy holder. Apart from the fee, the insurance company also profits from cancellations by retaining the accumulated profit participation during the deferment period.¹² Although the penalty may decrease with progression of the deferment period, the paper abstracts from these dynamics and assumes a constant average penalty of 2%, which is in line with industry practice. Figure ?? illustrates the effect of lapses.

Figure ?? about here

4.1.3 Scenario Generator and Mortality Tables

Implementation of the previously described regulation is a necessary condition for computing the MCEV of a generic life insurer and analyzing the value of the limited liability option. It is, however, not sufficient, and further assumptions are needed, which are described in the following.

Capital Market Scenario Generator. Most importantly, the scenario generator, determining each period's P&L, needs to be defined. With insurers being heavily invested in fixed income papers, the term structure model constitutes the centerpiece of the capital market model. For the sake of analytical tractability, the one factor Vasicek Model for the short rate process

$$dr_t = a(\bar{r} - r_t) dt + \sigma dW_t \quad (11)$$

is chosen, where \bar{r} is the average long-run rate and W_t is a standard Brownian motion. With the goal of discounting policyholder cash flows, Equation (??) is formulated in risk-neutral terms, as is standard in the derivative pricing literature.¹³ The model allows for a closed form solution of zero coupon bond prices $B(t, T)$ at time t with maturity T given by

$$B(t, T) = \exp \{-A(t, T)r_t + D(t, T)\}, \quad (12)$$

¹¹Although the insurer is only allowed to deduct a penalty fee, if such a fee has been agreed upon and has been quantified (§169 Abs. 5 VVG), insurers do not generally adhere to this according to a verdict of the Hamburg District Court in November 2009 (*Az.: 324 O 1116/07, 1136/07, 1153/07*).

¹²More concretely, only those amounts that have already been assigned to individual policyholders during the deferment period need to be paid out (§169 VVG).

¹³Risk-neutral parameters can be obtained by fitting the interest rate process to prices instead of rates. Table ?? shows the estimates based on term structure information provided by the *Bundesbank* between 1986 and 2010.

where

$$\begin{aligned} A(t, T) &= \frac{1 - e^{-a(T-t)}}{a} \\ D(t, T) &= \left(\bar{r} - \frac{\sigma^2}{2a} \right) [A(t, T) - (T - t)] - \frac{\sigma^2 A(t, T)^2}{4a}. \end{aligned}$$

The discount factors implied by the zero coupon bonds, then, allow the computation of coupon bond prices of any maturity. Naturally, these trade at par at issuance, but change their values in subsequent periods affecting the valuation reserve as well as the cash flows from divestment activities of the insurer. For the MCEV analysis, the insurance company is assumed to invest premium receipts into the period's new bond issues of 1, 5, 7, 10, 20, and 30 years maturity given a constant investment rule.

Furthermore, an equity index is available for investment whose risk-neutral dynamics are given by

$$dS_t = r_t S_t dt + \sigma_S S_t dB_t, \quad (13)$$

where B_t is another standard Brownian motion. By using the modeled short rate in Equation (??), the state of the economy, as measured by the interest rate level, carries over to stock market performance.

Mortality Tables. While it would be possible to model a large number of individual contracts, each based on a Poisson process with intensity corresponding to the policyholder's characteristics, it is convenient to aggregate contracts with similar attributes. This, in turn, means that one can estimate cash flows based on the expectation of the percentage of people in a given cohort being alive. Since this expectation refers to the entire population, it is independent of a particular insurer, and the standard mortality tables of the Association of German Actuaries (*Deutsche Aktuarvereinigung* (DAV)) can be used. Here, the DAV 2004 R table (see ? for details) is used, adjusted for 2009 as base year. The table contains the mortality as function of the year of birth together with the expected changes in mortality over subsequent years. Thus, the probability $TPX_{t-i, t+j}$ of being alive in year $t + j$ for someone borne in year $t - i$, for $i, j \geq 0$, can be computed as

$$TPX_{t-i, t+j} = \prod_{k=1}^j (1 - DAV_{t-i, t+k}), \quad (14)$$

where $DAV_{t-i, t+k}$ is the probability of dying in year $t + k$ of someone borne in year $t - i$.

4.2. Solvency

4.2.1 Solvency I

In accordance with Section ?? consider first the Solvency I framework, which is currently still applicable in the EU. The Solvency I rules have been established by EU Directive 2002/13/EC which has subsequently been translated into national law. In the UK, for instance, this is accomplished through the FSA's General Prudential sourcebook. For Germany, Solvency I capital requirements are determined by the *Solvabilitätsverordnung (SolvV)*.

The insurer's available capital, defined as the equity capital according to local GAAP (*Handelsgesetzbuch (HGB)*) plus the profit participation account during deferment (*SÜAF*) as well as the non-assigned part of the provision for premium refund (free *RfB*) (see §53c Abs. 3 VAG), has to exceed the capital requirement given by 4% of the actuarial reserve plus 0.3% of the capital at risk. Here, capital at risk is defined as the nominal sum of the policyholder payments minus actuarial reserve. Figure ?? illustrates the solvency check.

Figure ?? about here

4.2.2 Solvency II

While the Solvency I regime is easy to understand, the results of the analytical model have already indicated that capital requirements are independent of asset portfolio risk and that the regime, hence, provides little incentive for risk management. The Solvency II initiative is to overcome this by stipulating minimum amounts of capital for all types of risks based on the insurer's exposure. The quantitative requirements of Pillar I prescribe a target level of capital (solvency capital requirement (SCR)) corresponding to a 99.5% confidence level in a VaR framework at the one year horizon, as well as a minimum capital requirement (MCR) equal to an 85% confidence level. The 'two thresholds' system distinguishes between passive and active regulatory intervention.¹⁴ If the company's capital exceeds the SCR, the regulator simply ensures the correctness of the disclosed information. For available capital exceeding the MCR, but not satisfying the SCR, the regulator is to carry out further investigations and provide guidance back to a more prudent solvency level. Only for capital falling short of the MCR, will the regulator take over the insurance firm.¹⁵

Similarly to the Basel II capital accord, Pillar I is augmented by guidelines for corporate governance and risk management (Pillar II), and requirements for supervisory reporting and disclosure (Pillar III). With insurers, unlike banks, having to cover long-term liabilities, regulation needs to incorporate liability side risks, calling for a more sophisticated, market-consistent balance sheet approach compared to the Basel II framework, though.

¹⁴? elaborate further on the rationale for a 'double trigger' approach to insurance regulation.

¹⁵In the following, the discussion will solely focus on the SCR, but the analysis easily carries over to minimum requirements.

Just as the previously discussed solvency analysis in the analytical model, the insurer's solvency position can be assessed by comparing available and required capital. The former is given by the net-asset value of the market-consistent balance sheet (i.e. the MCEV at time 0). The SCR is formally defined as the amount of capital E_{SCR} that ensures that the one period loss $L = MCEV_{t=0} - PV(MCEV_{t=1})$ only exceeds the SCR with the maximally acceptable probability $\beta_{\text{max}} = 0.5\%$

$$\text{Prob} \left(MCEV_{t=0} - \frac{MCEV_{t=1}}{1 + r_f} > E_{\text{SCR}} \right) \leq \beta_{\text{max}}, \quad (15)$$

where r_f is the one year risk free rate. While the insurer would ideally determine the SCR based on the market-consistent valuation of assets and liabilities using an internal model, the method takes up significant resources, and the regulator, therefore, allows resorting to a standard model which is intended to approximate the loss distribution. Both methods are considered here and elaborated upon in the following.

Standard Model. In the standard model, solvency capital requirements are determined for each risk module (market, default, life, health, non-life) which are then aggregated using an approximating normal distribution for the loss distribution (see CEIOPS' Final L2 Advice available at <http://www.ceiops.eu/content/view/706/329> for details on the use of the standard formula). In particular, the L2 Advice (former Consultation Papers no. 47 & 70) proposes an application of market stress scenarios to determine the capital requirement based on changes in the net-asset value, where the asset and liability revaluations incorporate all optionalities and risk mitigation efforts. For example, the relative stresses to the interest rate level are proposed to be 46% in the downward and 55% in the upward direction for 5 year maturities. The total capital charge for interest rate risk is then the maximum change in net-asset value of various stresses. The capital charges for the other sub-modules of market risk are obtained in a similar fashion and then aggregated according to

$$\text{SCR}_{\text{Mkt}} = \sqrt{\sum_{i,j} \rho_{i,j}^{\text{Mkt}} \cdot \text{Mkt}_i \cdot \text{Mkt}_j}, \quad (16)$$

where i and j represent the various sub-modules and $\rho_{i,j}^{\text{Mkt}}$ is their correlation given in Table ???. Given the capital charges for each risk module, in similar spirit to Equation (??), the basic solvency capital requirement is given by

$$E_{\text{SCR,stan}} = \sqrt{\sum_{i,j} \rho_{i,j}^{\text{SCR}} \cdot \text{SCR}_i \cdot \text{SCR}_j}, \quad (17)$$

where i and j are the five sub-modules with correlation coefficients $\rho_{i,j}^{\text{SCR}}$ given in Table ???.

Tables ?? and ?? about here

Internal Model. Not only may the standard model improperly capture the desired confidence level based on the proposed stresses, but also the normality of the overall loss distribution and the associated constant correlations assumptions may lead to an inaccurate picture of the overall solvency position. Consequently one may have to turn to a multivariate approach to assess the impact of the various risk factors simultaneously, which can be achieved by obtaining today’s MCEV and the distribution of the MCEV at the one year horizon in accordance with Equation (??).

