Impulsivity and Social Security*

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Abstract

A leading rationale for having a social security program is that it remedies insufficient saving for retirement due to impulsive spending. Yet despite the fact that hyperbolic discounting has become the conventional way to model and represent impulsivity in economics research, a major challenge has emerged for the mandatory saving role of social security given that some recent research has documented that it is *impossible* for a social security program to counteract the insufficient saving that results from hyperbolic discounting. In contrast, I demonstrate both analytically and numerically that social security can remedy impulsive spending and that it can improve life-cycle well-being, if the idea of an impulsive consumer is conceptualized differently. This alternative specification of impulsivity is represented as an *intra*-temporal tension between saving optimally and saving too little, which fits within the general context of a naive "dual self" from psychology. These findings can provide theoretical support for the mandatory saving role of social security under the criterion of "new paternalism", outlined in Cremer and Pestieau (2011) among others.

Keywords: *social security, impulsivity, insufficient saving for retirement, time-inconsistent dynamic optimization, naiveté*

JEL Classification: H55, D91, C61

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A fundamental motivation for the adoption of governmental social-security systems has always been that many people just do not plan for their own future and will put themselves in penury in their old age unless someone forces or otherwise encourages them to save for retirement.

Robert J. Shiller
American Economic Review (2003, p.345)

It is common wisdom that people save too little. To compensate for this failure, most developed country governments heavily support the elderly during retirement... A key theoretical innovation permitting systematic analysis of time-inconsistent behavior is the recognition that individuals may maximize a utility function that is divorced from that representing "true welfare". Once this distinction is accepted, "saving too little" becomes a meaningful concept.

George A. Akerlof
In his acceptance lecture for receiving
The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2001

1 Introduction

The objective of this study is to examine whether or not a social security (mandatory saving) arrangement is able to help individuals achieve their long-run goals, and therefore be justified theoretically under the criterion of "new paternalism". But first, some context needs to be provided. The idea that people save too little, and that they will therefore benefit from a social security program, is not new and appears to be widely accepted. The abovementioned quotes by Nobel Laureates Robert Shiller and George Akerlof certainly reflect this sentiment.[1] Yet, the claim or proposition that individuals save inadequately and consequently need help does invite some legitimate questions: 1.) What does it mean to save too little? 2.) Given a notion of saving too little, why might people fall prey to it? and 3.) Can a social security (mandatory saving) program improve the outcomes or well-being of such individuals?

The first two questions are conceptually intertwined since the idea of whether or not a person saves too little depends on who is asking the question: the individual himself, or someone else. In searching for answers to such questions, economists have incorporated insights from other scientific disciplines like the field of psychology. This is because the traditional Neoclassical economics paradigm has ruled out (by assumption) the possibility of inadequate saving from the perspective of the individual himself, given that an individual who enters retirement with little or no savings has made choices that are the solutions to a higher-order utility maximization problem. Therefore, insufficient saving is a paradox because the idea of it contradicts the

[1] For similar statements see Musgrave (1968), Samuelson (1975), Musgrave and Musgrave (1984), Diamond (1977, 2002), Munnell (1977), Kotlikoff, Spivak, and Summers (1982), Feldstein (1985, 1988), Blinder (1988), Laibson (1998), Aaron (1999), Fehr (2002), Fenge and Pestieau (2005), McCaffery and Slemrod (2006), Venti (2006), Cremer, De Donder, Maldonado, and Pestieau (2008, 2009), Kaplow (2008), Pestieau and Possen (2008), Fehr (2009), Tenhunen and Tuomala (2010), Teulings (2010), Lazear (2011), Batina (2012), Boadway (2012) among many others.

assumptions of the theory of revealed preference.[2] Therefore, the only way that a mandatory saving arrangement can be theoretically justified in the Neoclassical paradigm is by assuming that an individual saves too little from the perspective of what a different person (i.e., a benevolent social planner or government) thinks is needed.[3] Cremer and Pestieau (2011) summarize this perspective under the criterion of "old paternalism" in which a mandatory saving program is justified by a government that imposes its preferences on its citizens. Indeed, they state, "The paternalistic notion of forced saving argument was often rejected because it rested on differences in discount rates between citizens and governments. More precisely, governments were assumed to be more future-oriented and more patient than their citizens" (pp.165-166). Cremer and Pestieau (2011) contrast old paternalism with that of "new paternalism" in which a government respects some notion of the individual's preferences: "A more modern view of paternalistic forced saving rests on a gap between individuals' long-run goals and their short-run behavior. This position appears to be more widely accepted... In other words, the present individual's choices [should] be corrected to make them time consistent" (p.166).

A potential candidate for when a mandatory saving arrangement might be rationalized under the criterion of new paternalism is if individuals are myopic or shortsighted while young. Such individuals might save too little if they fail to see or calculate what their resource needs will be during their future retirement years. Indeed, several studies have examined whether or not a social security program is able to improve well-being when individuals are literally myopic (e.g., Feldstein 1985; Docquier 2002; Cremer, De Donder, Maldonado and Pestieau 2007, 2009; Caliendo and Gahramanov 2009; Findley and Caliendo 2009; Caliendo and Findley 2013). Yet, if individuals are literally myopic or shortsighted (as is the case in these cited studies), then the mandatory saving rationale for social security still relies on the practice of old paternalism. This is due to the fact that shortsighted individuals do not have long-run goals by definition. As such, a government (i.e., benevolent social planner) effectively imposes its preferences on myopic individuals in the specification of the welfare criterion, given that the long-run preferences of myopic individuals are not even defined in some type of higher-order decision problem.

An alternative way to think about and represent inadequate saving is to distinguish the decision preferences of an individual from his or her true preferences. Indeed, researchers in the field of behaviorial economics have been exploring this broader (casual) definition of myopia that goes by the terms "impulsivity" or "time-inconsistent preferences", wherein an individual's actual choices or decisions diverge from his or her specified goals, plans, and earlier intentions. By definition, impulsive individuals attempt to achieve some type of long-run objective (such as lifetime utility maximization), yet such individuals undermine and invalidate their earlier saving intentions with their later consumption choices.

The contribution of this study is to demonstrate that a social security (mandatory saving) program is able to be justified theoretically under the criterion of new paternalism. Indeed, I analytically show by proposition that a social security arrangement is able to reallocate resources over the life cycle by restraining impulsive consumption during the working phase and by increasing consumption during retirement. I also show that a social security program succeeds at this for any magnitude of impulsivity, large or small. Moreover, I establish by proposition that the social security tax rate can be parameterized to perfectly restrain impulsivity such that

[2] See Akerlof (2002) for an expanded discussion of this idea.

[3] I am making this statement in the context of an economy that is dynamically efficient.

optimal consumption over the life cycle is replicated and maximal lifetime utility is attained. This means that social security is able to externally commit impulsive consumers to follow through with optimally formulated long-run plans or intentions, despite being impulsive in the absence of a mandatory saving arrangement. Lastly, I demonstrate numerically that social security is generally capable of restraining impulsivity for alternative arrangements about the financing of the program. These findings provide a theoretical basis for the principal justification of social security, thus supporting the "new paternalism" rationale for social security that is discussed by Cremer and Pestieau (2011) and others.

This alternative way of thinking about impulsivity is based on an older tradition in psychology of conceptualizing impulsivity as the inability to inhibit a "prepotent response" to an external stimulus (Madden and Johnson 2010). To be more precise, the individual in my model is a convex combination at any moment in time between a self who consumes and saves optimally and a self who wants to consume hand-to-mouth. The model is general enough to allow for different magnitudes and frequencies of impulsivity.[4] This dual selves specification for impulsivity represents an *intra*-temporal tension between saving optimally and saving too little, whereas hyperbolic discounting reflects an *inter*-temporal conflict between preferences from different ages over the life cycle.

