On the optimal introduction of a funded pension pillar

Abstract

Earlier studies argue that one cannot introduce funding into pension systems without inflicting loss on at least one cohort. We show that there exists a politically feasible instrument for a (nearly) Pareto-optimal reform from a pay-as-you-go to a funded pension system. Our instrument consists of redistribution via small implicit debt. We also compare computational algorithms with various degree of sophistication. Our instrument proves robust to a number of parametric and modeling choices.

JEL Codes: H55, E17, C60

Keywords: defined contribution, pay-as-you-go, funded, pension system reform

I INTRODUCTION AND MOTIVATION

Ever since the discussion between Samuelson (1958) and Aaron (1966) it has been a concern whether pension system reforms – especially the privatization of the social security – can be implemented without inflicting welfare loss on one or few cohorts. With the onset of the overlapping generations general equilibrium models following Auerbach and Kotlikoff (1987) the debate on Pareto-optimal pension reforms has gained momentum (see Kotlikoff et al. 1999, Diamond 2004, Joines 2005, Krueger and Kubler 2006, McGrattan and Prescott 2013, Roberts 2013, among others). Indeed, full cohort-by-cohort comparisons of welfare gains/losses is also key from the political economy perspective (see Holzman and Stiglitz 2001). Moreover, nearly all advanced economies face the need to reform their pension systems for demographic and fiscal reasons, whereas a majority will have to adapt the financing from the pay-as-you-go rule to alternative arrangements (e.g. Gruber and Wise 2007, Wise 2016, Coile et al. 2016)

While there is abundant literature arguing that privatization may in general be efficient, implementation without inflicting loss on any cohort remains an area of further research. The lack of clear policy prescriptions stems from the fact that the very understanding of the term "privatization" as well as its subsquent model implementation differ substantially betweeen studies. Some of the previous papers confound the financing of the pension system (pay-as-you-go, PAYG or funded) with the way pensions are computed (defined benefits or defined contributions), although in principle these two attributes of the pension systems are orthogonal. For example Breyer (1989), Hirte and Weber (1997), Gyárfás and Marquardt (2001) only change the method of financing, keeping other features of the balanced pay-as-you-go pension system at *status quo* whereas Browning (1975), Kotlikoff (1996) and McGrattan and Prescott (2013) refer to privatization as eliminating any obligatory pension system at all.

Clearly, if a pay-as-you-go system is to be replaced by a funded one, either the transition generations cover two pension benefits (theirs and their ancestors) or the government temporarily raises debt. For the welfare comparisons to make sense, the final and the initial fiscal stance need to be identical – a property that Kotlikoff (1996) names a zero-sum generational accounting. Hence, financing a reform with debt puts part of the transition costs on the future generations. One can easily identify whether or not a larger distortion over a shorter horizon is preferable to a smaller one spread over a longer horizon, but that does not help to find a method for phasing in the funding and phasing out the pay-as-you-go financing so that the welfare of no generation was deteriorated. In fact, Breyer (1989) argues that a transition from a PAYG system to a funded system that would incur no welfare loss on at least one cohort is impossible. This result holds both for the closed economy, where the interest rate arises endogenously, as well as for the small open economy with exogenously given interest rate.[1]

The loosing cohorts may be compensated *via* a lump-sum transfer, which is sometimes dubbed as a Lump Sum Redistribution Authority. Indeed, if overall the reform is welfare enhancing, there exists a solution to the inter-generational government optimization problem that allows fully compensating the cohorts that would lose otherwise. For example, Kotlikoff (1996), Hirte and Weber (1997) search for "politically feasible" transition paths by remunerating the losing cohorts with cohort-specific lump-sum transfers. They find that mixes of debt and income taxation and debt and consumption taxation used to finance the compensation scheme allow for a Pareto improvement. However, this approach relies on a variant of the lump-sum redistributive authority mechanism between multiple cohorts and with limited enforcement.

[1] The effects is smaller with incomplete insurance, as argued by Nishiyama and Smetters (2007). In a similar spirit Krueger and Kubler (2006) analyze the introduction of a PAYG social security instead of a private savings.

The literature has also analyzed compensation schemes alternative to lump sum redistribution. Belan and Pestieau (1999) exploits positive externalities to capital accumulation a la Lucas (1988), showing that full abolition of the pension system jointly with subsidizing private savings is strictly Pareto improving. Gyárfás and Marquardt (2001) extend this argument to an OLG setting. However, these are reforms resulting in no pension system at all, rather a voluntary – and subsidized – savings plan. In fact, authors argue that subsidy scheme can be phased out with no harm to welfare relative to a PAYG system. Some models demonstrate that abandoning PAYG may be universally welfare enhancing (e.g. Kotlikoff et al. 1999, McGrattan and Prescott 2013) because voluntary private savings is preferred to an actuarially equivalent obligatory scheme (optimal individual life-cycle savings path departs from a flat contribution rate).

However, private voluntary savings – subsidized or not – may be suboptimal not only for pragmatic reasons. There are two features which break the equivalence between voluntary and obligatory old-age savings. First, if mortality is not strictly zero during the life-cycle, individuals discount with both the time preference and probability of survival, which breaks the model equivalence between borrowing and saving in the life-cycle: it may still be rational to save for old-age consumption and borrow for current consumption. Second, in many countries instruments designed for old-age savings benefit from preferential tax treatment, caps, alternative investment schemes, etc., which breaks the equivalence between the net interest earned *within* the pension system as opposed to *outside* the pension system. For the same reasons, the conclusions from the models abolishing the pension system cannot be easily extended to apply to reforms, which consist of replacing a PAYG obligatory pension system with a funded obligatory pension system. To this end, Roberts (2013) develops a framework with transition from PAYG to funded financing, where higher accumulation due to pension system translates to temporarily lower interest rate, thus promoting the adoption of a new technology. Increased total factor productivity generates – for some parameter values – a series of supply side general equilibrium effects that make the transition cohorts better off, relative to PAYG scenario.

Summarizing the literature, it seems that from a policy perspective there remains a gap. Earlier findings suggest that it is conceptually infeasible to phase in funding and phase out PAYG financing in a way that would leave no cohort worse off. It is theoretically feasible to compensate the loosing cohorts and it is theoretically feasible to abolish pension systems, but such Pareto-improving paths are of low policy relevance for a number of practical reasons.

Given how many countries are facing the need to reform their PAYG systems, this paper attempts to fill the literature gap by providing two main contributions. First, we propose a politically feasible instrument for (nearly) Pareto-optimal transitions from a PAYG pension system to a funded system regardless of parametrization. In an OLG setup with decreasing mortality and exogenous technological progress we introduce a funded pillar to a defined contribution system. Our policy instrument consists of higher than balanced indexation schedule to the loosing cohorts, which permits financing the reform via implicit rather than explicit debt. Moreover, unlike for example Lump Sum Redistribution Authority – the instrument is politically feasible. Hence, the policy instrument becomes a decision variable for the problem of seeking an optimal path. Admittedly, the instrument/algorithm we provide minimizes losses to negligible levels, but does not eliminate them fully (hence: near Pareto-optimality). Consequently, our second contribution is to compare computational algorithms of various sophistication. We show that relatively unsophisticated algorithms provide results nearly as good a sophisticated ones, but overperform in terms of simplicity, thus possibly providing more feasibility in terms of policy.

Our findings suggest that the challenge of optimal implementation does require a contribution from computational economics: the preferable transition paths could not be derived theoretically and differ by the extent of regularity. We name the established equilibria nearly optimal for technical reasons. In fact, the welfare loss they imply is negligible and could be a computational artefact. Moreover the optimal paths are remarkably robust to alternative calibrations and simulation assumptions, suggesting that the algorithm/instrument may indeed provide relatively stable prescriptions for the long-term policy objectives. Moreover, our setup in obtaining path for the proposed instrument/algorithm is constructed in such a way as to constitute a maximum hurdle: our economy features an endogenous interest rate and decelerating technological progress. The funded pillar needs to be introduced within one generation, public debt cannot exceed certain thresholds even temporarily and the contribution rates are kept constant. The only benefit of the obligatory pension savings over private ones consists of capital income tax redemption, which replicates the feature of many pension systems across the globe (e.g. Gruber and Wise 2007).