Figure ?? about here

To operationalize the multivariate assessment of the solvency position, one approximates the distribution of the MCEV at $t = 1$ by its empirical counterpart which is obtained by simulating N real-world scenarios between $t = 0$ and $t = 1$ and computing the MCEV at each node. Figure ?? illustrates the nested simulations approach to determine the VaR at the one-year horizon. The solvency capital requirement is then obtained from the β_{\max} -quantile of the MCEV distribution and is given by

$$E_{\text{SCR,int}} = \text{MCEV}_{t=0} - \frac{\text{MCEV}_{t=1}^{\beta_{\max}}}{1 + r_f}. \quad (18)$$

Due to the significantly increased complexity for obtaining capital requirements using an internal model, one can expect insurance firms to only commit the required resources, if lower capital charges can be expected as result of the effort.

4.3. Model Output under the Solvency I Regime

Having outlined the complex and rather technical profit sharing rules that determine the policyholders’ payouts as function of the capital market scenario, the MCEV model now allows to address the questions of agency cost and whether the continuation value of operations in a multiperiod setup incentivizes the owner to avoid insolvency already under the simple Solvency I regime. Before doing so, it is useful to get acquainted with the model output and cash flow profiles for two representative policies, though. The first one is characterized by periodic contributions and a lump sum payout to the policyholder at the end of the deferment period. The second type pays out as an annuity until every policyholder is dead in expectation. Figure ?? shows the payouts and the key accounts of the actuarial reserve, and the profit participation for the two profiles.

Figure ?? about here

As already highlighted above, the MCEV is the net asset value adjusted for the present value of future profits which is obtained by discounting asset and liability cash flows. Despite all contract specifications – apart from payout profiles – and capital market realizations being equal, the difference in embedded values, with a higher value in the lump sum payout case, is hardly surprising; while in the annuity case, the insurer has to share the investment income until the last policyholder has

passed away, the investment returns are completely credited to the equity holder once the lump sum payment is made.

Having established some basic properties of model outputs, one may turn to the analysis of the insurer's value. To analyze whether the equity holder always maximizes his payoff by exercising his limited liability option, all subsequent results are reported for the case where the equity holder restores solvency by contributing additional capital as well as for the standard corporate finance case of surrendering the company upon breaching solvency rules. In terms of Figure ?? (Subplot (b)), the residual claimants either contribute $\lambda L_T^* - A_T$ to obtain the right to continue operations, or decide to receive $(\lambda - 1)L_T^*$ and close down the company.

4.3.1 Asset Allocation

For the purpose of the analysis an insurer with initial balance sheet size 100 is considered. While one may typically think of insurance companies taking risks on the liability side by writing policies, they are actually also subject to material (investment) risk on the asset side. One of these risk positions, over which management exerts discretion, is the share of assets invested into risky equity securities. Figure ?? shows the impact of varying the asset mix on the MCEV for insurers with low and high initial solvency ratios.¹⁶

Figure ?? about here

In both cases, the firm owners benefit from moving into equities compared to a pure fixed income asset portfolio. Beyond about 10% of assets under management allocated to equities, though, the increased volatility of the asset portfolio makes the policyholders' guarantee ever more valuable, and the cost of foregoing the continuation value of operations outweighs the benefit from short term risk shifting. Also note that a well capitalized firm exhibits a slightly stronger response to an initial increase of equity securities than a less well capitalized one. When it comes to differences between limited and unlimited liability of the equityholder¹⁷ real differences only emerge for prohibitively large shares of assets invested into equities. This coincides with the point where the owner's call option value begins to be dominated by the effect of increased volatility. Interestingly, restoring solvency in these cases increases the payoff to the owners by maintaining the right to participate in future returns. This suggests that even in the region where the standard positive effect of volatility on equity value dominates, the latter can be enhanced by restoring solvency.

Not only do these results indicate that in an insurance context, exercising the limited liability option is not always optimal, but the analysis also highlights the usefulness of the MCEV model in terms of assessing agency problems between policyholders and firm owners. Seen in light of the

¹⁶The solvency ratio is determined by available and required solvency capital. In 2009, the ratios of selected, large insurance companies ranged from 1.6 (Allianz) to 2.5 (Generali); these extremes also serve as benchmark for the initial solvency ratios underlying Figure ??.

¹⁷Unlimited liability here, and in the following, is understood as owners always contributing the capital required to restore solvency.

performance based fee structure,¹⁸ the altered payoff profile (see Figure ?? Subplot (b)) induces the firm to cautiously manage assets and to not engage in asset substitution, even in absence of a sophisticated solvency regime.

4.3.2 Interest Rate Environment

While equity exposure clearly is a non-negligible risk, it is generally not the most important one faced by a (German) life insurer. Based on CEIOPS (Committee of European Insurance and Occupational Pensions Supervisors) and *BaFin* (Federal Financial Supervisory Authority) publications, Figure ?? shows that interest rates constitute the most important risk for a life insurer.

Figure ?? about here

Figure ??, therefore, implies that rates risk which can again be analyzed using the MCEV model should be of particular concern when investigating the agency conflict between owners and policyholders. The impact of different interest rate environments on the insurer's value is depicted in Figure ?. Given the existence of guarantees, it is little surprising that the interest rate level materially affects the response of the MCEV to different rate scenarios. Given a calibration of the term structure to historical bond prices (Table ??) – implying a rather high mean interest rate – the effect of increasing the initial interest rate r_0 when simulating Equation (??) has a predictable, nearly linear impact on firm value with no differences between the limited and unlimited liability cases (Subplot (a)). This, however, completely changes when looking at the effect of the economy's mean interest rate level. Subplot (b) shows that insurers which operate in an interest rate environment below the guarantee level are significantly at risk of going bankrupt. Given the currently low rates regime, regulators are therefore rightfully concerned. The interesting and new implication of the picture is, however, that firm owners have an incentive to restore solvency for interest rate levels considerably below the guarantee rate. This increases the value of the firm compared to the residual value in case of bankruptcy and shows that exercising the limited liability option is not always the best strategy, which, in turn, is good news for policyholders. Only for mean rates close to zero is the owner better off liquidating the insurer. Figure ?? also shows how rates volatility affects the MCEV in high (Subplot (c)) and low (Subplot (d)) interest rate environments. When the owners do not have to worry about the rates level, the classical situation of volatility benefiting the residual claimants dominates guarantee and profit participation effects. These scenarios are of little concern to policyholders, though, because the danger of bankruptcy in a high interest rate regime is somewhat limited. Subplot (d), on the contrary, again shows the possibly devastating effect of a low interest rate regime. If exercising the limited liability option, the MCEV hardly exceeds the liquidation value and also higher volatilities can only marginally increase the number of non-bankruptcy scenarios.

¹⁸Recall that the insurance company is essentially remunerated like an asset manager receiving 10% of the returns, given the hurdle rate of the guarantee is exceeded.

With volatility increasing the chance of the residual claim being in the money, it generally pays off to restore solvency and retain the right to continue operations. Only for scenarios where volatility is so low that there is virtually no chance of the call option of the owner ending in the money is it advisable to liquidate the company.

Figure ?? about here

The effect of the rates level on the MCEV documented here is equivalent to the one shown in the insurers' MCEV reports; a lower rates level significantly reduces the fair equity value. On the other hand, the impact of volatility on value is not as clear cut as the reports suggest. Whereas the effect of increasing riskiness by investing more into equities exhibits the same diminishing effect on value as suggested by the reports, interest rate volatility actually increases the MCEV. As surprising as this result may seem, it was actually to be expected given that bond prices in a Vasicek framework positively respond to increases in volatility. Consequently an increase in the volatility parameter increases the asset portfolio value by more than the change in liability value due to a higher short rate volatility. While it should be taken for granted, this serves as a reminder that the MCEV methodology and results are always also a consequence of the modeling assumptions.

Overall, the MCEV sensitivity to the interest rate regime highlights the necessity to guard the asset portfolio from the impact of certain rate scenarios and thereby avoid running into the solvency problem in the first place. Therefore, the next Subsection discusses a few hedging possibilities in the insurance context.

4.3.3 Hedging Instruments

For the purpose of hedging interest rate exposures two widely used instruments in life insurance are described. The two discussed products are floored floaters and swaptions.

Floored Floater. A floored floater protects the investor from low interest rates while allowing full participation in rising rates. As such the investment vehicle reflects the liability structure of a life insurer who is subject to minimum guarantees and profit participation. Furthermore, the product offers positive convexity which makes the market value of the asset portfolio less vulnerable to rising rates and can help immunize firm equity performance to interest rate movements.

More formally, the buyer of the instrument obtains the right to exchange the floating rate against the specified floor for every period until maturity, such that the payoff is given by

$$\text{Cash Flow} = \text{Notional} \cdot \max(\text{Floor} - \text{Rate}, 0). \quad (19)$$

The portfolio to be valued consists of a floating rate bond and call options on zero coupon bonds with face value $\text{Floor} \cdot \text{Notional}$ which can be priced in closed form given the chosen interest rate model.

Swaptions. A swaption allows the investor to enter an interest rate swap at a specified date. The receiver swaption gives the right to buy the fixed rate bond and deliver the floater, and it protects the investor from falling rates. For valuation purposes, this is the right to buy a portfolio of zero coupon bonds that mimics a fixed rate bond. On the contrary, the payer swaption gives the right to deliver the fixed rate and receive the floating rate, protecting the investor in high interest rate scenarios and, thus, being less attractive in the insurance context.