It is also conceptually similar to that of Thaler and Shefrin (1981), who outline several features that a model of impulsive consumption would need to represent. More specifically, they imagine that an individual is composed of two distinct selves. One self of the individual is a "planner", and the other self is a "doer". The planner-self is concerned with lifetime utility, while the doer-self is only concerned with present circumstances. My stylized model exhibits this dual selves idea from the perspective that the representative individual formulates optimal consumption and saving plans (planner-self), yet the individual is also predisposed to consumption impulses that are at odds with optimally formulated plans (doer-self).[5]

Starting with the seminal study of Strotz (1955/1956), hyperbolic discounting has become a common way to represent impulsivity since a hyperbolic discount function engenders time-inconsistent preferences.[6] Indeed, it is an intuitive and popular view that the theory of hyperbolic discounting (as a representation of impulsivity and insufficient saving for retirement) is a prime setting in which a social security (mandatory savings) program is warranted as an external commitment device to achieve an individual's initial, long-run intentions. Yet, this popular perspective faces a serious obstacle given that a recent body of research has discovered that it is *impossible* to justify a social security program with hyperbolic discounting (İmrohoroğlu, İmrohoroğlu, and Joines 2003; Gul and Pesendorfer 2004; Caliendo 2011, 2013; Feigenbaum

[4]The idea of impulsive consumption (interpreted as deviations from planned consumption) is related to Milton Friedman's (1957) idea of *transitory consumption.* Modern interpretations of the *Permanent-Income Hypothesis* often omit this important feature of real-world behavior. See Speight (1989) and Deaton (1992) for more on transitory consumption.

[5]Footnote other dual-selves models... and distinguish.

[6]For more on hyperbolic discounting as a representation of impulsivity, see Ainslie (1975, 2001, 2005, 2009, 2010a, 2010b), Solnick, Kannenberg, Eckerman, and Waller (1980), Ainslie and Haslam (1992a, 1992b), Rachlin and Raineri (1992), Kirby and Herrnstein (1995), Kirby and Maraković (1995), Loewenstein (1996), Herrnstein (1997), Kirby (1997), Camerer (1999), Yi, Gatchalian, and Bickel (2006), Milkman, Rogers, and Bazerman (2008), Wittmann and Paulus (2008), Green and Myerson (2010), Hinvest and Anderson (2010), Madden and Johnson (2010), Ross (2010), Stevens and Stephens (2010), Knoll (2011), and Ottaviani and Vandone (2011), among many others.

2013; Guo and Caliendo 2014). This impossibility does not depend on whether the hyperbolic consumer is naive or sophisticated with respect to his time inconsistency problem.[7] More specifically, an actuarially fair social security program fails to restrain the impulsive spending that results from hyperbolic discounting, given that the actual life-cycle consumption path is unchanged across counterfactual states of the world with respect to the existence of a program. This is due to the fact that the hyperbolic consumer perfectly offsets the flow of mandatory savings via social security with an exact reduction in the flow of private savings. Moreover, a social security program with a negative net present value always makes a hyperbolic consumer strictly worse off, since consumption is everywhere lower with participation in such a program. To summarize this challenge for the mandatory savings role of social security, the intertemporal *Euler equation* for consumption is invariant to the existence of a social security program, meaning that participation in social security yields only level effects on the life-cycle consumption path of a hyperbolic consumer.[8] These recent findings have come as a surprise to many researchers (myself included) in the fields of behavioral economics and pensions.

My findings are orthogonal to the recent literature that social security unequivocally fails to be justified by the hyperbolic discounting representation of impulsivity. For purely expositional purposes, I depict this contrast in Figure 1 for the two alternative representations of impulsivity discussed here, my dual selves representation and hyperbolic discounting, where both types of consumers share otherwise identical features of the model economy. The dashed lines represent the time paths of the savings asset that each respective consumer-type would like to ideally follow from the perspective of age 25, or alternatively, the asset paths that could be followed if internal commitment is hypothetically possible. The solid lines are the time paths for the savings asset that each consumer-type actually generates being subject to impulsivity. What is interesting in Figure 1 is that the hyperbolic consumer intended to accumulate a much higher savings asset balance by retirement, yet this consumer-type fell far short of the goal because of time-inconsistent, impulsive decision making. At first glance, this appears to be a prime example of when a social security (mandatory savings) program might improve well-being through paternalistic commitment. Yet, the hyperbolic consumer is never better off with a social security program because mandatory savings via social security are exactly offset by a reduction in private savings, meaning that the actual life-cycle consumption path (subject to impulsivity) is invariant to the presence of a social security program.[9] The dual selves

[7]Concerning İmrohoroğlu, İmrohoroğlu, and Joines (2003), see specifically Section 3 (pp.758-762) even though they report that an unfunded social security program can sometimes increase welfare in their large-scale computational model with quasi-hyperbolic discounting. More specifically, the existence of any welfare gains from social security participation are not due to the presence of time-inconsistent preferences *per se*, but rather to other features of their quantitative model such as binding borrowing constraints and/or the general-equilibrium determination of prices.

[8]Consistent with the "old paternalism" criterion that is briefly summarized by Cremer and Pestieau (2011), an unfunded social security program can generate improvements to welfare if a paternalistic planner has a lower discount rate than what is possessed by the individual. This implies that the planner prefers an increasing age-consumption profile that is much steeper than what the individual had ever intended. Not only does this criterion for an unfunded social security program ignore the "long-run" preferences of the individual, this paternalistic perspective is problematic given that a steeply increasing age-consumption profile necessitates a larger interest-rate motive for saving. This means that social security will improve welfare only if it triggers significant reductions in the aggregate capital stock and in the national income of an economy.

[9]Such an outcome actually corresponds to the case of a social security program with a net present value that equals zero. For the case where social security has a net present value that is negative, the life-cycle consumption

impulsive consumer in my model also intended on having a higher savings asset balance by the age of retirement than what was actually accumulated due to impulsivity. Yet in contrast to hyperbolic discounting, social security does succeed at restraining the impulsivity of this individual, meaning that the actual life-cycle consumption path moves closer to (and can often overlay) the optimal consumption path in the presence of a mandatory savings program.[10] I now outline the details of the theoretical model of impulsivity in this study.

2 Impulsivity in a Continuous-Time Model

I model a representative individual who intends to follow the optimal consumption rule by setting period consumption equal to the annuity-value of lifetime wealth. Yet, the individual impulsively consumes more than what the optimally planned program actually prescribes. A defining feature of this stylized model is that the individual is free to reoptimize at every instant in time over the life cycle. Such time-inconsistent dynamic optimization is a response to unplanned, impulsive consumption that lowers the value of remaining lifetime resources.

Age is continuous and is indexed by t. The representative individual starts working at $t = 0$, retires at $t = T$, and dies at $t = \bar{T}$. The dates of retirement and death are exogenous. A constant after-tax income flow, $(1 - \theta)w$, is received for all $t \in [0, T]$, where social security taxes are contributed at rate θ with the full burden of taxation resting with the individual. Social security benefits are received at rate b for all $t \in \left[T, \bar{T}\right]$. All disposable income that is not consumed flows into the individual's savings asset, $k(t)$, which grows at the real rate of interest, r. I demonstrate in a later section of the paper that the main findings of this study are robust to a production economy setting where prices are determined endogenously in general equilibrium. Lastly, the representative individual has an exponential discount function, $e^{-\rho(t-t_0)}$, over a discounting delay of $t - t_0$ where ρ is the constant discount rate and where t_0 is the vantage point of decision making.