The paper is structured as follows. In section II we present the model and the reform scenario. In fact, there are a few countries which underwent a similar reform, for example Chile, the Baltic States, Sweden as well as Poland, whose data we use. Subsequently, section II.4 we describe the computational strategies. The details of the calibration are described in section III. We use the data for a country which actually underwent such a reform in 1999 – Poland. The results as well as analysis of their stability are discussed in section IV, while in the concluding paragraphs we derive the policy implications of this study.

II THE MODEL

There is a constant returns to scale Cobb-Douglas aggregate production function and constant labor-augmenting technical progress. Equilibrium factor prices are time-varying but deterministic.

Agents live for up to 80 periods, i.e. $j = 1, \dots, 80$, with $j = 1$ representing the actual age of 20, and are subject to mortality risk with survival probabilities known *ex ante*. Agents have two types of choices: inter-temporal between consumption $c_{j,t}$ and savings $s_{j,t}$, and intra-temporal between consumption and leisure $1 - l_{j,t}$, where $l_{j,t}$ denotes fraction of time supplied to the labor market by agent aged j in year t.

II.1 Production

Using capital and labor the producers provide a composite consumption good with the Cobb-Douglas production function $Y_t = K_t^{\alpha}(z_t L_t)^{1-\alpha}$. Firms solve the following problem:

$$\max_{(Y_t, K_t, L_t)} \quad Y_t - w_t L_t - (r_t + d) K_t \qquad (1)$$
$$\text{subject to} \quad Y_t = K_t^{\alpha} (z_t L_t)^{1-\alpha},$$

where L_t and K_t represent aggregate labor supply and capital stock. We allow for labor augmenting exogenous economic growth $\gamma_t = z_t / z_{t-1}$.[2] Note that if the return rate on capital is r_t then the rental rate must be $r_t + d$, where d denotes capital depreciation. Firm optimization naturally implies $w_t = (1-\alpha) K_t^{\alpha} z_t^{1-\alpha} L_t^{-\alpha}$ and $r_t + d = \alpha K_t^{\alpha-1} (z_t L_t)^{1-\alpha}$

[2] Bouzahzah et al. (2002) and Romaniuk (2009) discuss the sensitivity of OLG models to the assumptions concerning growth in the light of policy reforms and show that when analyzing pension systems, there is little or no effect of endogenizing productivity growth. Thus, for clarity, we model this economy as governed by exogenous productivity growth.

II.2 Consumers' choice

Consumers optimize lifetime utility derived from leisure and consumption:

$$U_{j,t} \quad = \quad u(c_{j,t}, 1-l_{j,t}) + \sum_{s=1}^{J-j} \beta^s \frac{\pi_{j+s,t+s}}{\pi_{j,t}} u\left(c_{j+s,t+s}, 1-l_{j+s,t+s}\right). \tag{2}$$

$$\text{with } u\left(c_{j,t}, 1-l_{j,t}\right) = c_{j,t}\left(1-l_{j,t}\right)^{\phi}. \tag{3}$$

The notation of π in equation (3) conveys the non-zero probability of dying between the age of j and $j+1$. Discounting takes into account time preference β and probability of survival π. At all points in time, consumers who survive until the age of $j = 79$ die with certitude when reaching the age of $J = 80$. Changes in fertility and longevity rates are operationalized in our model by decreasing across time the size of the 20-year old cohort as well as decreasing the mortality rates, respectively.

Consumers are free to chose their labor supply during the working period, but once they reach the age of $\bar{J}$ they retire. In exchange for supplying age-specific amount of labor $l_{j,t}$ consumers receive earned income of w_t, which is the market clearing marginal productivity of labor. The labor income tax τ^l and social security contributions τ^s are deducted from earned income. Interest earned on savings r_t is taxed with τ^k. In addition, there is a consumption tax, τ^c. Unintentional bequests are redistributed equally within cohort as $beq_{j,t}$. Thus, for working age population ($j < \bar{J}$) the budget constraint at time t is given by:

$$(1+\tau_t^c)c_{j,t} + s_{j,t} = (1-\tau_t^s)(1-\tau_t^l)w_t l_{j,t} + (1+(1-\tau_t^k)r_t)s_{j-1,t-1} + beq_{j,t} \tag{4}$$

whereas for the retired population ($j \geq \bar{J}$) it takes the form of:

$$(1+\tau_t^c)c_{j,t} + s_{j,t} = (1-\tau_t^l)b_{j,t}^{\iota} + (1+(1-\tau_t^k)r_t)s_{j-1,t-1} + beq_{j,t}, \tag{5}$$

where $b_{j,t}^{\iota}$ denotes pension benefit from system ι for person of age j in time t. Appendix A provides the analytical solution to the consumer problem. The market clearing conditions and general equilibrium definition can be found in Appendix B.

II.3 Pension system and the government

The government taxes consumption and income from capital and labor, issues and services debt, purchases goods (fixed exogenous share of GDP), and pays (a part of) retirement benefits. Pension systems are indexed by ι, which corresponds to either defined contribution PAYG (frequently referred to as Notionally Defined Contribution - NDC) or funded defined contribution system, i.e. $\iota \in \{NDC, FDC\}$. The economy starts from the defined contribution pay-as-you-go system (the initial steady state). The baseline scenario consists of a transition to a defined contribution pay-as-you-go system, whereas the reform scenario consists of a defined contribution system which is partially funded.

Defined contribution PAYG (NDC) The DC pension system collects contributions and uses them to finance the contemporaneous benefits, but records them as individual stock of pension savings and at retirement converts this stock to an annuity. For simplicity we denote by τ^{NDC} the obligatory contribution that goes into the PAYG system, whereas b^{NDC} denote benefits from this component of the pension system. The benefit at age $\bar{J}$ is given by:

$$b^{NDC}_{\bar{J},t} = \frac{\sum_{s=1}^{\bar{J}-1}\left[\Pi_{i=1}^{s}(1+r^{NDC}_{t-\bar{J}+i-1})\right]\tau^{NDC}_{t-\bar{J}+s-1}w_{t-\bar{J}+s-1}l_{s,t-\bar{J}+s-1}}{\prod_{s=\bar{J}}^{J}\pi_{s,t}} \tag{6}$$

where r^{NDC} denotes the indexation rate in the obligatory notional defined contribution pillar, which is defined by the payroll growth rate.

Defined contribution funded (FDC) In the funded pillar the collected contributions form a stock of capital. Private accrued savings are converted to an annuity at retirement. The benefit at age $\bar{J}$ is given by:

$$b^{FDC}_{\bar{J},t} = \frac{\sum_{s=1}^{\bar{J}-1}\left[\Pi_{i=1}^{s}(1+r^{FDC}_{t-\bar{J}+i-1})\right]\tau^{FDC}_{t-\bar{J}+s-1}w_{t-\bar{J}+s-1}l_{s,t-\bar{J}+s-1}}{\prod_{s=\bar{J}}^{J}\pi_{s,t}}.$$

There is no capital income tax on obligatory savings, whereas the voluntary savings are subject to τ^k, which replicates the feature in many pension systems, including the US, Switzerland, UK, France and with no exceptions all the countries which in the 1990s wave reformed their pension systems following the guidelines by The World Bank (see Holzman and Stiglitz 2001, Gruber and Wise 2008).

Pension system gap The DC systems are by construction balanced, i.e. the benefits paid out to an agent equal the contributions she has made to the system, whether the benefits are funded or financed contemporaneously. However, depending on the size and life expectancy of all cohorts living at time t, is is possible that the system has revenues in excess/short of current needs. We thus specify, that this is the government that collects social security contributions and pays out pensions.

$$pensiongap_t = \tau_t^{NDC} w_t L_t - \sum_{j=\bar{J}}^{J} N_{j,t} b_{j,t}^{NDC} \tag{7}$$

The government Naturally, in addition to balancing the social security, the government collects taxes on earnings, interest and consumption and spends a fixed share of GDP on unproductive (but necessary) consumption. Given that the government is indebted, it naturally also services the outstanding debt.

$$T_t = \sum_{j=1}^{J} \left[\tau_t^c c_{j,t} + \tau_t^k r_t s_{j,t} + \tau_t^l (1 - \tau_t^s) w_t l_{j,t} + \tau_t^l b_{j,t}^{\iota} \right] N_{j,t} \tag{8}$$

$$G_t + pensiongap_t + r_t D_{t-1} = T_t + (D_t - D_{t-1}) \tag{9}$$

with $G_t = \gamma_g Y_t$. We set debt D_t to GDP ratio at the fixed level of 45%, which was the actual value of debt to GDP ratio in 1999.