4.4. Model Output under the Solvency II Regime

Moving to the Solvency II regime, capital requirements in the standard and internal models can be computed. Based on the Δ NAV approach, the standard model prescribes an SCR of $E_{\text{SCR,stan}} = 55.70$ for the generic life insurer of this Section, given zero equity investments. This is significantly larger than the Solvency I requirement. The main driver of this result is the effect of the rates level on the value of the asset portfolio.

For the internal model, recall that the SCR is based on the empirical distribution of the MCEV at the one year horizon which is shown in Figure . Just as previously conjectured, the resulting SCR with $E_{\text{SCR,int}} = 22.66$ in accordance with Equation (??) is significantly lower, justifying the use of an internal model.

Figure ?? about here

5. Systemic Risk

Since life insurers in a given country usually have a very similar risk profile – Figure ?? shows the risk decomposition of a typical German life insurer – the outlined analysis is not only interesting from a firm value perspective, but may also help to shed some light on systemic risk within the (life) insurance sector. Market risk can be identified as one of the main risk sources, and it is, therefore, not surprising that Japan’s low interest rate environment led to the failure of seven life insurers around the turn of the millennium due to the insurers not being able to earn the minimum guarantee return (?). In light of the crisis regulators should be alarmed by a tightening band between risk free returns and promised guarantee levels. As such, the German regulator *BaFin* has warned against low interest rate pressures at life insurers in early 2010.¹⁹ While the tightening between guarantee levels and the risk free rate is a sure reason for concern – especially since average guarantee levels for an insurer are typically higher than the current minimum (3.38% for Allianz in 2008, for instance) – insurers are reluctant to decrease the return credited to policyholders due to competitive pressures. Since this may mean that policholder returns exceed the return on the reference asset portfolio, management action can actually add to the solvency risk of the industry. At the same time the near invariance of the credited returns (Figure ?? shows the distributions for the years 2008-2010) leaves

¹⁹ *Handelsblatt*, “Zinswarnung für Lebensversicherungen (Interest Rate Warning for Life Insurers).” January 18, 2010.

scope for lowering returns in case of solvency risk of a particular company before the regulator needs to take more serious measures.

Figure ?? about here

Although market risk has been identified as the most important source of risk, the risk decomposition in Figure ?? shows that also lapse risk matters. Since insurance contracts have typically been an extremely illiquid investment, one may think that the insurer can take a long term perspective to its asset-liability management. In practice, though, an annual lapse rate of only 2% already implies that more than half the contracts are cancelled before the end of the deferment period, despite the, at times, hefty penalties charged. Given the positive impact on profits (compare Figure ??) this is actually welcomed by insurers, and the recent trend of life policy securitization²⁰ may dampen profits, possibly introducing a new systemic risk component to the industry in the short run before pricing can be adjusted to the new lapse environment.²¹

Lastly, also the failure of one of the few reinsurers can increase systemic risk. Furthermore, the industry is subject to longevity risk due to actual mortality rates being overestimated by the standard tables.

With systemic risk being non-diversifiable, the regulator has a clear mandate to ensure a functioning life insurance industry. This is especially important in modern societies where governments emphasize the need to plan more and more for old age privately. Otherwise, solvency risk may cause guarantees to be worthless when they are most needed (i.e. in a sustained economic downturn). In such a case, also the existence of a back up fund²² may only partially cushion the losses to policyholders.

6. Concluding Remarks

The paper investigates the agency conflict between policyholders and firm owners under different solvency regimes. Despite the embedded options of insurance liabilities, which the equity holder is short, the limited liability call can dominate equity value in bad states of the world, such that, in absence of proper regulation, the classical agency problem between debt and equity holders prevails.

After documenting the existence of an incentive for excessive risk taking in absence of regulation, the paper develops a simple model to capture the different Solvency regimes and analyzes the interplay between prevailing regulation and optimal response of the insurance firm. The results suggest that Solvency II eliminates the unfair subsidy of equity holders at the expense of policyholders in bad states of the world. By imposing an implicit restriction on asset performance through the link

²⁰*New York Times*, “Wall Street Pursues Profit in Bundles of Life Insurance.” September 6, 2009.

²¹Securitization here means that funds purchase life policies from *policyholders* who would like to liquidate their contracts. After the transfer, the fund services the policies and receives the death benefit once the original holder dies.

²²*Sicherungsfonds* as required by §124 VAG in Germany

of capital requirements to asset performance, The new regime makes policyholder protection compatible with the shareholder incentive of equity value maximization and thereby positively impacts welfare. As part of the analysis, the paper is the first to consider Solvency II in a setup going back to ??.

In the multiperiod MCEV setup, it is further shown that the value of future operations reduces the owner's incentive for excessive risk taking already for simple Solvency regimes. Here, the paper develops a realistic valuation model recognizing the embedded optionality in life insurance based on the market-consistent embedded value methodology which allows to address the corporate finance optimal firm value question. Given guarantees and sharing rules based on regulation and local GAAP, both approaches use insights from option pricing to obtain the market value of assets, liabilities, and equity capital.

The paper also raises interesting questions for future work. A focus will be the more elaborate analysis of Solvency II in the MCEV setup. Also the effect of hedging and the use of derivatives on equity value in light of the agency conflict could be interesting points of attention.

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Appendix A. Solving the Simple Model for Profit Participation and Guarantees

For the sake of analytical tractability, assume the asset dynamics to be of the form

$$dA_t = \mu A_t dt + \sigma A_t dW_t, \quad (\text{A.1})$$

where W_t is a standard Brownian motion. In that case it is straightforward to show that the liability payoff in Equation (??) admits the following solution

$$L_t^{\text{fair}} = A_t [\delta\alpha\Phi(d_1) + \Phi(-d_3)] + L_T^* e^{-r(T-t)} [\Phi(d_4) - \Phi(d_2)], \quad (\text{A.2})$$

where r is the constant risk free rate, $\Phi(\cdot)$ is the cumulative normal distribution, and

$$\begin{aligned} d_1 &= \frac{\ln \frac{\delta\alpha A_t}{L_T^*} + \left(r + \frac{1}{2}\sigma^2\right) (T-t)}{\sigma\sqrt{T-t}} \\ d_2 &= d_1 - \sigma\sqrt{T-t} \\ d_3 &= \frac{\ln \frac{A_t}{L_T^*} + \left(r + \frac{1}{2}\sigma^2\right) (T-t)}{\sigma\sqrt{T-t}} \\ d_4 &= d_3 - \sigma\sqrt{T-t}. \end{aligned}$$

Analogously, the fair equity value is given by

$$E_t^{\text{fair}} = A_t [\Phi(d_3) - \delta\alpha\Phi(d_1)] + L_T^* e^{-r(T-t)} [\Phi(d_2) - \Phi(d_4)]. \quad (\text{A.3})$$

The impact of volatility on liability and equity values follows from the partial derivatives of (??) and (??), respectively,

$$\frac{\partial L_t}{\partial \sigma} = A_t \left[\delta\alpha\phi(d_1) \frac{\partial d_1}{\partial \sigma} + \phi(-d_3) \frac{\partial(-d_3)}{\partial \sigma} \right] + L_T^* e^{-r(T-t)} \left[\phi(d_4) \frac{\partial d_4}{\partial \sigma} - \phi(d_2) \frac{\partial d_2}{\partial \sigma} \right] \quad (\text{A.4})$$

$$\frac{\partial E_t}{\partial \sigma} = A_t \left[\phi(d_3) \frac{\partial d_3}{\partial \sigma} - \delta\alpha\phi(d_1) \frac{\partial d_1}{\partial \sigma} \right] + L_T^* e^{-r(T-t)} \left[\phi(d_2) \frac{\partial d_2}{\partial \sigma} - \phi(d_4) \frac{\partial d_4}{\partial \sigma} \right], \quad (\text{A.5})$$

where $\phi(\cdot)$ is the standard normal pdf and

$$\begin{aligned} \frac{\partial d_1}{\partial \sigma} &= \sqrt{T-t} \left(1 - \frac{1}{\sigma^2(T-t)} \left(\ln \left(\frac{\delta\alpha A_t}{L_T^*} \right) + \left(r + \frac{1}{2}\sigma^2 \right) (T-t) \right) \right) \\ \frac{\partial d_2}{\partial \sigma} &= \frac{\partial d_1}{\partial \sigma} - \sqrt{T-t} \\ \frac{\partial d_3}{\partial \sigma} &= \sqrt{T-t} \left(1 - \frac{1}{\sigma^2(T-t)} \left(\ln \left(\frac{A_t}{L_T^*} \right) + \left(r + \frac{1}{2}\sigma^2 \right) (T-t) \right) \right) \\ \frac{\partial d_4}{\partial \sigma} &= \frac{\partial d_3}{\partial \sigma} - \sqrt{T-t}. \end{aligned}$$

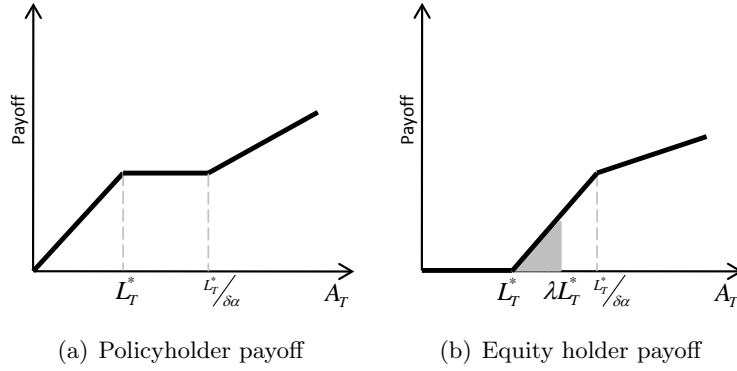


Figure 1: Life insurance company payouts at maturity. Panel (a) shows the payoff to policyholders resulting from the minimum guarantee L_T^* and profit sharing for $A_T > \frac{L_T^*}{\delta\alpha}$. Panel (b) depicts the terminal payoff to equity holders as residual claimants. The constant $\lambda \geq 1$ determines the buffer (gray area) that the regulator requires to be satisfied. δ denotes the portion of profits allocated to policyholders, and α is the share of the time zero asset value to cover the actuarial reserve.