2.1 Working Phase of the Life Cycle

With the amount of impulsive (unplanned) consumption designated as $\mathcal{I}(t_0)$, the representative individual's composite consumption at any instant $t_0 \in [0, T]$ is defined as

$$c_i(t_0) = c^*(t_0; t_0) + \mathcal{I}(t_0), \tag{1}$$

where $c^*(t; t_0)$ is the consumption program that the individual perceives to be optimal (from the perspective of the planning instant t_0) and intends to follow for all $t \in \left[t_0, \bar{T}\right]$.[11] But since the model is cast in continuous-time, $c^*(t; t_0)$ in fact represents the solution to a continuum

path of the hyperbolic consumer is everywhere lower at any point in the life cycle, meaning that social security participation makes the hyperbolic consumer strictly worse off.

[10] The hyperbolic consumer is always more *impatient* than the impulsive consumer at the parameterization depicted by Figure 1. This is because the area under the hyperbolic function is less than the area under the exponential discount function of the impulsive consumer for all ages. See Myerson, Green, and Warusawitharana (2001) and Green and Myerson (2010) for more on the idea that the area under a discount function is inversely related to overall impatience.

[11] The model is general enough to examine impulses at discrete intervals. Namely, $c_i(t_0) = c^*(t_0; t_0) + \xi\mathcal{I}(t_0)$ given the indicator variable $\xi \in \{0, 1\}$.

of interior optimal control problems that the time-inconsistent individual solves at each and every mass-zero planning instant in time on $[0, T]$, where just one of the infinitely many control problems that is solved is specified as

$$\max \int_{t_0}^{\bar{T}} e^{-\rho(t-t_0)} u[c(t)]\, dt, \tag{2}$$

subject to

$$\frac{dk(t)}{dt} = rk(t) + (1-\theta)w - c(t), \text{ for } t \in [t_0, T], \tag{3}$$

$$\frac{dk(t)}{dt} = rk(t) + b - c(t), \text{ for } t \in \left[T, \bar{T}\right], \tag{4}$$

$$k\left(t_0; [\mathcal{I}(v)]_0^{t_0}\right) = \int_0^{t_0} \left[(1-\theta)w - c^*(v;v) - \mathcal{I}(v)\right] e^{r(t_0-v)} dv, \tag{5}$$

$$k(\bar{T}) = 0, \tag{6}$$

where $u[c(t)]$ is the instantaneous utility function with the properties $u_c[c(t)] > 0$ and $u_{cc}[c(t)] < 0$ and where v is a dummy variable of integration. As denoted by (5), the initial condition on this interior control problem reflects the fact that the optimal solution $c^*(t; t_0)$ for $t \in \left[t_0, \bar{T}\right]$ will actually account for all of the past impulsivity on the interval $[0, t_0]$. Equations (2)–(6) represent just one of the many interior control problems that is solved at each planning instant $t_0 \in [0, T]$. Such time-inconsistent dynamic optimization is the result of the fact that the representative individual fails to follow through with optimally formulated plans due to experiencing an unplanned impulse to spend $\mathcal{I}(\cdot)$ in violation of optimal intentions. This means that time-inconsistent, impulsive behavior is not anticipated by assumption.[12]

The dynamic optimization problem given by (2)–(6) can be solved via the *Maximum Principle* for two-stage optimal control problems with a fixed-endpoint condition.[13] The maximum condition, costate equations, and switchpoint condition will together yield

$$e^{-\rho(t-t_0)} u_c[c(t)] = \lambda(t_0) e^{r(t_0-t)}, \tag{7}$$

given a continuously differentiable costate variable $\lambda(t)$, where $\lambda(t_0)$ is a constant of integration. Given the properties of the instantaneous utility function, the marginal utility of consumption, $u_c[c(t)]$, is a one-to-one mapping from consumption. Therefore, $u_c[c(t)]$ has an inverse $u_c^{-1}[c(t)]$, and the planned consumption path is

$$c^*(t; t_0) = u_c^{-1}\left[\lambda(t_0) e^{(\rho-r)t + (r-\rho)t_0}\right], \text{ for } t \in [t_0, \bar{T}], \tag{8}$$

from the perspective of any planning instant $t_0 \in [0, T]$. Combining (8) with (3)–(6) yields the intended asset path

$$k(t) = k\left(t_0; [\mathcal{I}(v)]_0^{t_0}\right) e^{r(t-t_0)} + \int_{t_0}^{t} \left((1-\theta)w - u_c^{-1}\left[\lambda(t_0) e^{(\rho-r)v + (r-\rho)t_0}\right]\right) e^{r(t-v)} dv, fort \in [t_0, T], \tag{9}$$

[12] The modeling of impulsive behavior, meaning the divergence between intentions and actual behavior, requires that individuals naively fail to account for their own time inconsistency. See O'Donoghue and Rabin (1999b, 1999c, 2000, 2003), Caillaud and Jullien (2000), Prelec (2004), Beshears, Choi, Laibson, and Madrian (2008), D'Orlando and Sanfilippo (2010), and Herweg and Müller (2011) for more on this idea.

[13] See Tomiyama (1985) for more on the technical details of two-stage optimal control problems.

$$k(t)=\int_{\bar{T}}^{t}\left(b-u_c^{-1}\left[\lambda(t_0)e^{(\rho-r)v+(r-\rho)t_0}\right]\right)e^{r(t-v)}dv,\text{ for }t\in\left[T,\bar{T}\right],\tag{10}$$

from the perspective of any $t_0 \in [0,T]$. Evaluating (9) and (10) at $t=T$ and then equating yields

$$k\left(t_0;[\mathcal{I}(v)]_0^{t_0}\right)e^{-rt_0}+\int_{t_0}^{T}(1-\theta)we^{-rv}dv+\int_{T}^{\bar{T}}be^{-rv}dv=\int_{t_0}^{\bar{T}}u_c^{-1}\left[\lambda(t_0)e^{(\rho-r)v+(r-\rho)t_0}\right]e^{-rv}dv\tag{11}$$

which definitizes $\lambda(t_0)$ in (8).

$c^*(t;t_0)$, as given by (8) with $\lambda(t_0)$ identified, is the consumption program that the individual perceives to be optimal and intends to follow for all $t\in\left[t_0,\bar{T}\right]$ while standing from the perspective of $t_0\in[0,T]$. Yet, a defining feature of the model is that the individual also experiences a consumption impulse wherein he consumes an additional unplanned impulse amount $\mathcal{I}(t_0)$. Therefore, the actual consumption of the individual at the planning moment t_0 is denoted by (1) where $c^*(t_0;t_0)$ is the result of evaluating $t=t_0$ in (8). Yet, it should be mentioned again that t_0 represents any arbitrary planning point in time on the interval $[0,T]$. This suggests that the actual consumption of the individual for all $t\in[0,T]$ can be found by replacing all t_0 in (1) with t. This gives

$$c_i(t)=c^*(t;t)+\mathcal{I}(t),\text{ for }t\in[0,T],\tag{12}$$

where

$$c^*(t;t)=u_c^{-1}\left[\lambda(t)\right]\tag{13}$$

is the result of setting $t_0=t$ in (8) given $\lambda(t)$ which solves (11) with $t_0=t$,

$$k\left(t;[\mathcal{I}(v)]_0^{t}\right)e^{-rt}+\int_{t}^{T}(1-\theta)we^{-rv}dv+\int_{T}^{\bar{T}}be^{-rv}dv=\int_{t}^{\bar{T}}u_c^{-1}\left[\lambda(t)e^{(\rho-r)v+(r-\rho)t}\right]e^{-rv}dv.\tag{14}$$

It is important to recognize that (12) identifies the individual's actual consumption at each and every age $t\in[0,T]$. But it is also important to recognize that this expression accounts for the entire history of impulsivity on the interval $[0,t]$ during the working years via (13) and (14). As such, consumption at time t is in fact a function of the actual savings asset at time t, which in turn is a function of the time path of actual consumption on the interval $[0,t]$ via

$$\frac{dk(t)}{dt}=rk(t)+(1-\theta)w-c_i(t).\tag{15}$$

Therefore, to completely identify the actual consumption of the representative individual at each and every age $t\in[0,T]$, the system of equations needs to be solved: (12) given (13) and (14), along with (15) given $k(0)=0$.