The reform: forming the funded pillar In the baseline scenario the economy runs a defined contribution system financed on the pay-as-you-go basis. Thus, agents expect the pension system to gradually get balanced. This implies diverting funds from PAYG pillar and increasing the $pensiongap_t$, relative to the baseline scenario. This in turn, leads to higher fiscal pressure, following equation (9).

To mitigate the costs of financing this transition we propose an indexation of pension benefits in excess of r^{NDC}. In fact, we index both contributions and post-retirement benefits in the notional pillar with $r^{NDC'} > r^{NDC}$, which denotes the *indexation in excess* of the rules specified in (6) i.e. a pure defined contribution system. This way the transition cohorts receive from the pension system slightly more than they contributed, but the fiscal costs surfaces only gradually and is delayed to the periods when these cohorts actually retire.

The indexation factor $r^{NDC'}$ can be either year specific or cohort specific. In the case of the former, any given cohort may receive a different lifetime compensation scheme than either of the adjacent cohorts. This approach seems especially attractive from the policy standpoint, since year-specific changes are typically easily enacted. In the case of the latter, cohort specific compensation, it is somehow analogous to the earlier literature, because it offers a cohort specific transfer, similar to the LSRA. Yet, it is not a lump-sum transfer of the welfare surplus but rather a directed compensation (only to net out the welfare loss) to the selected cohorts. Thus, it is possibly second-best from a welfare perspective (with LSRA offering superior welfare outcomes due to the lack of general equilibrium effects), but a politically more feasible option.

Welfare accounting As is standard in the literature, with the utility of j aged agent in period t is defined as in equation (3). Denote allocation and welfare in the baseline scenario (no funded pillar) with superscript B and in the reform scenario (funded pillar) with the superscript R. Then the consumption equivalent of the reform is the following:

$$U_{1,t}((1+\mu_t)\tilde{c}_t^B, \tilde{l}_t^B) = U_{1,t}(\tilde{c}_t^R, \tilde{l}_t^R) \tag{10}$$

where $\tilde{c}_t = (c_{1,t}, c_{2,t+1}, ..., c_{J,t+J-1})$ and $\tilde{l}_t = (l_{1,t}, l_{2,t+1}, ..., l_{J,t+J-1})$. A positive value of μ_t signifies welfare improvement for cohort born in period t.

Model solving The model is solved twice: for the baseline and for the reform scenario. First, using the Gauss-Seidel algorithm we compute the initial and the final steady state. In the first steady state the share of pension savings that goes into the funded pillar is equal to strictly 0.

In the final steady state 33% of the contribution goes to the funded pillar.[3] Indexation may only operate between $t + 2$ and $t + 42$, thus initial and final steady states are unaffectd.

The transition path is found iteratively as a path between the initial and the final steady state, using the Gaus-Seidel algorithm as well. Initially, a path of capital is filled linearly between the final and the initial steady state. Subsequent iterations update the value of capital stock in every period following the model. The length of this path is such that the population stabilizes, which in our setting commences after 140 periods. Once the difference in capital stock between iteration stabilizes, the algorithm reaches convergence. For each given path of indexations (each possible reform path) we compute welfare equivalents. We then employ algorithms to optimize the path of indexations such that the number of loosing cohorts and the overall loss were minimized.

II.4 Seeking the optimal transition path

The goal of the system is to find Pareto-optimal indexation path for introducing the funded pillar to an economy with originally a pay-as-you-go pension system. We thus seek for such a schedule of indexation $r^{NDC'}$ such that would leave no cohort worse off. To do so we employ two different classes of algorithms: rule-of-thumb minimizer and genetic. The former minimizes the largest loss among all cohorts. The latter minimizes the number of loosing cohorts and the overall welfare loss. We set a starting vector of indexations $r^{NDC'} = 0$ and a vector of τ^{FDC} gradually, linearly converging to the final steady state value in $t = 42$. The instrument may operate in two versions: year-specific and cohort specific.

[3]The actual value of this parameter has been provided by the Polish legislation, but remains irrelevant for the proper functioning of the algorithm.

Minimizer We employ the minimax criterion: minimization of the loss of the biggest loser of the reform. We thus limit the severity of the losses to a threshold of .001% of lifetime consumption. The year-specific version of the algorithm iteratively identifies those cohorts whose welfare equivalent is lower than the threshold and adjusts incrementally indexation and share of contributions that go to the funded pillar for the last 20 years of life of the oldest of them. Indexation increases welfare "forward", and so the cohorts somewhat older shall also experience an increase in welfare. We do so either until the threshold loss is hit (or until the algorithm tries 1 000 000 paths, whichever comes first). The cohort-specific version of the algorithm is identical, but focuses on cohorts rather than years of life: procedure identifies the most hurt cohort and then slightly increases the indexation for this cohort.

Genetic algorithm If a given path of indexations yields higher number of cohorts better off and higher overall welfare, this path is considered better than an alternative. We formulate the following heuristic. We design possible starting indexation paths as periodically constant intervals of various values. We obtain consumption equivalents and rank the starting paths. We then employ the genetic algorithms: for a given path with the best outcome thus far, the algorithm combines it with second best etc, to test if any given combination of the two results in better outcomes. If not, the algorithm moves on to the next genetic combination of a given path. To assure cross-breeding and pedigree, limited randomization occurs (randomly chosen indexation values $r^{NDC'}$ and/or share of contribution that go to the funded pillar are replaced by a path smaller by 0.005).

III CALIBRATION

The calibration replicates micro- and macroeconomic features of the Polish economy in 1999. The demographic structure and projection is retrieved from the EU's Economic Policy Committee Working Group on Aging Populations and Sustainability (henceforth AWG) together with the projected path of the labor-augmenting total factor productivity growth. The demographic projection covers years 1999-2050, and we assume that after 2050 both fertility and mortality stay constant, thus demography stabilizes after year 2130.[4] The projected TFP growth rate at start replicates the data (yielding 3.8% per year) and is assumed to slowly converge to the value of 1.7% by the year 2040.[5]

The capital share of income is assumed at the standard 0.33 level, and the annual depreciation rate is assumed at 5%. The discounting factor β and depreciation rate d are calibrated to yield the share of investment in GDP at the level of 21%[6] and the interest rate of approximately 7%. While this value may seem high, note that it applies to the economy still in transition with high TFP growth. Nishiyama and Smetters (2007) find the corresponding interest rate of 6.2% for the US.[7] The shadow price of the implicit debt is the indexation of pensions[8] – pure in the case of the baseline scenario and augmented for cohorts or years in the case of the reform. The leisure preference parameter ϕ was calibrated to replicate the labor force participation rate of 56.8%.

The share of government expenditure in GDP is assumed to be constant both in the steady

[4]See Liu et al. (2005) for an analysis with endogenous longevity and an interplay between pension systems and longevity.

[5]See Boucekkine et al. (2002), Bouzahzah et al. (2002) for OLG models with endogenous growth. While macroeconomic effects may differ, typically policy reform yields analogous welfare effects with and without endogenous growth (Del Rey and Lopez-Garcia 2012, Cipriani and Makris 2012).

[6]Depending on the period over which the average is taken, it ranges from 20.8% for a 10 year average (5 years before and 5 years after the reform), 23.1% for 2 year window and 24.1% in the year of the reform. The average for the period between 1995 (first reliable post-transition data) and 2010 amounts to 20.7%.

[7]The average real annual rate of return at the level of 7.5% was achieved by the open pension funds with a balanced portfolio strategy in the period 1999-2009.

[8]See Kuhle (2010) for an extensive comparison of implicit and explicit pension debt with idiosyncratic shocks.