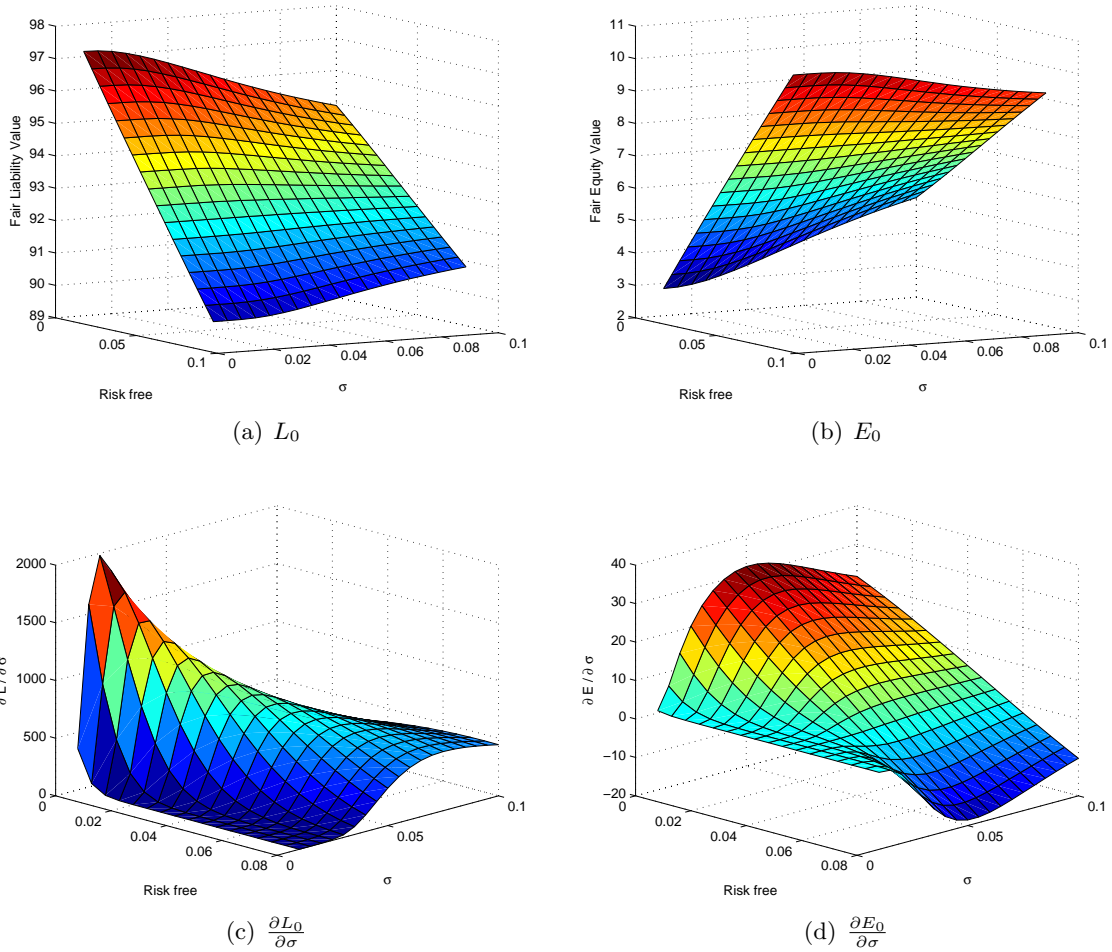
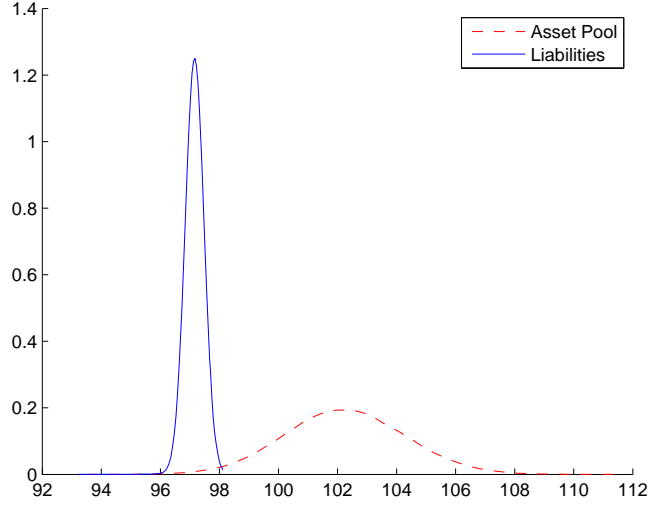
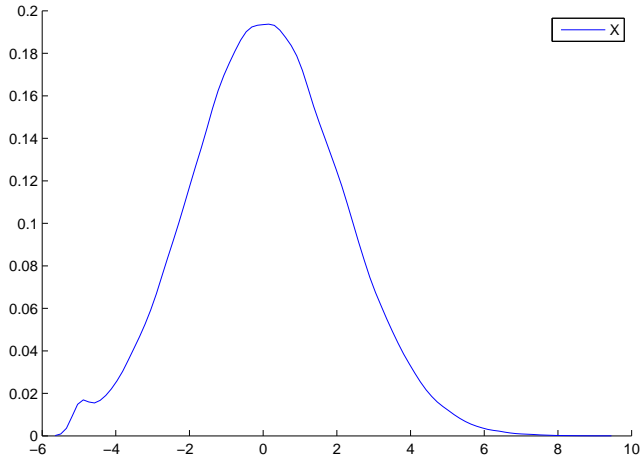


Figure 2: Effect of rates and volatility levels on fair equity and liability values. Panels (a) and (b) show the effects of varying the risk free rate as well as the volatility level on fair liability and equity values based on the closed form solutions of ??, respectively. Panels (c) and (d) report the corresponding sensitivities to a volatility change (option Vega) in a given rates and volatility scenario. The initial balance sheet size A_0 equals 100, and $\alpha = 95\%$; $r^* = 2.25\%$, $\delta = 90\%$.

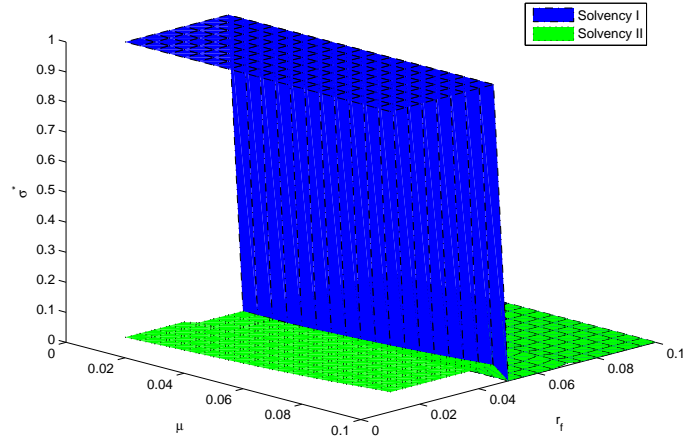


(a) Terminal Distributions of A_T and L_T

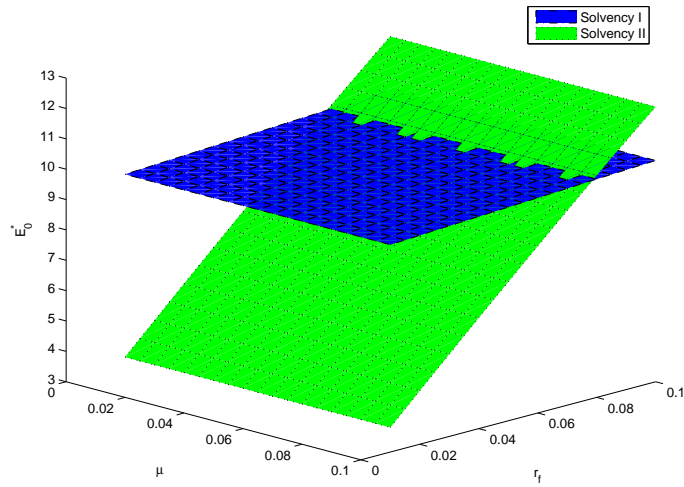


(b) Empirical Distribution of X

Figure 3: Terminal distributions determining the probability of default. Subplot (a) shows the terminal distributions of the asset pool and liability payoff at the one year horizon based on assets following a geometric Brownian motion with drift $\mu = 2.25\%$ and $\sigma = 2\%$; the liability payoff is given by $L_T(A_T) = [\delta\alpha A_T - L_T^*]^+ + L_T^* - [L_T^* - A_T]^+$. Subplot (b) gives the empirical distribution of $X = A_0\tilde{r} - L_0g(\tilde{r})$ determining the economic ruin probability $\text{Prob}(X < -E)$. The initial balance sheet size A_0 equals 100, and $\alpha = 95\%$; $r^* = 2.25\%$, $\delta = 90\%$.