2.2 Retirement Phase of the Life Cycle

The objective of this study is to examine whether or not a social security (mandatory savings) program can mitigate the adverse effects of impulsive spending on the accumulation of assets for retirement. For purposes of tractability (and in addition to the fact that a detailed examination of impulsive spending by retirees on the decumulation of retirement assets is outside the scope of this study and corresponding literature), I assume that the representative individual adheres to the optimal consumption and saving rule during the retirement phase. This suggests that the individual consumes the annuity-value of remaining lifetime resources.[14] The actual consumption path during retirement is therefore time-consistent, and it can be found by setting $t_0 = T$ in (8) and (11),

$$c_i(t) = u_c^{-1}\left[\lambda(T)e^{(\rho-r)t+(r-\rho)T}\right], \text{ for } t \in [T, \bar{T}], \tag{16}$$

where

$$k\left(T; [\mathcal{I}(v)]_0^T\right)e^{-rT} + \int_T^{\bar{T}} be^{-rv}dv = \int_T^{\bar{T}} u_c^{-1}\left[\lambda(T)e^{(\rho-r)v+(r-\rho)T}\right]e^{-rv}dv \tag{17}$$

identifies $\lambda(T)$ in (16).

2.3 Characteristics of Social Security in the Model

Two alternative arrangements for financing a social security program will be examined in this study: a fully-funded (actuarially fair) program and a pay-as-you-go program. Under fully-funded financing of social security in the model, a mandatory savings account is operated by government in which a constant benefit is received during retirement,

$$b_f = \frac{\int_0^T \theta we^{-rt}dt}{\int_T^{\bar{T}} e^{-rt}dt}, \text{ for } t \in [T, \bar{T}], \tag{18}$$

noting that fully-funded mandatory savings earn interest at the market rate, r.

Aggregate benefits received by retirees must equal aggregate taxes collected in an unfunded or pay-as-you-go arrangement. This is represented as $(\bar{T} - T)b = \theta wT$, where the length of the working phase is the number of workers and the length of the retirement phase is the number of retirees in the economy, assuming a stationary population. Solving for b gives the pay-as-you-go benefits rule,

$$b_p = \theta wT/(\bar{T} - T), \text{ for } t \in [T, \bar{T}]. \tag{19}$$

As a basic reminder, the internal rate of return on the pay-as-you-go arrangement in the model is equal to zero, since the rates of wage growth and population growth are both equal to zero by assumption. Therefore, the pay-as-you-go arrangement is actuarially unfair (negative net present value), unless the interest rate is set to zero in the model economy, resulting in $b_p = b_f$.

3 Impulsivity and Social Security

3.1 Stylized, Simplifying Assumptions

In the previous section of the paper, the model is left in general form. Here, I impose some stylized assumptions on the model in order to analytically derive some key theoretical findings

[14]Such an assumption is consistent with a small body of empirical evidence which indicates that retirees smooth consumption after an initial downward correction at retirement (Banks, Blundell, and Tanner 1998; Bernheim, Skinner, and Weinberg 2001; Browning and Crossley 2001).

concerning the ability of a social security (mandatory savings) program to restrain impulsivity. I relax these assumptions in later sections of the manuscript, and I demonstrate quantitatively that the principal findings of this study remain intact. First, I set $\rho = r$ which implies that the intended growth rate of consumption is equal to zero. This assumption centers the analysis in this section of the paper on the well-known case where consumption smoothing is optimal. And perhaps more importantly, this assumption means that the theoretical findings regarding the optimality of social security will generalize to any utility function with the properties $u_c[c(t)] > 0$ and $u_{cc}[c(t)] < 0$. In Section ???, I show that the main findings of this section are robust when $\rho \gtrless r$, even in a general-equilibrium setting with productive labor and capital.

Second, I set $r = 0$ for analytical convenience, although I show in a subsequent section that the findings of this study are robust to exogenously-imposed and endogenously-determined parameterizations where $r > 0$. Here, the assumption of $r = 0$ isolates the mandatory saving role of social security in the model by controlling for inefficiencies in financing the program (i.e., $b_p = b_f$ such that the pay-as-you-go financing scheme has a zero net present value). But more importantly, the assumption of $r = 0$ allows me to examine in a tractable manner whether or not a social security program can successfully act as a commitment device that restrains impulsivity. As a reminder to the reader, a commitment device is generally considered to be successful if it assists (commits) an individual to follow through with initial intentions. In the context of the model in this section of the manuscript, a commitment device performs perfectly if it helps an impulsive consumer to achieve a perfectly smooth consumption path over the entire life cycle, given that the very first plan of the impulsive consumer prescribes following the optimal consumption rule.

As a benchmark of comparison, recall that an actuarially fair social security program fails at providing any type of commitment to counter the impulsive spending that results from hyperbolic discounting. In other words, it is impossible for a mandatory savings program to improve the well-being of a hyperbolic consumer, since the actual life-cycle consumption path is unchanged across counterfactual states of the world with respect to the existence of a program. This is due to the fact that the hyperbolic consumer perfectly offsets the flow of mandatory savings via social security with an exact reduction in the flow of private savings (e.g., İmrohoroğlu, İmrohoroğlu, and Joines 2003; Caliendo 2011, 2013).

Third, I specify some particular functional forms for $\mathcal{I}(t)$. Two easy, alternative ways to think about and represent impulsive spending are

$$\mathcal{I}(t) \equiv \psi \left[(1-\theta)w - c^*(t;t) \right], \tag{20}$$

and

$$\mathcal{I}(t) \equiv \mu(1-\theta)w. \tag{20$'$}$$

According to (20) a fraction $\psi \geq 0$ of the intended (optimal) savings flow is consumed when the individual is impulsive, and according to (20$'$) a fraction $\mu \geq 0$ of the disposable wage is impulsively consumed. Now it should be mentioned that (20) and (20$'$) are not the only ways to mathematically represent $\mathcal{I}(t)$, yet they are simple and intuitive. However, a potential complication might occur when using (20), given that a subset of the parameter space on $r - \rho$ exists such that the individual will impulsively borrow less (save more) than what was intended.[15] Yet to analytically provide some intuition about impulsivity and social security, in

[15] The case of impulsive borrowing can still be examined if $\psi > 1$.

this section I have imposed $\rho = r$ which focuses the analysis on the prominent case in which consumption smoothing is optimal. This circumvents the potential complication of impulsive saving when (20) is used. In contrast, there are no complications when (20′) is employed as the definition of impulsivity, which allows me to study (in later sections) whether or not social security can improve well-being for the important cases of $\rho \gtrless r$ and when the individual can readily borrow.