Table 1: Calibrated parameters

Parameters		General	Lower β	Higher ϕ
α	capital share of income	0.33	0.33	0.33
d	depreciation rate	0.05	0.05	0.05
β	discounting factor	0.9735	0.9700	0.9735
ϕ	preference for leisure	0.825	0.825	1
γ_g	share of govt expenditure in GDP	20%	20%	20%
D/Y	share of public debt to GDP	45%	45%	45%
τ^k	capital income tax	19%	19%	19%
τ^l	labor income tax	11%	11%	11%
τ^ι	effective social security contribution	6.2%	6.2%	6.2%
Outcome values (first steady state)				
$(dk)/y$	share of investment in GDP	21%	20.3%	21%
b/y	share of pensions in GDP	5.0%	5.0%	4.6%
r	interest rate	7.2%	7.5%	7.1%
	labor force participation rate	56.9%	56.5%	52%
τ^l	labor income tax	17.4%	16.4%	16.4%

states as well as along the transition at the level of 20%. The initial debt to GDP ratio is set at the level of 45%, corresponding to the data. We set the capital income tax rate at the *de iure* level of 19% as there are no exemptions. Note however, that the return on assets in the funded pension funds is tax-free. The labor income tax τ^l was set at 11%, which matches the rate of revenues from this tax in aggregate consumption in 1999. The social security contribution rate was calibrated to replicate the resulting pensions to GDP ratio of 5%. We allow the consumption tax rate to vary freely in order to close the government budget constraint. The resulting 17.4% is slightly higher than the 16-17% ratio of labor income tax revenues to total taxable labor income in the late 1990s in Poland, and is understood to capture the revenues from other non-modeled taxes.

In addition to this general calibration, we provide a variety of sensitivity checks. We rerun simulations for scenarios with alternative productivity, fertility and mortality paths as well as alternative preferences calibration in order to verify if the optimal paths are susceptible to the parametrization. While productivity and demographics do not affect calibration outcomes, preferences do. Table 1 reports all calibrations used in this study.

IV RESULTS

The basic criterion our model tries to satisfy is the following: given the general equilibrium effects of introducing the funded pillar (i.e. a change in pensions), is there a compensation in additional indexation that alltogether yields superior outcomes despite increased taxation. The interest rate which accrues the individual obligatory savings in the funded pillar of the pension system is a market interest rate $r^{FDC} > r^{NDC}$, which means that pensions with a funded pillar are higher. However, pension gap in the PAYG system inflicts a welfare cost.

Our funded pillar is an actual pension system pillar – not private voluntary savings. There are two main differences between the obligatory and voluntary savings in our model. First, obligatory savings are determined at the rate τ^{FDC} and thus do not follow the life-cycle, whereas the private savings $s_{j,t}$ can even be negative at any given point in life as long as the agents are not indebted at $J = 80$. Indeed, the life-time variability of private savings is one of the sources of the welfare gain in earlier studies which abolish the pension system at all, e.g. Kotlikoff (1996), McGrattan and Prescott (2013). Second, there is no capital gain income tax.

We present the results of the estimation from the calibration described in Section III. We show results for the cohort-specific indexation computed with the minimizer algorithm and for both time and cohort-specific indexation computed with the genetic algorithm. Subsequently we move to the sensitivity analysis.

IV.1 Standard calibration

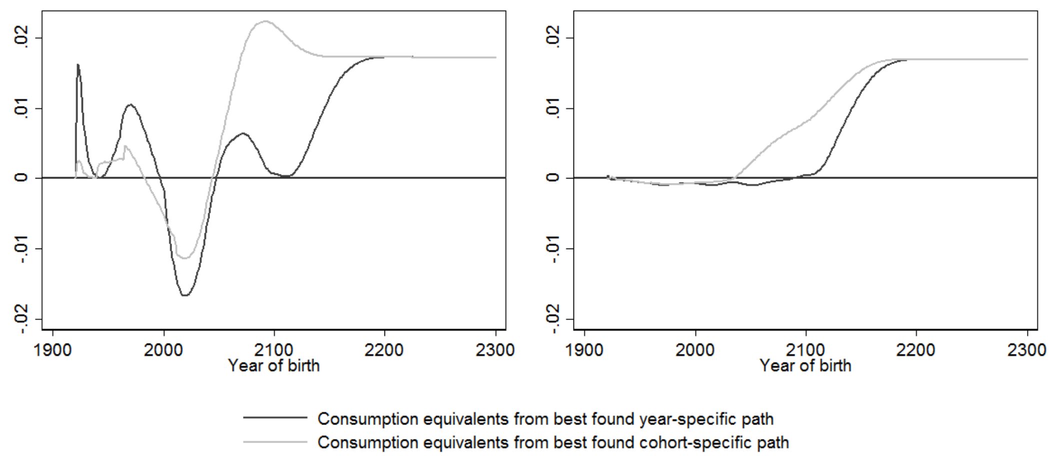
Comparing the outcomes, cohort-specific indexation has a larger potential to smoothen consumption, but clearly the minimizer algorithm requires less iterations and delivers better welfare outcomes, see Figure 1. The losses in the minimizer algorithm are computationally negligible, as the overall welfare loss is about an order of magnitude of the convergence error in the iterative process of finding equilibrium transition path. While we cannot demonstrate that all cohorts *gain* from the reform, we argue that the loss of 0.001% of lifetime consumption could be regarded as irrelevant by rational agents in e.g. voting models.

The benefit of the minimizer algorithm is revealed also by the paths of indexation and contribution rates. Minimizer delivers more monotonic per-cohort or per-year transfers to compensate for a more stable forming of the capital pillar, see Figure 2. On the other hand, the compensations are higher than delivered by the genetic algorithm. Thus, it appears that larger scope for fine tuning the welfare effects does not translate to a lower number of 'losing' cohorts, when compared to the per-year indexation. Yet, it translates to higher extent of between-cohort redistribution. Although cohort-specific indexation delivers better welfare outcomes for minimizer and lower maximum loss for the genetic algorithm, its is also less effectively utilizing the scope of adjustments. Despite a large number of iterations, the forming of the funded pillar is rather sudden in genetic algorithms, with subsequent adjustments in indexation equally abrupt.[9]

In the conservative parametrization we find the indexation path to compensate mostly the future cohorts, who start bearing the costs associated paying 'excessively' high pensions to the

[9] We also run simulations seeking optimal indexation with ad-hoc gradual introduction of the funded pillar (for the reforms which take 33 years, 66 years and 132 years to complete). The slower the reform, the smaller the welfare loss, but the path of indexation and consumption equivalents are the same. For faster reform, the indexation instrument concentrates the welfare costs in fewer cohorts.

Figure 1: Welfare effects - consumption equivalents



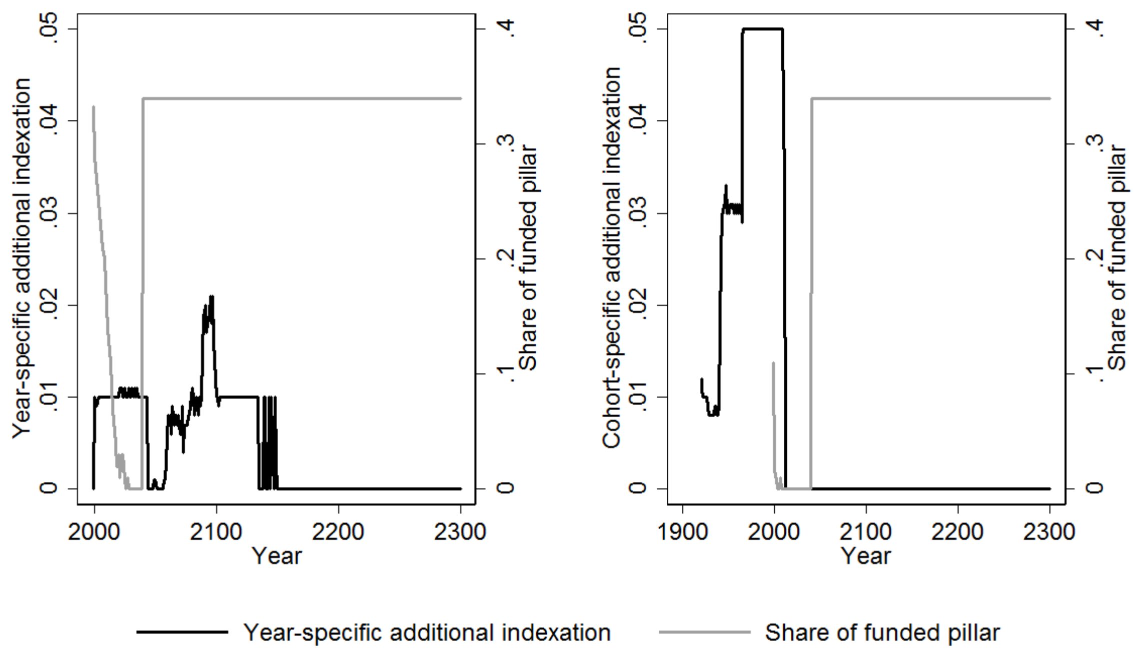
Notes: results from 10 000 iterations for the minimizer algorithm, genetic algorithm required over 20 000 iterations and did not reach the required target for the maximum welfare loss.