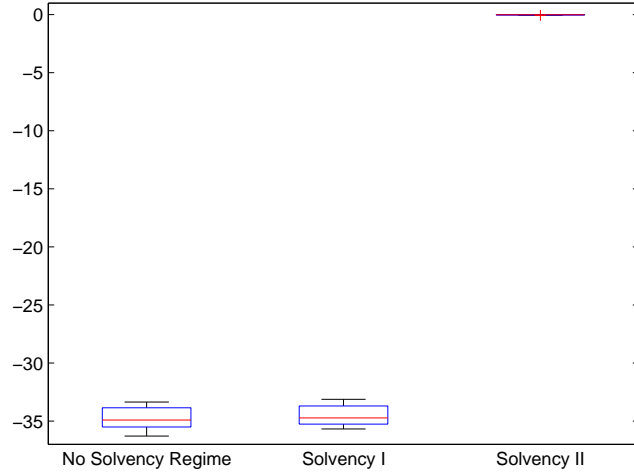


(a) σ^*

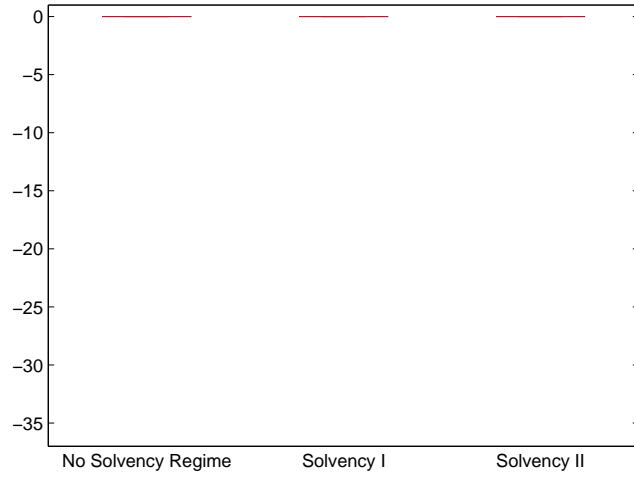


(b) E_0^{fair}

Figure 4: Optimal σ and E_0^{fair} surfaces. Given a solvency regime and its implied constraint for required equity capital, the Subplot (a) shows the equity value maximizing choice of σ , as proxy for the desired riskiness of the asset portfolio, as function of the economy's risk free rate r_f and the drift μ of the asset portfolio. Subplot (b) depicts the corresponding fair equity value. The initial balance sheet size A_0 equals 100, and $\alpha = 95\%$; $r^* = 2.25\%$, $\delta = 90\%$.



(a) State Loss $r_f = 2.5\%$



(b) State Loss for $r_f = 5.0\%$

Figure 5: State loss for selected interest rate scenarios and different solvency regimes. The boxplots show the shortfall associated with the insurance company not earning the minimum guarantee for the Solvency I & II cases as well as the case of no capital requirements. The initial balance sheet size A_0 equals 100, and $\alpha = 95\%$; $r^* = 2.25\%$, $\delta = 90\%$.

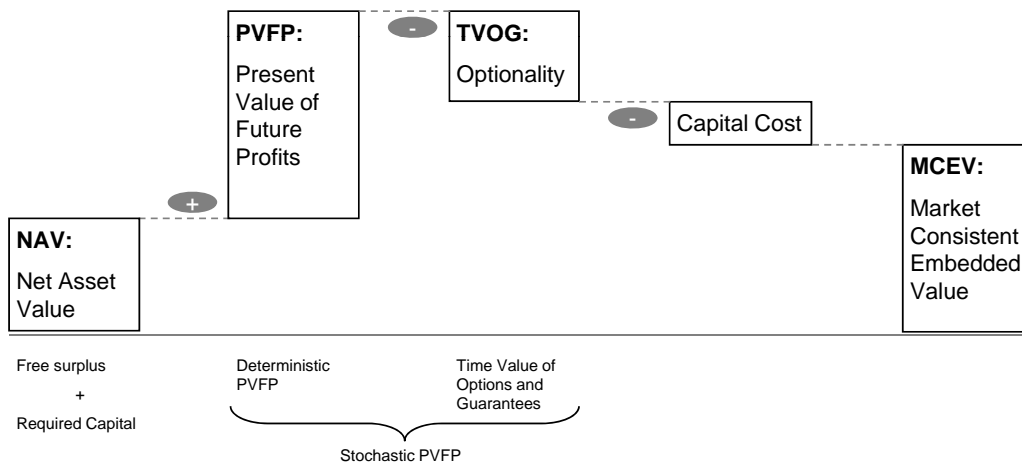


Figure 6: Definition of the Market-Consistent Embedded Value according to the ?. The Figure shows the decomposition of the MCEV into capital cost, time value of options and guarantees, deterministic present value of future profits of the business in force, and assets over and above those matching the insurance liabilities.

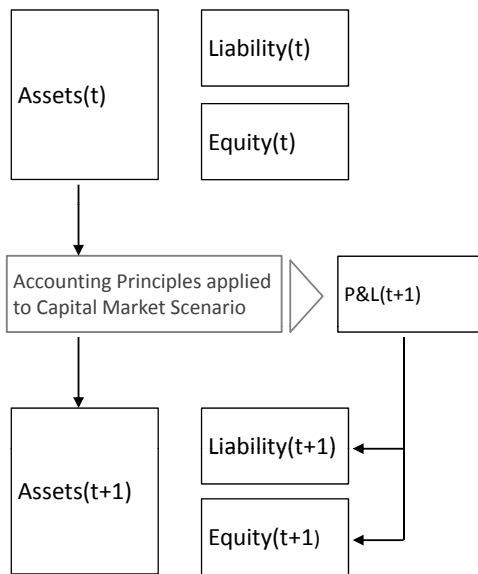


Figure 7: Mapping of capital market scenario into balance sheet. Figure shows time t and $t + 1$ balance sheets of a generic life insurer. Time t assets are invested in the capital market which generates returns that the accounting principle maps into the profit and loss account (P&L) and into the $t + 1$ balance sheet. The accounting value of the liabilities at maturity determines the cash flows to policyholders.

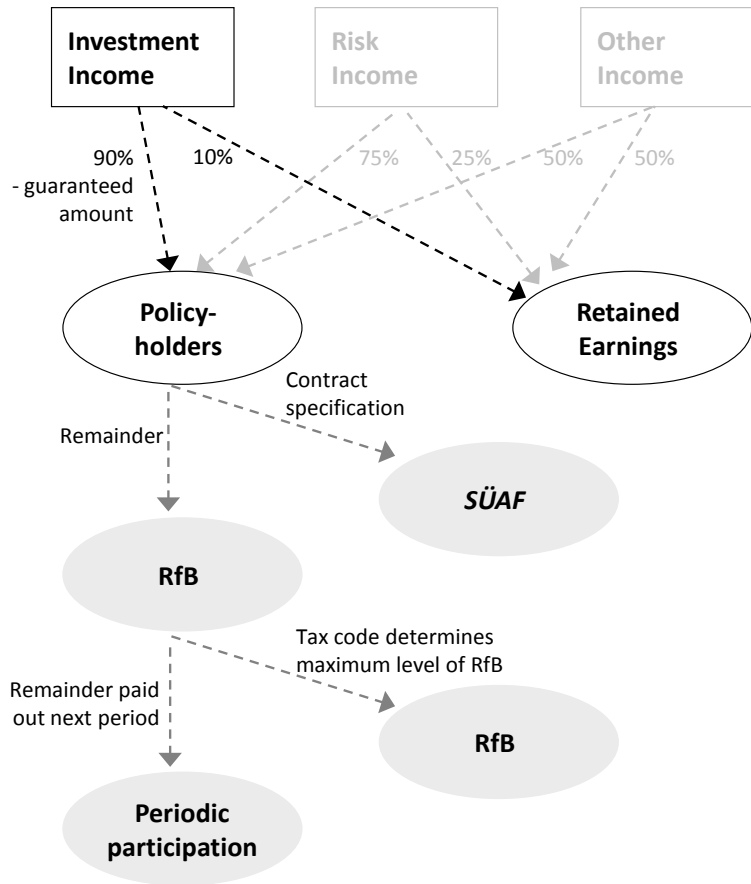


Figure 8: Life insurance income sharing between policyholders and firm owners in Germany. The Figure shows how income is attributed to equity- and policyholders. During the deferment period, the latter are credited a contractually specified amount that is accumulated in the *Schlussüberschussanteilsfond (SÜAF)*. The remainder is maintained in the provision for premium refund (*Rückstellung für Beitragsrückerstattung (RfB)*).