Compared to hyperbolic discounting where impulsivity is modeled as an *inter*-temporal conflict between preferences from different vantage points over the life cycle, impulsivity in this model reflects an *intra*-temporal tension between following optimally formulated plans and succumbing to impulsive spending. This fits within the general context of a naive "dual self". Indeed, with (20) actual consumption can be arranged as $c_i(t) = (1-\psi)c^*(t;t) + \psi(1-\theta)w$ for $t \in [0,T]$, suggesting that an impulsive consumer is a convex combination at a moment in time between optimal consumption ($\psi = 0$) and hand-to-mouth consumption ($\psi = 1$). With (20′) actual consumption is $c_i(t) = c^*(t;t) + \mu(1-\theta)w$ for $t \in [0,T]$, meaning that an individual consumes an irrational fraction μ of disposable wages in addition to the amount of disposable wages that is consumed by following the optimal rule.

3.2 Analytical Findings

Proposition 1, Corollary 2, and Proposition 2 address the case when impulsivity defined with (20). Alternatively, Corollaries 1, 3, and 4 correspond to the case when impulsivity is defined by (20′).

Proposition 1. *Given* $c_i(t) = c^*(t;t) + \psi\left[(1-\theta)w - c^*(t;t)\right]$ for $t \in [0,T]$, *a social security (mandatory savings) program reallocates resources over the life cycle by restraining impulsive consumption during the working phase and by increasing consumption during the retirement phase.*

Proof. Given the assumptions outlined above, write the analytical expressions for the actual consumption and savings asset paths upon solving the system of equations, (12) given (13) and (14) with (15) given $k(0) = 0$, in addition to (16) and (17),

$$\begin{aligned} c_i(t) &= \frac{(1-\psi)\left\{k\left(t;[\mathcal{I}(v)]_0^t\right) + \int_t^T (1-\theta)w dv + \int_T^{\bar{T}} b dv\right\}}{\bar{T}-t} + \psi(1-\theta)w \\ &= z(t)k\left(t;[\mathcal{I}(v)]_0^t\right) + z(t)q(t) + \psi(1-\theta)w, \text{ for } t \in [0,T], \end{aligned} \tag{21}$$

$$k\left(t;[\mathcal{I}(v)]_0^t\right) = (1-\psi)(1-\theta)w\int_0^t e^{\int_t^v (1-\psi)(\bar{T}-j)^{-1}dj}dv - \int_0^t z(v)q(v)e^{\int_t^v (1-\psi)(\bar{T}-j)^{-1}dj}dv, \text{ for } t \in [0,T], \tag{22}$$

$$c_i(t) = \frac{k\left(T;[\mathcal{I}(v)]_0^T\right) + \int_T^{\bar{T}} b dv}{\bar{T}-T}, \text{ for } t \in [T,\bar{T}]. \tag{23}$$

where $k\left(T;[\mathcal{I}(v)]_0^T\right)$ is identified by evaluating (22) at $t = T$, and where

$$z(t) \equiv \frac{1-\psi}{\bar{T}-t}, \qquad q(t) \equiv \int_t^T (1-\theta)w dv + \int_T^{\bar{T}} b dv = w(T-t) + \theta w t.$$

As outlined in Appendix A, rewrite (22) as (22′)

$$\begin{aligned} k(t;\cdot) &= \frac{(1-\psi)(1-\theta)w}{(\bar{T}-t)^{\psi-1}}\left(\frac{\bar{T}^{\psi}-(\bar{T}-t)^{\psi}}{\psi}+\frac{t(\bar{T}-t)^{\psi-1}}{1-\psi}+\frac{(\bar{T}-t)^{\psi}-\bar{T}^{\psi}}{\psi(1-\psi)}\right) \\ &\quad + wT\left[\left(\frac{\bar{T}-t}{\bar{T}}\right)^{1-\psi}-1\right], \text{ for } t\in[0,T], \end{aligned} \tag{22′}$$

where the notation for the history dependence of impulsivity has been suppressed.

Differentiate (21) with respect to the social security tax rate,

$$\frac{\partial c_i(t)}{\partial\theta} = z(t)\frac{\partial k(t;\cdot)}{\partial\theta} + z(t)\frac{\partial q(t)}{\partial\theta} - \psi w, \text{ for } t\in[0,T], \tag{24}$$

where

$$\frac{\partial q(t)}{\partial\theta} = wt, \text{ for } t\in[0,T], \tag{25}$$

and where

$$\frac{\partial k(t;\cdot)}{\partial\theta} = -\left(\frac{\bar{T}^{\psi}-(\bar{T}-t)^{\psi}}{\psi}+\frac{t(\bar{T}-t)^{\psi-1}}{1-\psi}+\frac{(\bar{T}-t)^{\psi}-\bar{T}^{\psi}}{\psi(1-\psi)}\right)\frac{w(1-\psi)}{(\bar{T}-t)^{\psi-1}}, \text{ for } t\in[0,T]. \tag{26}$$

Evaluate (24) at $t=0$,

$$\left.\frac{\partial c_i(t)}{\partial\theta}\right|_{t=0} = -\psi w < 0, \tag{27}$$

which denotes that social security restrains the initial consumption of an impulsive consumer. Now, insert (25) and (26) into (24) and algebraically rearrange,

$$\frac{\partial c_i(t)}{\partial\theta} = w\left[(1-\psi)\left(\frac{\bar{T}}{\bar{T}-t}\right)^{\psi}-1\right], \text{ for } t\in[0,T]. \tag{28}$$

Set (28) equal to zero and solve for t, the unique age during the working phase at which there is an intersection in the consumption profiles across counterfactual states of the model with respect to the existence of a social security program,

$$t_x = \bar{T}\left[1-(1-\psi)^{\frac{1}{\psi}}\right]. \tag{29}$$

Note that the existence of a social security program restrains consumption for all $t\in[0,t_x)$ during the working phase of the life cycle, given that the sign of (27) is negative and also given the fact that t_x is unique.

Focusing on the retirement phase of the life cycle, differentiate (23) with respect to the social security tax rate,

$$\frac{\partial c_i(t)}{\partial\theta} = \left(\frac{1}{\bar{T}-T}\right)\frac{\partial k(T;\cdot)}{\partial\theta} + \frac{wT}{\bar{T}-T}, \text{ for } t\in[T,\bar{T}], \tag{30}$$

given $b = \int_0^T \theta w dv \Big/ \int_T^{\bar{T}} dv$ and given

$$\frac{\partial k(T;\cdot)}{\partial \theta} = -\left(\frac{\bar{T}^{\psi} - (\bar{T}-T)^{\psi}}{\psi} + \frac{T(\bar{T}-T)^{\psi-1}}{1-\psi} + \frac{(\bar{T}-T)^{\psi} - \bar{T}^{\psi}}{\psi(1-\psi)} \right) \frac{w(1-\psi)}{(\bar{T}-T)^{\psi-1}}. \tag{31}$$

Insert (31) into (30) and algebraically rearrange,

$$\frac{\partial c_i(t)}{\partial \theta} = w \left(\frac{\bar{T}^{\psi} - (\bar{T}-T)^{\psi}}{(\bar{T}-T)^{\psi}} \right) > 0, \text{ for } t \in [T, \bar{T}]. \tag{32}$$

Therefore, social security increases consumption all throughout the retirement phase of the life cycle. ■