initial transition generations. Thus, indexation is relatively low at first, because at this stage the majority of the shift in the contributions is compensated by taxing a relatively large base. However, when the future pension benefits needs to be adjusted for the additional indexation, the welfare costs appear. We report the adjustment in taxes, which signifies the timing of periods in which the rules for higher pension benefits get activated in Figure D.6 in the Appendix. Generally, both the taxes and the deficit in the pension system adjust contemporaneously. Yet, although the *timing* is clearly related, there is no clear translation between the the behavior of the indexation indexation and the *strength* of the fiscal adjustment. This suggests that the (nearly) optimal introduction of the funded pillar requires cushioning in a way that could not be derived analytically (nor guess-timated) without employing computational algorithms.

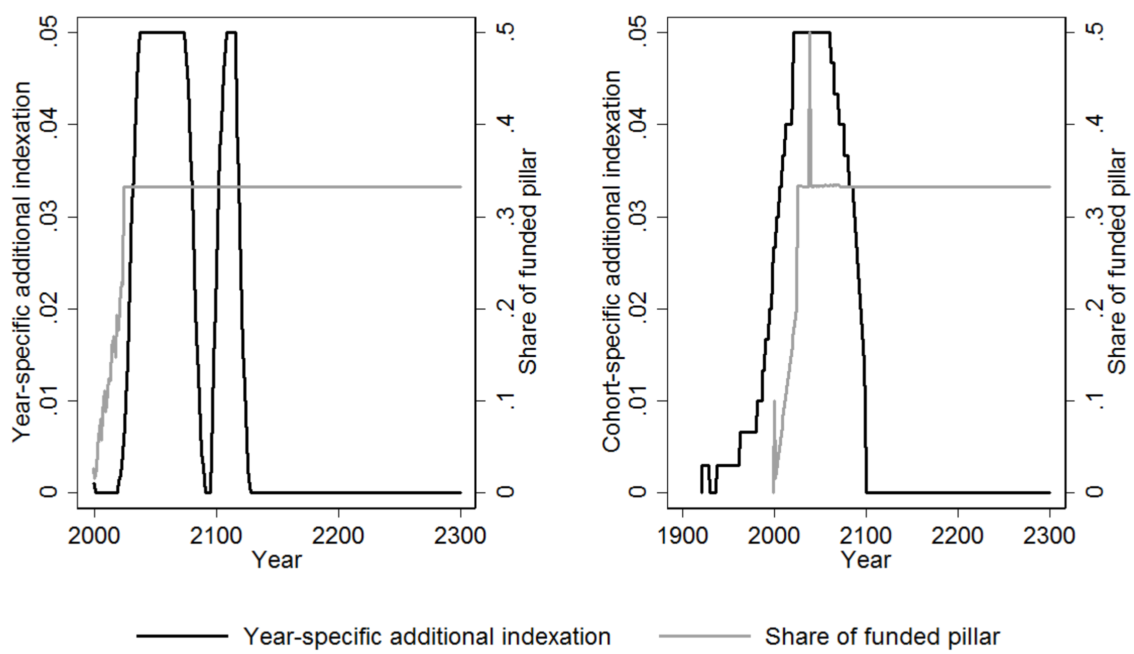
The basic intuition for the results is as follows. For the reforms paths which burden a larger number of adjacent cohorts, the instrument / algorithm may be unable to provide significant improvements because the periods of original fiscal adjustment and the consequences of 'excessive'

Figure 2: Optimal instrument

(a) genetic algorithm



(b) minimizer



indexation begin to overlap. In addition, faster reforms generate larger costs, which in turn require higher compensation. The two effects are likely to propagate each other. Thus, one should expect that neither for the very fast nor for the very slow reforms an optimal path can be found. As far as delaying is concerned, for the reforms which are enacted later, finding an optimal path should be more feasible, because all affected cohorts are able to fully internalize the change in the pension system.

In the above results a number of exogenous assumptions may be affecting the results. First, the reform starts immediately, which may interact with the demographic transition and the relative situation of the initial old. Second, initially the population is not old yet, so the tax base is wider, thus allowing for a smaller scope of the tax adjustment. Also, the initial technological progress rate is relatively high, which favors the implicit debt over the explicit one. Thus, we test if the algorithm is able to find a similarly welfare enhancing path if these calibrations are relaxed towards more conservative ones.

IV.2 Testing the robustness to alternative exogenous assumptions

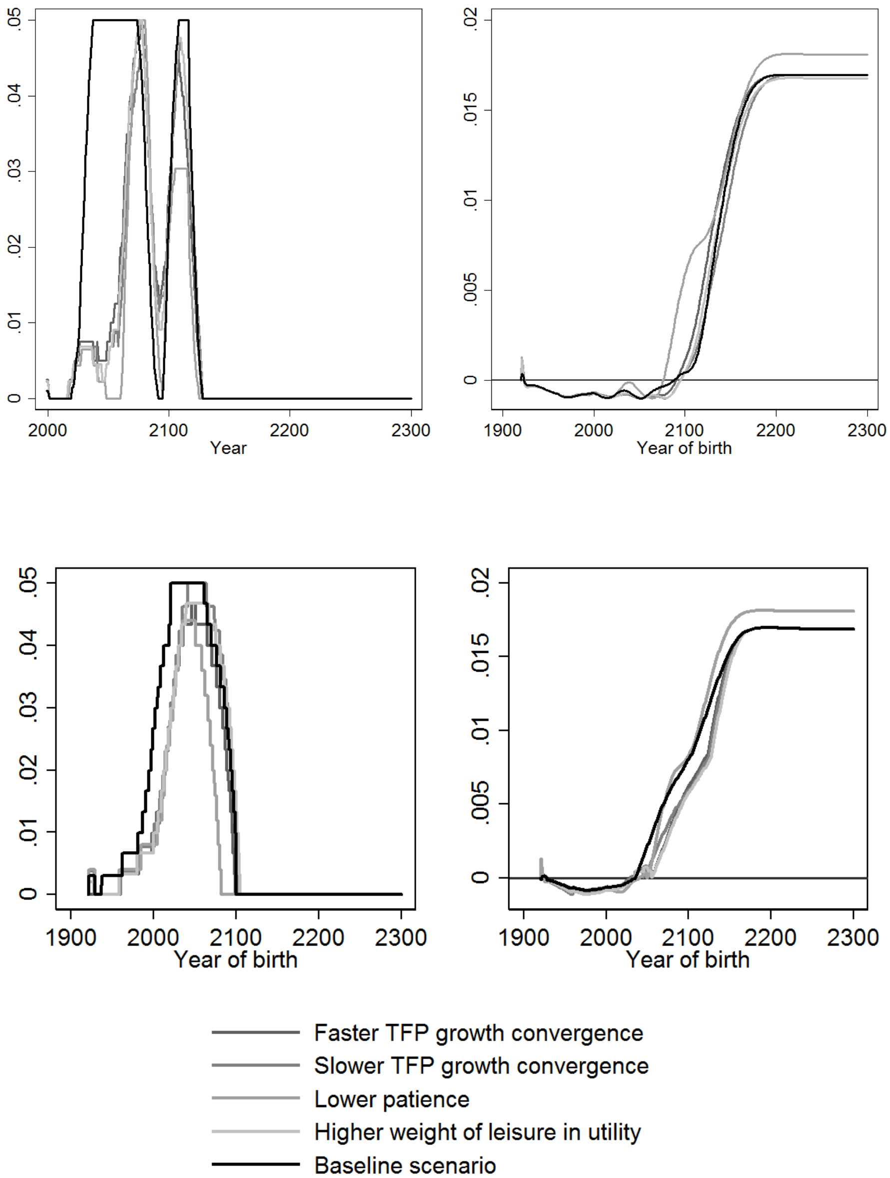
For the conservative specification presented above we develop a series of alternative calibrations which permit to test if the algorithm proposed provides stable results. In each of the analysis we change one of the exogenous parameters (or their paths), keeping all other calibration parameters unchanged. The robustness checks concern: utility function parameters, exogenous fertility and technological progress paths. For the utility function parameters we check two cases: lower patience δ and lower preference for consumption $1-\phi$. With lower patience agents are likely to prefer contemporaneous consumption to future pension benefits. Thus, instantaneous tax adjustments will require higher indexation to maintain the same welfare, which sets the bar for optimum transition path higher than in the baseline scenario. Similar reasoning holds for higher preference for leisure – agents will be less likely to adjust upwards labor supply and downward consumption, thus increased contemporaneous taxation will be more costly than in the baseline calibration in terms of welfare, see Table 1 in Section III for the details of these alternative calibrations.