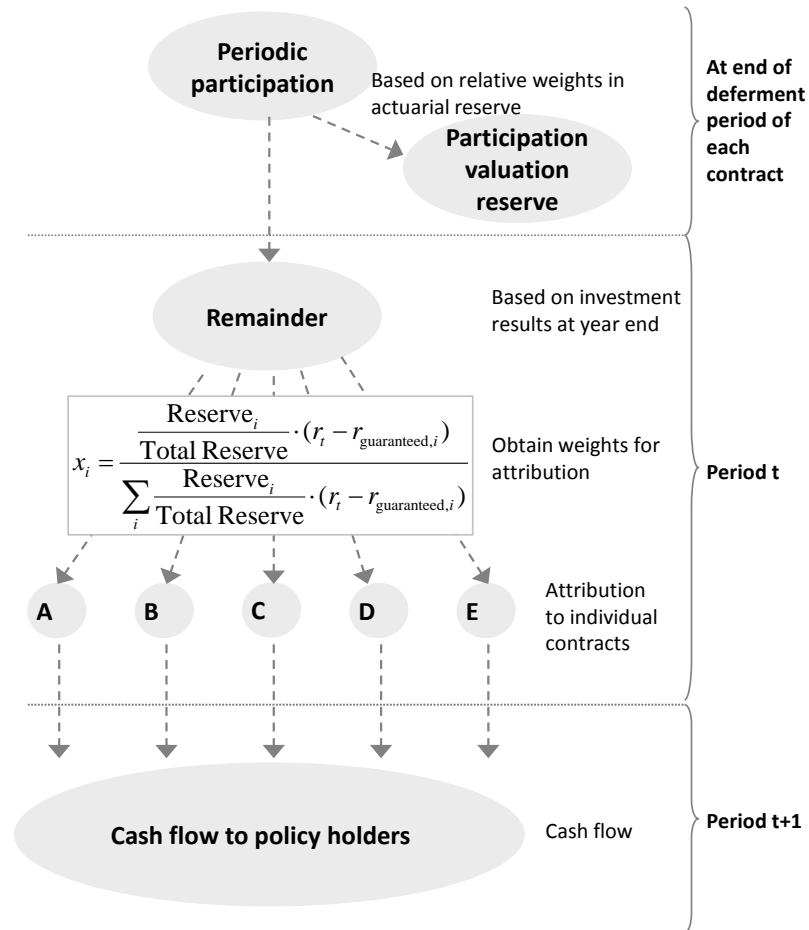


Figure 9: Surplus attribution and valuation reserve. The Figure shows the allocation of available surplus to policyholders at the end of their deferment period and all other policyholders based on a weighting that takes, both, the level of the individual’s actuarial reserve as well as the minimum guaranteed return into account.

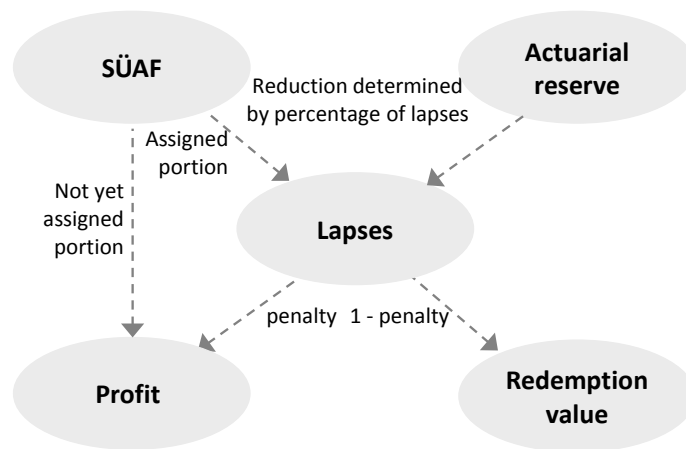


Figure 10: Effect of lapses on policyholder cash flows and profits. Lapses reduce the account for contractually specified profit participation during the deferment period (*Schlussüberschussanteilsfond (SÜAF)*). Only the already assigned portion constitutes part of the redemption value. The remainder credited to the insurer. Also the actuarial reserve corresponding to the canceled contract only goes back to the policyholder after the insurer has subtracted a cancellation fee.

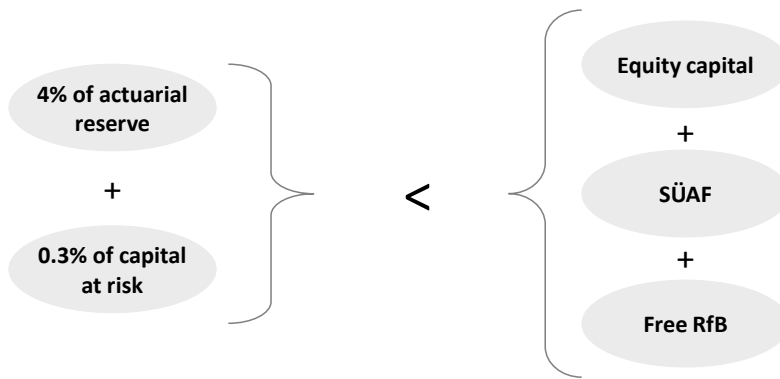


Figure 11: Solvency check under Solvency I regime. The left side of the Figure illustrates the decomposition of the required capital into portions of the actuarial reserve and capital at risk which is defined as the nominal sum of the policyholder payments minus actuarial reserve. On the right, the available capital is given as the sum of equity capital according to local GAAP, the profit participation account during deferment ($SÜAF$), and the non-assigned part of the provision for premium refund (free RfB).

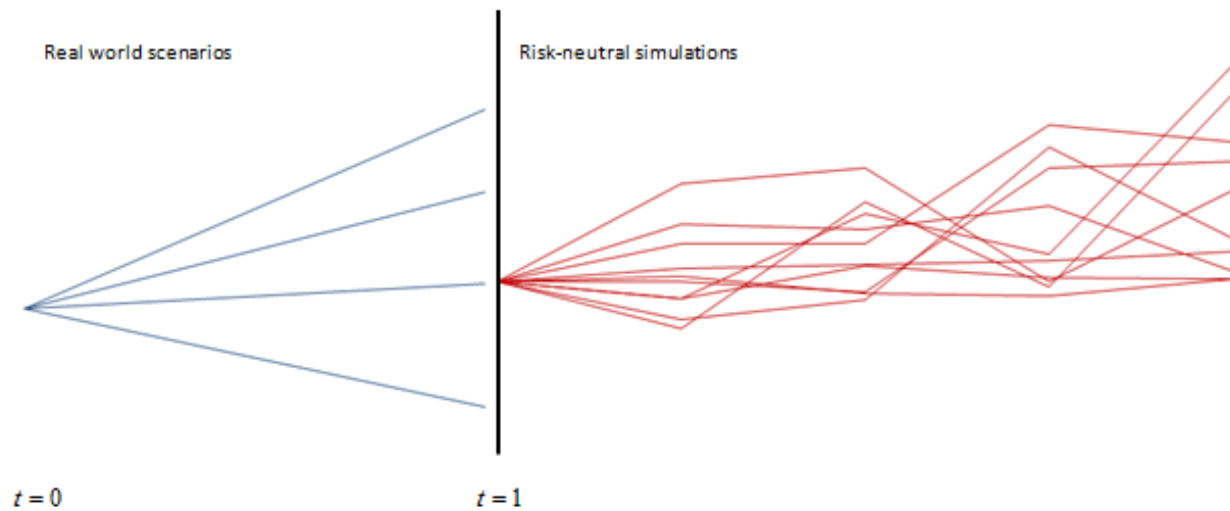
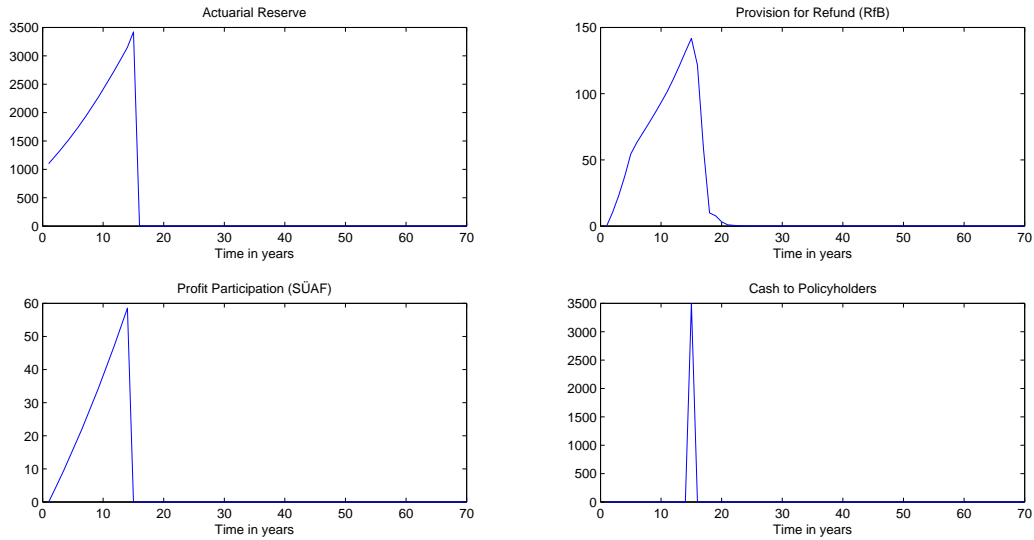
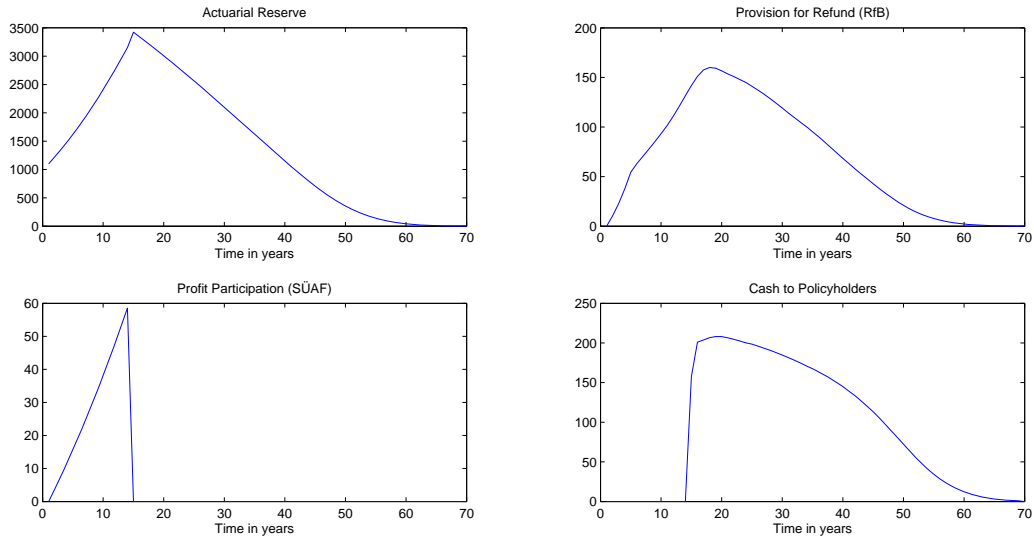


Figure 12: Determination of the Value-at-Risk at the one-year horizon for Solvency II capital requirements using an internal model. The Figure illustrates the mechanics of obtaining a distribution of the MCEV at the one year horizon which is needed to determine the capital requirement. Based on N real world scenarios, the MCEV is computed at each node giving the empirical distribution of the VaR.