Corollary 1. *Given* $c_i(t) = c^*(t;t) + \mu(1-\theta)w$ for $t \in [0, T]$, *a social security (mandatory savings) program reallocates resources over the life cycle by restraining impulsive consumption during the working phase and by increasing consumption during the retirement phase.*

Proof. See Appendix B. ■

Proposition 2. *Given* $c_i(t) = c^*(t;t) + \psi\left[(1-\theta)w - c^*(t;t)\right]$ for $t \in [0, T]$, s*ocial security successfully commits (paternalistically) and reallocates consumption over the life cycle for any degree of impulsivity, large or small.*

Proof. Solve the inequality in (27) for ψ, which yields $\psi > 0$. This indicates that initial consumption is restrained by social security if impulsivity exists at any degree. Now insert (31) into (30) and rewrite given the strict inequality,

$$\frac{(1-\psi)(\bar{T}-T)^{1-\psi}}{T} \left(\frac{\bar{T}^{\psi} - (\bar{T}-T)^{\psi}}{\psi} + \frac{T(\bar{T}-T)^{\psi-1}}{1-\psi} + \frac{(\bar{T}-T)^{\psi} - \bar{T}^{\psi}}{\psi(1-\psi)} \right) < 1 \tag{33}$$

Distribute inside the parentheses on the left-hand side of the inequality in (33) and rewrite further,

$$\frac{(1-\psi)(\bar{T}-T)^{1-\psi}}{T} \left(\frac{\bar{T}^{\psi} - (\bar{T}-T)^{\psi}}{\psi} \right) - \frac{(\bar{T}-T)^{1-\psi}}{T} \left(\frac{\bar{T}^{\psi} - (\bar{T}-T)^{\psi}}{\psi} \right) < 0. \tag{34}$$

This simplifies to $\psi > 0$, meaning that consumption will be everywhere higher during retirement in a state of the world with social security if the individual possesses any degree of impulsivity. ■

Corollary 2. *Given* $c_i(t) = c^*(t;t) + \mu(1-\theta)w$ for $t \in [0, T]$, s*ocial security successfully commits (paternalistically) and reallocates consumption over the life cycle for any degree of impulsivity, large or small.*

Proof. See Appendix B. ■

Proposition 3. *Given* $c_i(t) = c^*(t;t) + \psi\left[(1-\theta)w - c^*(t;t)\right]$ for $t \in [0, T]$, a *social security program can be linearly parameterized to perfectly restrain impulsivity such that optimal consumption over the life cycle is replicated and maximal utility is attained.*

Proof. Set $t_0 = 0$ in (8) and (11), which gives the optimal consumption path in a state of the world where mandatory savings in unnecessary due to the absence of impulsivity. Given the additional assumptions here, this can be written as

$$c^*(t)|_{\theta=0} = wT/\bar{T}, \text{ for } t \in [0, \bar{T}]. \tag{35}$$

This consumption program is time-consistent and it achieves maximal life-cycle utility. The savings rate that corresponds to this optimal state is

$$\begin{aligned} s^* &= \frac{w - c^*(t)|_{\theta=0}}{w} \\ &= (\bar{T} - T)/\bar{T}. \end{aligned} \tag{36}$$

Set impulsive consumption in a world with mandatory savings equal to optimal consumption when mandatory savings in unnecessary because of no impulsivity, meaning set $c_i(t)|_{\theta>0} = c^*(t)|_{\theta=0}$, or

$$z(t)k(t;\cdot) + z(t)q(t) + \psi(1-\theta)w = wT/\bar{T}. \tag{37}$$

Make the appropriate substitutions into (37), algebraically simplify, and then solve for the tax rate that holds at this equality,

$$\theta^* = (\bar{T} - T)/\bar{T}. \tag{38}$$

Note that (38) is exactly equal to the optimal savings rate in a state of the world where mandatory savings is unnecessary due to the absence of impulsivity. Therefore, a social security (mandatory savings) program can be parameterized on behalf of impulsive consumers to replicate optimal consumption and to realize maximal life-cycle utility, meaning the level of utility attained by a non-impulsive individual who consumes optimally over the entire life cycle. ■

Corollary 3. *Given* $c_i(t) = c^*(t;t) + \mu(1-\theta)w$ for $t \in [0, T]$, a *social security program can be linearly parameterized to perfectly restrain impulsivity such that optimal consumption over the life cycle is replicated and maximal utility is attained.*

Proof. See Appendix B. ■

3.3 Discussion

The ability of a social security (mandatory savings) program to commit consumption resources to retirement and to improve the well-being hinges on the fact that it needs to have a material effect on the consumption impulse, meaning $d\mathcal{I}(t)/d\theta < 0$ is needed. It is easy to verify that this holds for (20) and (20′). Therefore, what is important is to see whether or not social security improves well-being depends on whether people impulsively spend from their savings flow or from their disposable income flow. If these are reasonable sources of funds to finance impulsive spending, then social security has the ability to reduce these sources of impulsive spending and then return them during retirement.

4 Numerical Exercises and Robustness

4.1 Parameter Values and Representative Life-Cycle Consumption Profiles

The theoretical findings above denote that an actuarially fair mandatory savings program can successfully act as an external commitment device to increase the well-being of impulsive consumers. Indeed, such a program can be parameterized on behalf of impulsive consumers to replicate the level of utility attained by individuals who consume optimally. In this section, I numerically examine the robustness of these results to different assumptions about the financing of the mandatory savings program and to assumptions about how well-being is evaluated. As a preview, the principal findings of this study are generally robust.

Unless stated otherwise, I assume the following parameter values in the numerical exercises below. I set $T = 40$ and $\bar{T} = 55$ in order to represent an individual who starts work at age 25, retires at 65, and passes away at 80. This implies a worker-to-retiree ratio of 2.7, which is only relevant when examining the case of a pay-as-you-go social security program. A ratio of 2.7 is close to the current value of approximately 3 in the United States (Goss 2010). I entertain values for the real rate of return, r, from a range of 0 to 2 percent. I assume a normalization of $w = 1$ since alternative numerical values have only scale or level effects on consumption and saving. Lastly, I assume $u[c(t)] = \ln c(t)$ given empirical evidence and convention (Attanasio 1999). This assumption that period utility is logarithmic is for purposes of numerical demonstration only.

For purely expositional purposes, I numerically simulate some representative age-consumption profiles that are depicted in Figure 2. The interest rate is set at $r = 0.02$ in this depiction. The dashed line is the consumption path that a non-impulsive consumer optimally follows over the life cycle. The thick solid line is the profile of an impulsive consumer ($\psi = 0.4$) who participates in an actuarially fair social security program at a tax rate of 10.6 percent. And lastly, the thin solid line is the age-consumption profile of an otherwise identical impulsive consumer who does not participate in social security. It is clear from Figure 2 that social security restrains impulsivity by making consumption increasingly more "smooth", meaning that social security reduces impulsive consumption during the working phase and increases consumption during retirement. Indeed, an actuarially fair social security program could be parameterized such that the life-cycle consumption path of the impulsive consumer will exactly overlay the consumption path that the non-impulsive consumer optimally follows (see Proposition 2 above). Such optimal consumption smoothing is achieved via social security participation at a tax rate of 17.5 percent, given the underlying values for the other parameters in the model.