For the exogenous fertility, the demographic projection used in the baseline calibration assumes that it will be gradually decreasing. Yet, the fertility rate is already extremely low in the initial steady state – at 1.4. Moreover, decreasing size of the population implies that initially there is a much higher tax base, whereas in the future indexation needs to be provided for by a much smaller population. To see of the algorithm is robust to this parametrization, we also include a scenario with a constant rather than decreasing fertility. In practice, this implies that the population stabilizes earlier in the simulation. Please note that this scenario does not affect the initial steady state calibration.

Finally, we also consider two alternative paths for the catching up. While in the final steady state the economy is always projected to experience technological progress of 1.7% per annum, the starting point is actually at 4% per annum. The speed of convergence may matter for the ability to find an optimum path in the interaction with the speed of implementing the reform. Thus, we include a faster convergence and a slower convergence, see Figure C.5.

The results of these sensitivity checks are reported in Figures 3 and 4 and suggest that while the actual values of 'excessive indexation' differ between the calibrations, the time variation is largely the same. This is indicative of the property that the obtained (nearly) optimal reform paths are fairly stable for a given schedule of introducing the funded pillar.

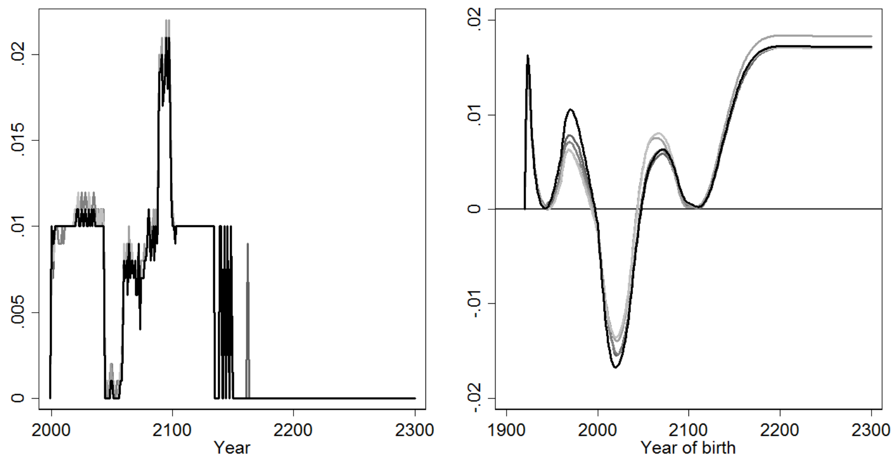
Figure 3: Welfare effects of reform with $indexation_t$ for the alternative calibrations, minimizer: year-specific (top) and cohort-specific (bottom)



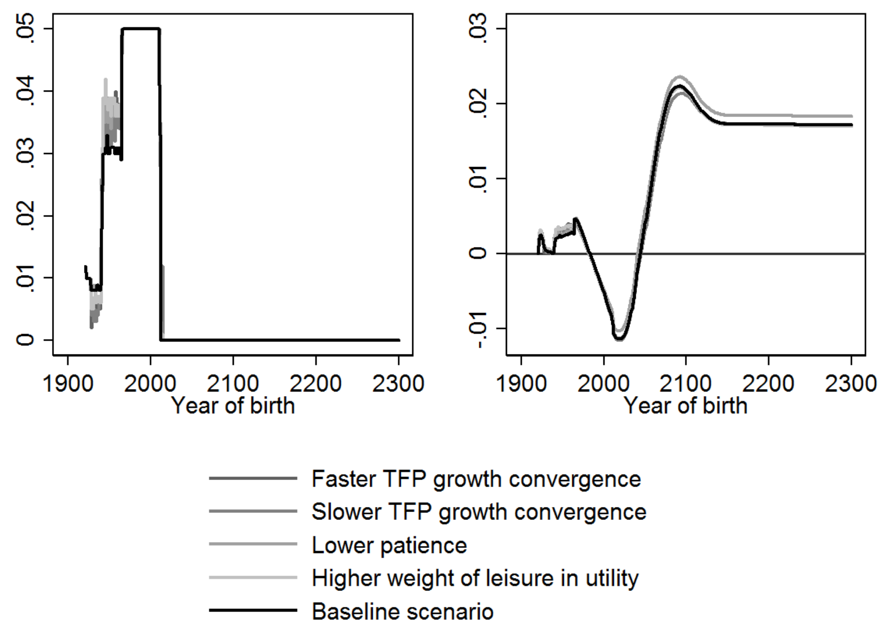
V CONCLUSIONS

While the introduction of the funded pillar may well be gradual, the fiscal costs still need to be short of gains from higher pensions and output for assuring Pareto-optimality. Thus, forming a funded

Figure 4: Welfare effects of reform with $indexation_t$ for the alternative calibrations, genetic: year-specific (top) and cohort-specific (bottom)



(a) genetic, year-specific $indexation_t$



(b) genetic, cohort-specific $indexation_t$

pillar when PAYG scheme exists requires either of the three: increase in tax-rate, an increase in public debt or a decrease in other public spending. It is customary to treat non-pension public expenditure as a fixed variable in the analyses of the pension systems (typically it is modeled as pure waste and thus changing its value directly alternates welfare accounting). Increase in taxes concentrates the costs of the reform on transition cohorts, so it cannot produce a Pareto-optimal reform. Thus, one is left with public debt. However, in real world the public debt costs, its levels are regulated by often national laws and international treaties, whereas systematic increase in public debt can translate to financial and/or currency crisis before the economy reaches the new steady state.

When reforming the pension system, it is a natural policy objective to leave little or no voters worse off. Our objective in this paper has been to find a transition path from a pay-as-you-go scheme to a funded scheme that leaves no cohort worse off after the reform in the defined contribution setting. Earlier literature suggests that one cannot introduce a funded pension system in a Pareto-optimal way. Implied policy recommendation ranged between cohort lump sum redistribution or abandoning any pension system whatsoever. We propose an alternative approach, by shifting the costs of the reform to the future generations by the means of implicit rather than explicit debt. We develop a model of an OLG economy, which starts from a PAYG DC system and reforms to a (partially) funded DC. We propose to finance the gap generated by the introduction of a funded pillar with an implicit rather than explicit debt: indexation of pensions to the loosing cohorts in excess of actuarial fairness. The advantage of relying on implicit debt is that it reduces slightly the costs of servicing the debt for the future generations.