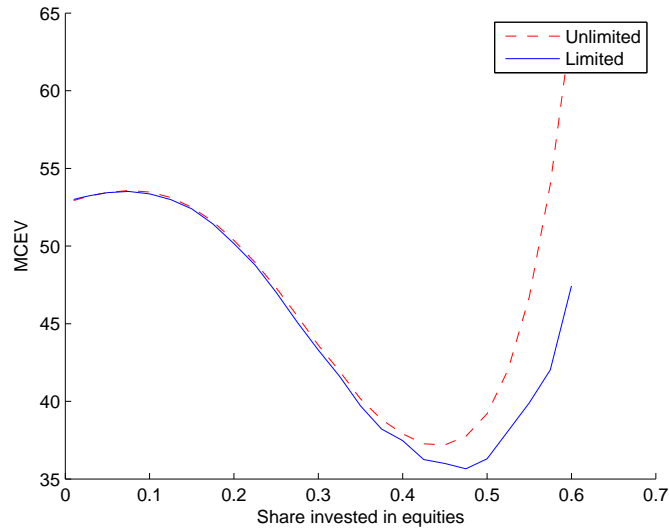


(a) Lump Sum Payout

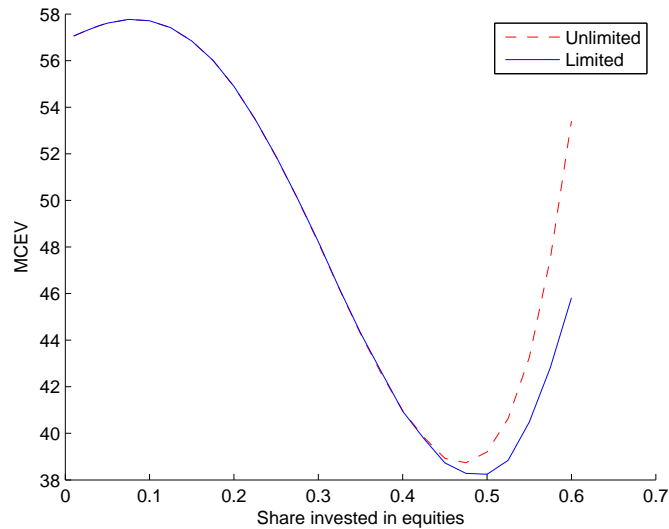


(b) Annuity Payout

Figure 13: Selected balance sheet items and policyholder cash flows for the two representative payout profiles of lump sum payment (Subplot (a)) and annuity payout (Subplot (b)). For both contract types the policy is entered into at the age of 40 with periodical payment of 100 during each period until retirement at 65. The minimum guarantee rate is set equal to 2.25%. The insurer is assumed to invest all available cash into the period's newly issued 10 year government bond and, when in need of cash, divest the bonds closest to (and still above) par in order to minimize the balance sheet impact. The number of Monte Carlo loops equals 1,000.



(a) Low Initial Solvency Ratio



(b) High Initial Solvency Ratio

Figure 14: MCEV as function of risky equity investments. The Figure shows the MCEV for different amounts invested into equity. Subplot (a) is based on an initial solvency ratio of 1.6. Subplot (b) uses an initial solvency ratio of 2.5. The blue, solid line shows the insurer's value when the firm is liquidated upon breaching the solvency requirement (see Figure ??). The MCEV development when the owner contributes additional capital in case of deteriorating solvency is given by the red, dashed line. All scenarios are based on an initial balance sheet size of 100.

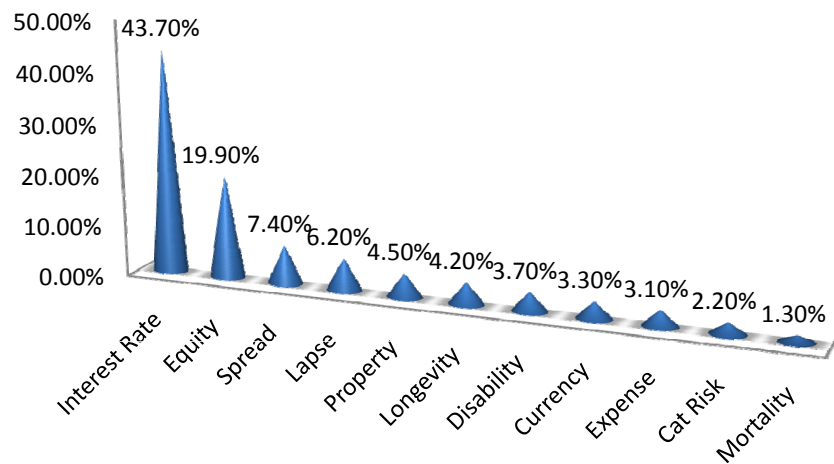
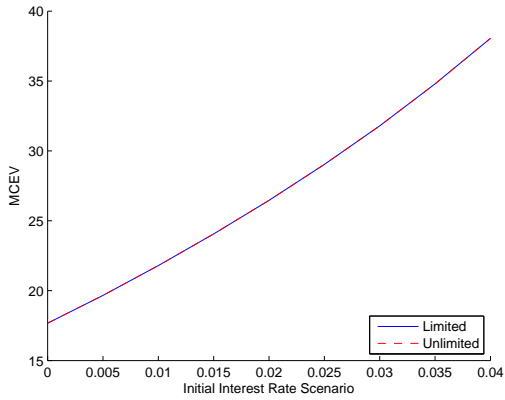
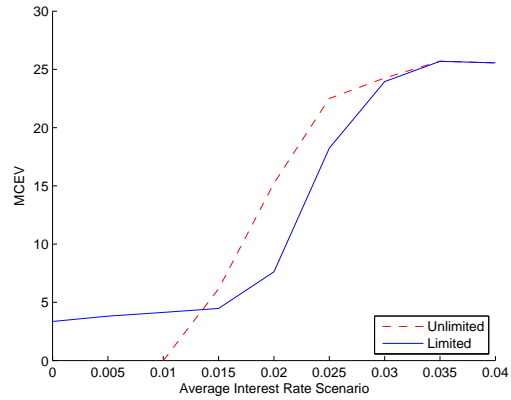


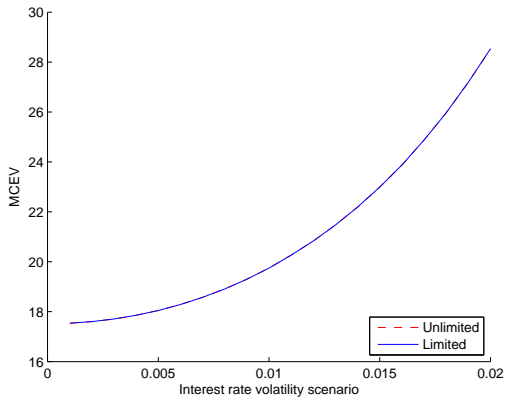
Figure 15: Risk sources of a generic German life insurer based on ? and ? publications



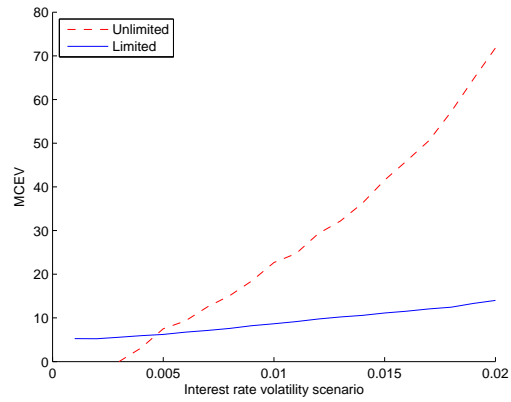
(a) Initial Interest Rate



(b) Mean Interest Rate



(c) Rates Volatility & High Interest Rate Regime



(d) Rates Volatility & Low Interest Rate Regime

Figure 16: MCEV as function of the interest rate environment. Subplot (a) shows the impact of an initially low interest r_t . Subplot (b) relates the MCEV to the average interest rate level, \bar{r} . Subplot (c) highlights the impact of interest rate volatility on the MCEV when the average interest rate of the economy is high ($\bar{r} = 6.8\%$). Subplot (d) shows the same sensitivity with respect to volatility for a low interest rate regime ($\bar{r} = 2\%$). The blue, solid line shows the insurer's value when the firm is liquidated upon breaching the solvency requirement (see Figure ??). The MCEV development when the owner contributes additional capital in case of deteriorating solvency is given by the red, dashed line. All scenarios are based on an initial solvency ratio of 1.6 and a balance sheet size of 100, as well as an equity investment share of 5%.