4.2 A Counterexample

4.3 Value of Social Security Participation

For general purposes, the social security tax rate that maximizes the lifetime utility of an impulsive consumer is defined as

$$\theta^* \equiv \underset{\theta \in [0,1]}{\arg\max} \left\{ \int_0^{\bar{T}} e^{-\rho t} u[c_i(t)] dt \right\}, \tag{39}$$

where the private discount rate is also the discount rate used by a social planner in evaluating true lifetime well-being.[16,17] It should be mentioned that the measurement of lifetime well-being in (39) is consistent with the statement by George Akerlof in his Nobel Prize acceptance lecture (quoted at the beginning of this manuscript), namely "that individuals may maximize a utility function that is divorced from that representing true welfare". It should also be noted that (38) is a special case of (39) given the simplifying assumptions in Section 3.

I also define a "compensating variation" welfare metric that quantifies the value of social security participation for impulsive consumers. The compensating variation is the particular value of Γ that solves the following equation,

$$\int_0^{\bar{T}} e^{-\eta t} u \left[(1+\Gamma) \; c_i(t)|_{\theta=0} \right] dt = \int_0^{\bar{T}} e^{-\eta t} u \left[c_i(t)|_{\theta=\theta^*} \right] dt. \tag{40}$$

The right-hand side of (40) is the lifetime utility of an impulsive consumer who participates in a social security program subject to (39), and the left-hand side of (40) is the lifetime utility of an otherwise identical impulsive consumer who does not participate in social security. The compensating variation, Γ, measures the percentage increase in period consumption that would need to be given to the impulsive consumer who does not participate in social security in order to raise his lifetime utility to that of the impulsive consumer who does participate at the optimal rate of taxation. Thus, the compensating variation measures a consumption-monetized value for optimal participation by impulsive consumers in a social security program.

I report in Table 1 the computed values for the optimal tax rate and for the compensating variation given different degrees of impulsivity, ψ, and for different assumptions about the interest rate, r, and the social discount rate, ρ. Note that results reported in Table 1 correspond to the case of a fully-funded (actuarially fair) social security program, meaning that the net present value of social security participation is equal to zero. The top row of Table 1 exactly corresponds to the result in Proposition 2 above, where $\theta^* = (\bar{T} - T)/\bar{T} = s^* = 27.3\%$ since the

[16] For more on paternalism and welfare in behavioral economic analysis, see Samuelson (1975), Harsanyi (1977), Feldstein (1985), Kahneman, Wakker, and Sarin (1997), O'Donoghue and Rabin (1999a), Akerlof (2002), Docquier (2002), Kanbur, Pirttilä, and Tuomala (2006), Rubinstein (2006), Hurst and Willen (2007), Cremer, De Donder, Maldonado, and Pestieau (2008, 2009), Pestieau and Possen (2008), Cremer and Pestieau (2011), Boadway (2012), and Winter, Schlafmann, and Rodepeter (2012), and Love (2013), among many others.

[17] A social discount rate of $\rho > 0$ is inconsistent with the criterion of Ramsey (1928) which precludes social discounting of the future on the basis of pure time preference. Ramsey describes such practices as "ethically indefensible" that results from a "failure of the imagination". In this paper I assume that there is no difference between the private and social rates of time preference, in order to see if a mandatory savings program can be justified with the "New Paternalism" criterion.

interest rate equals zero in this case (this result is invariant to whether a paternalistic planner discounts or not). For cases of $r > 0$ while preserving actuarial fairness of the program, it still happens to be the case numerically that $\theta^* = s^*$, although the exact mathematical form of θ^* has not yet been identified analytically. It is evident from Table 1 that participation in an actuarially fair social security program can yield significant increases in the well-being of impulsive consumers, as measured by the compensating variation. Moreover, improvements in lifetime well-being exist for all degrees of impulsivity, which is consistent with the Corollary in Section 3 above. And lastly, the measured increases in well-being are predictably smaller for cases where $\rho > 0$.

I also report the optimal tax rate and compensating variation for the case of participation in an unfunded or pay-as-you-go social security program. These values are reported in Table 2. It should again be mentioned that the internal rate of return on the unfunded arrangement equals zero in the model on account that the rates of wage growth and population growth are both equal to zero. As such, this financing arrangement has a negative net present value (actuarially unfair) unless the interest rate equals zero in the model economy, as reflected by the fact that the top row in Table 2 is identical to the corresponding top row in Table 1 for the case of actuarial fairness. Yet, what is of particular interest here are the cases of $r > 0$, more specifically when a program with a negative net present value is still able to increase the well-being of impulsive consumers. Indeed, the findings in Table 2 indicate that participation in social security with inefficiencies in the actuarial financing of the program can still be valuable if the degree of impulsivity is large enough, despite the fact that the unfunded program in the model becomes more inefficient (the net present value becomes more negative) as r increases. The results in Table 2 contrast starkly with the case of hyperbolic discounting, where participation in an actuarially unfair social security program always lowers lifetime well-being regardless of the degree of time inconsistency.

5 Robustness

I now examine the robustness of the simple quantitative findings of the previous section to two additional assumptions. First, I examine the possibility that consumption impulses may be experienced randomly. Second, I study a competitive general-equilibrium setting with productive labor and capital.

5.1 Random Consumption Impulses

Until now, it has been assumed for analytical convenience that the consumption impulse is continuously experienced during the working phase. Here, I entertain the possibility that consumption impulses may be experienced randomly. Actual consumption is $c_i(t_0) = c^*(t_0; t_0) + \xi \mathcal{I}(t_0)$ where the indicator ξ is distributed randomly according to a Bernoulli distribution. The mass function of this simple distribution is $m(n; x) = x^n(1-x)^{1-n}$ for support $n \in \{0, 1\}$ with a probability of success, x.

5.2 General-Equilibrium Determination of Prices and an Alternative Impulse

The total stock of workers in the model economy is equal to the length of the work phase,

$$L = T.$$

The aggregate demand for capital is

$$K = \int_0^{\bar{T}} k(t)\ dt + \int_0^{\bar{T}} B(t)\ dt$$

where $k(t)$ is the life-cycle savings profile and where $B(t) = \int_0^t \theta w e^{r(t-v)} dv$ for $t \in [0, T]$ and $B(t) = \int_t^{\bar{T}} b_f e^{r(t-v)} dv$ for $t \in [T, \bar{T}]$ is the life-cycle profile of the individual's social security account if government is operating a fully-funded program.[18] The life-cycle profiles for private and mandatory savings can be treated as the cross-sectional profiles for private and government claims on capital since the rate of economic growth is zero in the model by assumption. Aggregate output or income is produced by a constant-returns-to-scale technology of the form

$$Y = K^\alpha L^{1-\alpha}$$

where α is the constant share of capital in national income. Factors of production are priced competitively

$$r = \frac{\partial Y}{\partial K} - \delta = \alpha Y/K - \delta,$$
$$w = \frac{\partial Y}{\partial L} = (1-\alpha)Y/L,$$

given a rate of physical capital depreciation, δ.

6 Summary

A recent literature reports that it is impossible for a social security (mandatory savings) program to improve the well-being of impulsive consumers, if the method of modeling impulsivity is that of hyperbolic discounting. Based on an alternative way of conceptualizing impulsivity in psychology, I demonstrate that a "dual selves" version of impulsivity can succeed at justifying the existence of a social security program. Indeed, I analytically show that a social security program can act as a perfect commitment device that successfully restrains impulsive spending and helps such consumers to attain maximal lifetime utility (the level of utility achieved by non-impulsive consumers who follow the optimal consumption and saving rule). This improvement in well-being results for any degree of impulsivity, large or small. I also demonstrate numerical robustness to an alternative assumption that inefficiencies exist in the financing of a social

[18] Under fully-funded financing, the social security account operated by government on behalf of the individual evolves according to the following laws of motion: $dB(t)/dt = rB(t) + \theta w$ for $t \in [0, T]$ and $dB(t)/dt = rB(t) - b_f$ for $t \in [T, \bar{T}]$. Solving the boundary-value problem for $B(0) = 0$ and $B(\bar{T}) = 0$ yields the time path for $B(t)$. Of course, $B(t) = 0$ for all t if government is operating a pay-as-you-go program.

security program. In total, this study provides a theoretical basis for the primary justification of a social security program, namely that of countering insufficient saving for retirement due to behavioral impulsivity.