We show that the indexation scheme more generous than would have been actuarially fair posits a politically feasible instrument of between cohort redistribution and allows to effectively eliminate any welfare loss relative to the *status quo* of the PAYG financing of the social security. Additional advantage of the proposed policy instrument stems from the fact that the very instrument is a state variable in the search for the optimal transition path: indexation scheme is a path seeking algorithm.

We run a series of simulations searching the indexation scheme that leaves (nearly) no cohort worse off. While our analysis is calibrated to a specific case of a country which implemented social security privatization, we provide a variety of sensitivity analyses, altering the set of assumptions concerning the demographic scenarios, preferences and exogenous technological progress path. The proposed policy instrument – and at the same time optimal transition path – remains stable across those exogenous scenarios.

We hope that our study contributes to furthering research in designing optimal transition to funded pension systems. Admittedly, our results are based on a number of premises. First, with the Cobb-Douglas utility function, there is only minor reaction of labor to changes in the pension system. Relying on a different utility function – such as Greenwood et al. (1988) – could also matter for the ability of the algorithm to find the Pareto-optimal transition path. Second, while we assume a fairly swift introduction of the funded pillar, redistribution may continue for as long as a lifetime. Under such circumstances, agents internalizing the link between the contributions to the pension system and the subsequent pension benefits would respond differently than agents who do not see such link. Providing this extension is likely to result in a slight behavioral response in terms of labor supply decisions. However, one should not expect a substantial change in conclusions. Pensions as well as voluntary savings are predominantly driven by the perspective of increasing longevity. Thus, our results are likely to survive even those extensions to the model. Finally, although earlier studies show that the gains from funded pillar are smaller in models with idiosyncratic income shocks, this conclusion was mainly driven by the DC nature of the funded systems, not by the funding itself. The most promising avenue for further research seems to concern aggregate uncertainty: agents may want to account for policy change or unknown changes to demographics as well as technological progress.

References

Aaron, H.: 1966, The social insurance paradox, *Canadian Journal of Economics and Political Science/Revue canadienne de economiques et science politique* **32**(03), 371–374.

Auerbach, A. J. and Kotlikoff, L. J.: 1987, *Dynamic fiscal policy*, Cambridge University Press.

Belan, P. and Pestieau, P.: 1999, Privatizing Social Security: A Critical Assessment, *The Geneva Papers on Risk and Insurance. Issues and Practice* **24**(1), 114–130.

Boucekkine, R., De la Croix, D. and Licandro, O.: 2002, Vintage human capital, demographic trends, and endogenous growth, *Journal of Economic Theory* **104**(2), 340–375.

Bouzahzah, M., De la Croix, D. and Docquier, F.: 2002, Policy reforms and growth in computable OLG economies, *Journal of Economic Dynamics and Control* **26**(12), 2093–2113.

Breyer, F.: 1989, On the Intergenerational Pareto Efficiency of Pay-as-you-go Financed Pension Systems, *Journal of Institutional and Theoretical Economics (JITE) / Zeitschrift fur die gesamte Staatswissenschaft* **145**(4), 643–658.

Browning, E. K.: 1975, Why the social insurance budget is too large in a democracy, *Economic Inquiry* **13**(3), 373–388.

Cipriani, G. P. and Makris, M.: 2012, PAYG pensions and human capital accumulation: some unpleasant arithmetic, *The Manchester School* **80**(4), 429–446.

Coile, C., Milligan, K. and Wise, D. A.: 2016, *Introduction to: "Social Security Programs and Retirement Around the World: The Capacity to Work at Older Ages"*, University of Chicago Press.

Del Rey, E. and Lopez-Garcia, M.-A.: 2012, On welfare criteria and optimality in an endogenous growth model, *Journal of Public Economic Theory* **14**(6), 927–943.

Diamond, P.: 2004, Social Security, *American Economic Review* **94**(1), 1–24.

Greenwood, J., Hercowitz, Z. and Huffman, G. W.: 1988, Investment, capacity utilization, and the real business cycle, *American Economic Review* **78**(3), 402–417.

Gruber, J. and Wise, D. A.: 2007, *Social Security Programs and Retirement around the World: Fiscal Implications of Reform*, University of Chicago Press.

Gruber, J. and Wise, D. A.: 2008, *Social security and retirement around the world*, University of Chicago Press.

Gyárfás, G. and Marquardt, M.: 2001, Pareto improving transition from a pay-as-you-go to a fully funded pension system in a model of endogenous growth, *Journal of Population Economics* **14**(3), 445–453.

Hirte, G. and Weber, R.: 1997, Pareto improving transition from a pay-as-you-go to a fully funded system - is it politically feasible?, *FinanzArchiv / Public Finance Analysis* **54**(3), 303–330.

Holzman, R. and Stiglitz, J. E.: 2001, *New ideas about old age security*, Washington, DC: World Bank.

Joines, D. H.: 2005, Pareto-Improving Social Security Reform, *2005 Meeting Paper 396*, Society for Economic Dynamics.

Kotlikoff, L. J.: 1996, Privatizing Social Security: How It Works and Why It Matters, *Tax Policy and the Economy* pp. 1–32.

Kotlikoff, L. J., Smetters, K. and Walliser, J.: 1999, Privatizing social security in the united states – comparing the options, *Review of Economic Dynamics* **2**(3), 532–574.

Krueger, D. and Kubler, F.: 2006, Pareto-Improving Social Security Reform when Financial Markets Are Incomplete, *American Economic Review* **96**(3), 737–755.

Kuhle, W.: 2010, The optimum structure for government debt, *Technical report*, Munich Center for the Economics of Aging (MEA) at the Max Planck Institute for Social Law and Social Policy.

Liu, L., Rettenmaier, A. J., Saving, T. R. et al.: 2005, Longevity and public old-age pensions, *Economic Inquiry* **43**(2), 247–262.

Lucas, R. E.: 1988, On the mechanics of economic development, *Journal of Monetary Economics* **22**(1), 3 – 42.

McGrattan, E. R. and Prescott, E. C.: 2013, On financing retirement with an aging population, *NBER Working Paper 18760*, National Bureau of Economic Research.

Nishiyama, S. and Smetters, K.: 2007, Does Social Security Privatization Produce Efficiency Gains?, *Quarterly Journal of Economics* **122**(4), 1677–1719.

Roberts, M. A.: 2013, Pareto-improving Pension Reform Through Technological Implementation, *Scottish Journal of Political Economy* **60**(3), 317–342.

Romaniuk, K.: 2009, Pareto Improving Transition from a Pay-as-You-Go to a Fully Funded Pension System in a Model of Endogenous Growth: The Impact of Uncertainty, *SSRN Scholarly Paper ID 1613251*, Social Science Research Network.

Samuelson, P. A.: 1958, An exact consumption-loan model of interest with or without the social contrivance of money, *The Journal of Political Economy* **66**(6), 467–482.

Wise, D. A.: 2016, *Social Security Programs and Retirement Around the World: The Capacity to Work at Older Ages*, University of Chicago Press.