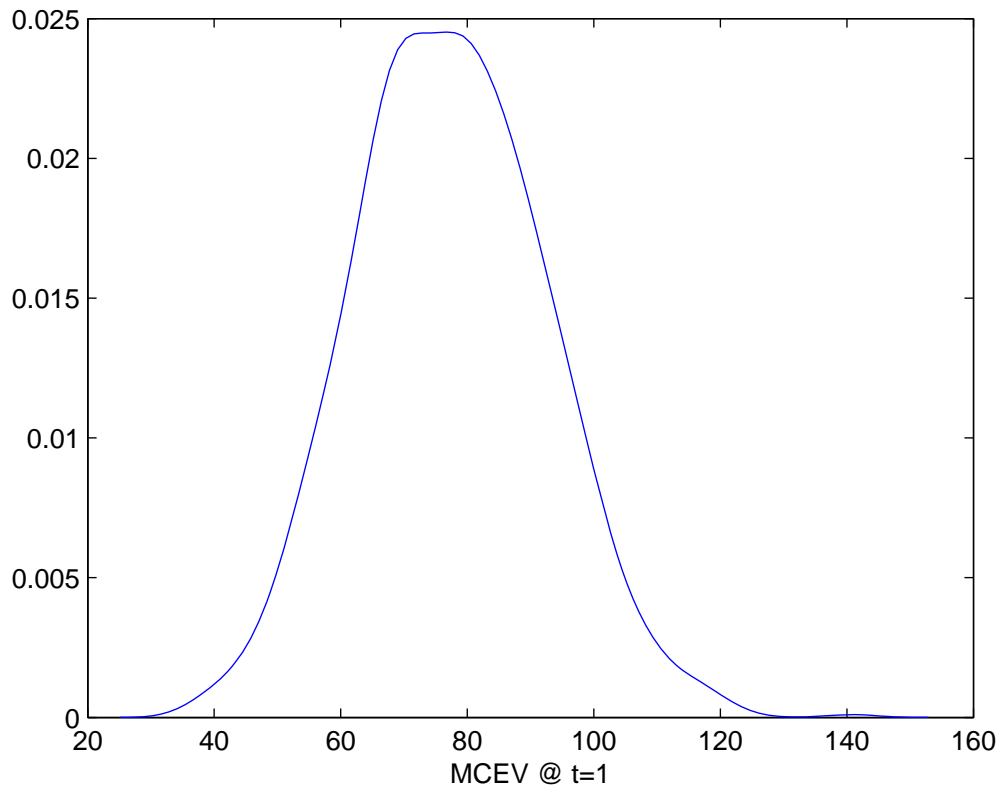


Figure 17: Empirical MCEV distribution at $t = 1$. The Figure shows the distribution of the MCEV at time $t = 1$, which is needed to determine the VaR corresponding to the prespecified confidence level $1 - \beta_{\max}$ in the Solvency II framework for internal models. The graph is based on $N = 1,000$ real world scenarios and real world short rate parameters $a = 0.08$, $\bar{r} = 0.0434$, and $\sigma = 0.001$.

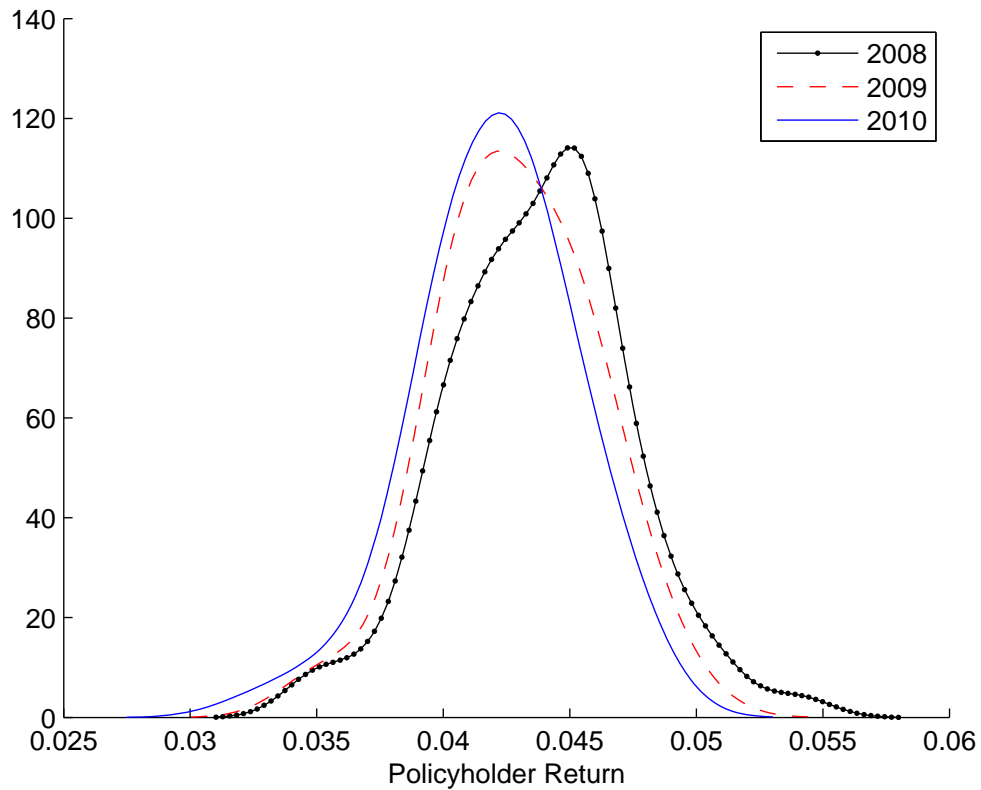


Figure 18: Distributions of returns credited to policyholders in Germany in the years 2008 till 2010. The Figure shows density plots of the policyholders' actual returns (*Überschussbeteiligung*) for the years 2008, 2009, and 2010 based on the 77 biggest life insurers operating in Germany. The mean returns for the three years were 4.20%, 4.29%, and 4.38%, respectively with corresponding standard deviations of 0.3%, 0.3%, and 0.35%. (Source: ?)

Table 1: Balance sheet of a generic life insurer

Assets		Liabilities & Equity	
Assets	A_0	Liabilities	$L_0 = \alpha A_0$
		Equity	$E_0 = (1 - \alpha)A_0$
Total	A_0	Total	A_0

Notes: Time 0 balance sheet of a generic life insurer where assets are normalized to 100. $(1 - \alpha)$ denotes the portion of assets financed by equity and αA_0 represents the actuarial reserve for policyholder claims as of $t = 0$.

Table 2: Solvency Regimes and Regulatory Capital in Analytical Model

μ	σ	Solvency I		Solvency II		Economic VaR		Corr(A_T, L_T)
		E_{SCR}	λ	$E_{\text{SCR,stan}}$	λ	$E_{\text{SCR.int}}$	λ	
1.00%	2.00%	3.80	131.58%	5.93	84.35%	5.00	100.00%	35.07%
2.25%	2.00%	3.80	131.58%	4.61	108.39%	4.58	109.21%	16.65%
5.00%	2.00%	3.80	131.58%	1.94	258.14%	1.86	268.80%	2.71%
2.25%	0.00%	3.80	131.58%	0.00	NaN	0.00	NaN	-100.00%
2.25%	1.00%	3.80	131.58%	2.27	219.84%	2.26	221.04%	0.00%
2.25%	2.00%	3.80	131.58%	4.61	108.39%	4.58	109.21%	16.65%
2.25%	3.00%	3.80	131.58%	6.70	74.59%	5.00	100.00%	37.94%

Notes: The table shows the required capital (E_{SCR}) and solvency ratios λ implied by the Solvency I and II regimes, as well as the ones implied by the true economic value at risk. Required capital under Solvency I is computed as 4% of the liabilities at $t = 0$. The required capital under Solvency II is computed as the capital that ensures $\text{Prob}(X < -E_{\text{SCR,stan}}) = 1\%$, where $X \sim N(\mu_X, \sigma_X)$. The economic value-at-risk (VaR) relaxes the assumption of X following a normal distribution and gives $E_{\text{SCR.int}}$ based on the empirical distribution of X . The initial balance sheet size A_0 equals 100, and $\alpha = 95\%$; $r^* = 2.25\%$, $\delta = 90\%$. The asset pool is assumed to evolve according to a geometric Brownian motion with instantaneous drift μ and diffusion coefficient σ .

Table 3: Correlation matrix for combining capital charges of the market risk sub-modules using the standard formula

	Rates	Equity	Property	Spread	Currency	Concentration
Rates	1					
Equity	0.5	1				
Property	0.5	0.75	1			
Spread	0.5	0.75	0.5	1		
Currency	0.5	0.5	0.5	0.5	1	
Concentration	0.5	0.5	0.5	0.5	0.5	1

Notes: The table shows the correlation coefficients $\rho_{i,j}^{\text{Mkt}}$ for computing the capital charges of each market risk sub-module to arrive at the market risk module's capital charge in the standard model setup as proposed by CEIOPS' L2 Advice, Article 111(d), available at <http://www.ceiops.eu/content/view/706/329>.

Table 4: Correlation matrix for combining capital charges of the various risk modules using the standard formula

	Market	Default	Life	Health	Non-Life
Market	1				
Default	0.25	1			
Life	0.25	0.25	1		
Health	0.25	0.25	0.25	1	
Non-Life	0.25	0.5	0	0	1

Notes: The table shows the correlation coefficients $\rho_{i,j}^{\text{SCR}}$ for computing the solvency requirement in the standard model setup as proposed by CEIOPS' L2 Advice, Article 111(d), available at <http://www.ceiops.eu/content/view/706/329>.

Table A1: Vasicek Parameter Estimates

a	\bar{r}	σ
0.1495	0.0686	0.0081

Notes: Calibrated parameters of the Vasicek model. The estimates are based on monthly bond prices implied by the term structure published by the *Bundesbank* for one through 15 years maturity between 1986 and 2010.