Appendix A. Consumer problem solution

For $j < \bar{J}$:

$$
\begin{aligned}
c_{j,t} &= \frac{(1+(1-\tau_t^k)r_t)s_{j-1,t-1} + (1-\tau_t^s)(1-\tau_t^l)w_t + beq_{j,t} + \Omega_{j,t} + \Gamma_{j,t}}{(1+\tau_t^c)\left[\sum_{s=0}^{\bar{J}-j-1}\left((1+\phi)\,\beta^s \frac{\pi_{j+s,t+s}}{\pi_{j,t}}\right) + \sum_{s=\bar{J}-j}^{J-j}\left(\beta^s \frac{\pi_{j+s,t+s}}{\pi_{j,t}}\right)\right]} \qquad \text{(A.11)} \\
l_{j,t} &= 1 - \frac{\phi(1+\tau_t^c)c_{j,t}}{(1-\tau_t^s)(1-\tau_t^l)w_t} \\
s_{j,t} &= (1-\tau_t^s)(1-\tau_t^l)w_t l_{j,t} + (1+(1-\tau_t^k)r_t)s_{j-1,t-1} - (1+\tau_t^c)c_{j,t},
\end{aligned}
$$

with

$$
\begin{aligned}
\Omega_{j,t} &= \sum_{s=1}^{\bar{J}-j-1} \frac{(1-\tau_{t+s}^s)(1-\tau_{t+s}^l)w_{t+s}l_{j+s,t+s} + beq_{j+s,t+s}}{\prod_{i=1}^{s}(1+(1-\tau_{t+i}^k)r_{t+i})} \\
\Gamma_{j,t} &= \sum_{s=\bar{J}-j}^{J-j} \frac{(1-\tau_{t+s}^l)b_{j+s,t+s}^{\iota} + beq_{j+s,t+s}}{\prod_{i=1}^{s}(1+(1-\tau_{t+i}^k)r_{t+i})}.
\end{aligned}
$$

For $j \geq \bar{J}$:

$$
\begin{aligned}
c_{j,t} &= \frac{(1+(1-\tau_t^k)r_t)s_{j-1,t-1} + (1-\tau_t^l)b_{j,t}^{\iota} + beq_{j,t} + \Gamma_{j,t}}{(1+\tau_t^c)\left[\sum_{s=\bar{J}-j}^{J-j}\left(\beta^s \frac{\pi_{j+s,t+s}}{\pi_{j,t}}\right)\right]} \qquad \text{(A.12)} \\
l_{j,t} &= 1 - \frac{\phi(1+\tau_t^c)c_{j,t}}{(1-\tau_t^s)(1-\tau_t^l)w_t} \\
s_{j,t} &= (1-\tau_t^l)b_{j,t}^{\iota} + (1+(1-\tau_t^k)r_t)s_{j-1,t-1} - (1+\tau_t^c)c_{j,t},
\end{aligned}
$$

with

$$
\Gamma_{j,t} = \sum_{s=\bar{J}-j}^{J-j} \frac{(1-\tau_{t+s}^l)b_{j+s,t+s}^{\iota} + beq_{j+s,t+s}}{\prod_{i=1}^{s}(1+(1-\tau_{t+i}^k)r_{t+i})}.
$$

The numerators in equations (A.12) and (A.13) represent the present discounted value of the remaining lifetime income.

Appendix B. Market clearing conditions and general equilibrium

The goods market clearing condition is standardly defined as

$$\sum_{j=1}^{J} N_{j,t}c_{j,t} + G_t + K_{t+1} = Y_t + (1-d)K_t, \tag{B.13}$$

where we denote the size of the j-aged cohort at period t as $N_{j,t}$. This equation is equivalent to stating that at each point in time the price for capital and labor would be set such, that the demand for the goods from the consumers, the government and the producers would be met. This necessitates clearing in the labor and in the capital markets. Thus labor is supplied and capital accumulates according to:

$$L_t = \sum_{j=1}^{\bar{J}-1} N_{j,t}l_{j,t} \qquad \text{and} \qquad K_{t+1} = (1-d)K_t + \sum_{j=1}^{J} N_{j,t}\hat{s}_{j,t}, \tag{B.14}$$

where $\hat{s}_{j,t}$ denotes private savings $s_{j,t}$ as well as accrued obligatory contributions in the funded pillar of the pension system.

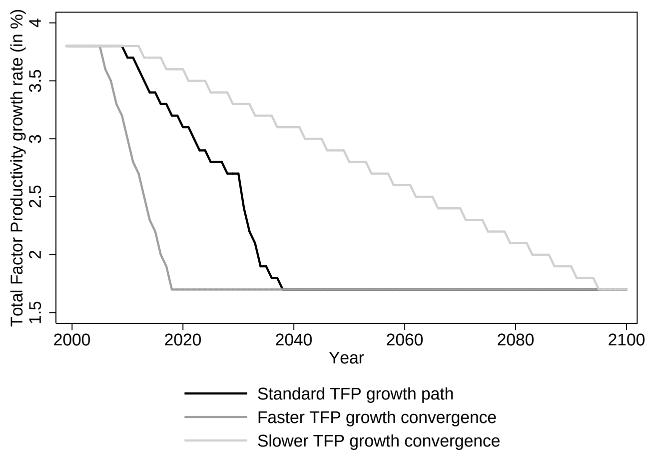
An equilibrium is an allocation $\{(c_{1,t},...,c_{J,t}),(s_{1,t},...,s_{J,t}),(l_{1,t},...,l_{J,t}),K_t,Y_t,L_t\}_{t=0}^{\infty}$ and prices $\{w_t,r_t\}_{t=0}^{\infty}$ such that

- for all $t \geq 0$, for all $j \in [1,J]$ $((c_{j,t},...,c_{J,t+J-j}),(s_{j,t},...,s_{J,t+J-j}),(l_{1,t},...,l_{J,t+J-j})$ solves the problem of an agent j for all t, given prices;

- (K_t, Y_t, L_t) solves the firm's problem (1);
- government sector is balanced, i.e. (8) - (9) are satisfied;
- markets clear, i.e. (B.13)-(B.14) are satisfied.

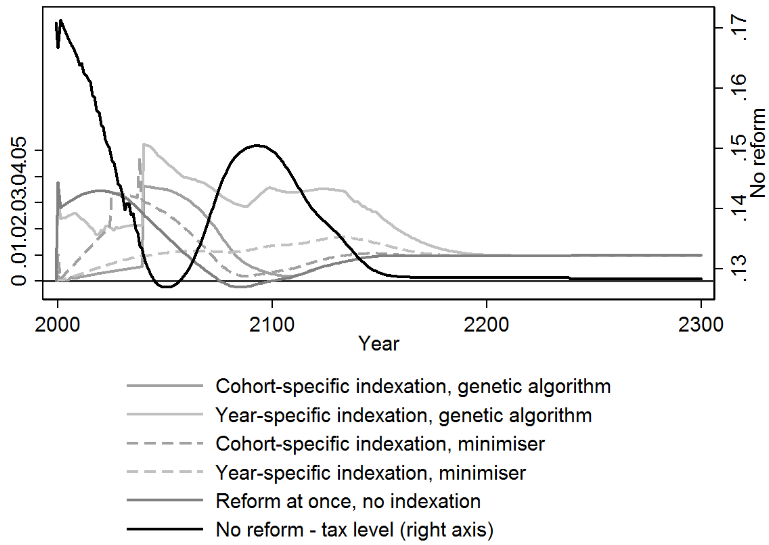
Appendix C. Alternative calibrations

Figure C.5: Alternative technological progress rates (γ)

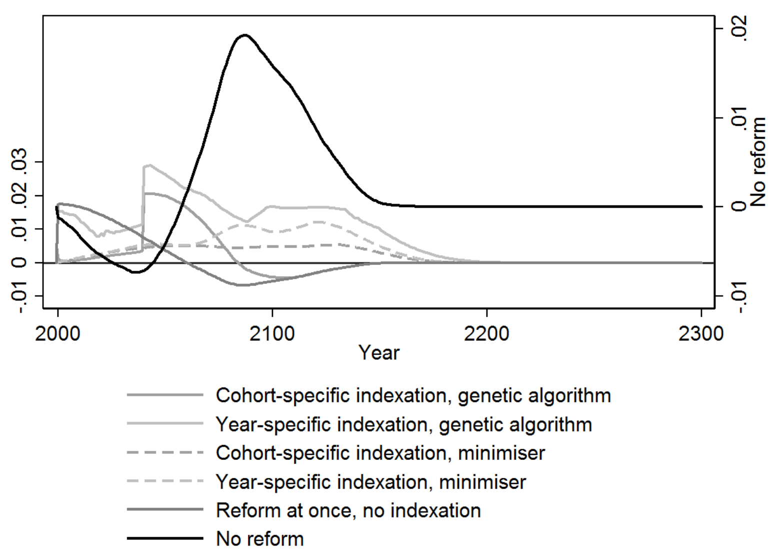


Appendix D. Macroeconomic adjustments

Figure D.6: Changing pension system balance and tax adjustments



(a) Taxes in baseline scenario of no indexation and the reform scenarios with cohort and year specific instrument



(b) Deficit in the pension system as a share of GDP in the baseline scenario and in the reform scenarios

Figure D.7: Capital with no funded pillar and deviations (in per cent)