Appendix A

Suppressing the notation for the history dependence of impulsivity, rewrite (22) with integration

$$k(t;\cdot) = (1-\psi)(1-\theta)w \int_0^t e^{(1-\psi)\left(\ln[\bar{T}-t]-\ln[\bar{T}-v]\right)} dv - \int_0^t z(v)q(v)e^{(1-\psi)\left(\ln[\bar{T}-t]-\ln[\bar{T}-v]\right)} dv. \quad \text{(A1)}$$

Substitute in for $z(v)$ and $q(v)$ and algebraically simplify

$$\begin{aligned} k(t;\cdot) &= \frac{(1-\psi)(1-\theta)w}{(\bar{T}-t)^{\psi-1}} \left(\int_0^t (\bar{T}-v)^{\psi-1} dv + \int_0^t v(\bar{T}-v)^{\psi-2} dv \right) \\ &+ \frac{(1-\psi)wT}{(\bar{T}-t)^{\psi-1}} \int_t^0 (\bar{T}-v)^{\psi-2} dv. \end{aligned} \quad \text{(A2)}$$

Now, rewrite using integration by parts

$$\begin{aligned} k(t;\cdot) &= \frac{(1-\psi)(1-\theta)w}{(\bar{T}-t)^{\psi-1}} \left(\int_0^t (\bar{T}-v)^{\psi-1} dv + \frac{t(\bar{T}-t)^{\psi-1}}{1-\psi} - \frac{1}{1-\psi} \int_0^t (\bar{T}-v)^{\psi-1} dv \right) \\ &+ \frac{(1-\psi)wT}{(\bar{T}-t)^{\psi-1}} \int_t^0 (\bar{T}-v)^{\psi-2} dv. \end{aligned} \quad \text{(A3)}$$

Perform the integration in (A3) to yield (22′).

Appendix B

Proof of Corollary 1. Write the analytical expressions for the actual consumption and savings asset paths upon solving the system of equations, (12) given (13) and (14) with (15) given $k(0) = 0$, in addition to (16) and (17),

$$\begin{aligned} c_i(t) &= \frac{k(t) + \int_t^T (1-\theta)w dv + \int_T^{\bar{T}} b dv}{\bar{T}-t} + \psi(1-\theta)w, \quad \text{for } t \in [0,T], \\ &= \frac{1}{\bar{T}-t}k(t) + \frac{1}{\bar{T}-t}q(t) + \psi(1-\theta)w, \quad \text{for } t \in [0,T], \end{aligned} \quad \text{(A4)}$$

where

$$q(t) \equiv \int_t^T (1-\theta)w dv + \int_T^{\bar{T}} b dv = w(T-t) + \theta w t$$

is used to compress notation. Insert (A4) into $dk(t)/dt = (1-\theta)w - c_i(t)$ and integrate for the initial condition, $k(0) = 0$. This yields

$$\begin{aligned} k(t) &= \int_0^t \left[(1-\psi)(1-\theta)w - q(v)/(\bar{T}-v)\right] e^{\int_t^v (\bar{T}-j)^{-1} dj} dv, \\ &= (\bar{T}-t)(1-\theta)w \left((1-\psi) \ln\left[\frac{\bar{T}}{\bar{T}-t}\right] + \frac{t}{\bar{T}-t} + \ln\left[\frac{\bar{T}-t}{\bar{T}}\right] \right) - \frac{wtT}{\bar{T}}, \quad \text{for } t \in [0,T], \end{aligned} \quad \text{(A5)}$$

Rewrite the retirement consumption profile of an individual who was impulsive during the working years,

$$c_i(t) = \frac{k(T) + \int_T^{\bar{T}} bdv}{\bar{T} - T}, \quad \text{for } t \in [T, \bar{T}]. \tag{A6}$$

Differentiate (A4) with respect to the social security tax rate,

$$\frac{\partial c_i(t)}{\partial \theta} = \left(\frac{1}{\bar{T} - t}\right) \frac{\partial k(t)}{\partial \theta} + \left(\frac{1}{\bar{T} - t}\right) \frac{\partial q(t)}{\partial \theta} - \psi w, \quad \text{for } t \in [0, T], \tag{A7}$$

where

$$\frac{\partial q(t)}{\partial \theta} = wt, \tag{A8}$$

and

$$\frac{\partial k(t)}{\partial \theta} = -w \left((1 - \psi) \int_0^t e^{\int_t^v (\bar{T} - j)^{-1} dj} dv + \int_0^t \frac{v}{\bar{T} - v} e^{\int_t^v (\bar{T} - j)^{-1} dj} dv \right), \tag{A9}$$

or, alternatively

$$\frac{\partial k(t)}{\partial \theta} = -(\bar{T} - t) w \left((1 - \psi) \ln \left[\frac{\bar{T}}{\bar{T} - t}\right] + \frac{t}{\bar{T} - t} + \ln \left[\frac{\bar{T} - t}{\bar{T}}\right] \right). \tag{A9$'$}$$

Evaluate (A7) at $t = 0$,

$$\left.\frac{\partial c_i(t)}{\partial \theta}\right|_{t=0} = -\psi w < 0, \tag{A10}$$

which denotes that social security constrains the initial consumption of an impulsive consumer. Now, insert (A8) and (A9$'$) into (A7) and algebraically rearrange,

$$\frac{\partial c_i(t)}{\partial \theta} = \psi w \left(\ln \left[\frac{\bar{T}}{\bar{T} - t}\right] - 1 \right). \tag{A11}$$

Set (A11) equal to zero and solve for t, the unique age during the working phase at which there is an intersection in the consumption profiles across alternative states of the model with respect to the existence of a social security program,

$$t_x = \bar{T} \left(\frac{e^1 - 1}{e^1} \right). \tag{A12}$$

Note that the existence of a social security program reduces or constrains consumption for all $t < t_x$ during the working phase, since social security reduces initial consumption with (A10) and also given the fact that t_x is unique.

Focusing on the retirement phase of the life cycle, differentiate (A6) with respect to the social security tax rate,

$$\frac{\partial c_i(t)}{\partial \theta} = \left(\frac{1}{\bar{T} - T}\right) \frac{\partial k(T)}{\partial \theta} + \frac{wT}{\bar{T} - T}, \quad \text{for } t \in [T, \bar{T}], \tag{A13}$$

given $b = \int_0^T \theta w dv \Big/ \int_T^{\bar{T}} dv$ and given

$$\frac{\partial k(T)}{\partial \theta} = -(\bar{T} - T) w \left((1 - \psi) \ln \left[\frac{\bar{T}}{\bar{T} - T}\right] + \frac{T}{\bar{T} - T} + \ln \left[\frac{\bar{T} - T}{\bar{T}}\right] \right). \tag{A14}$$

Insert (A14) into (A13) and algebraically rearrange,

$$\frac{\partial c_i(t)}{\partial \theta} = \psi w \ln\left[\frac{\bar{T}}{\bar{T}-T}\right] > 0, \quad \text{for } t \in [T, \bar{T}]. \tag{A15}$$

Therefore, social security acts as a commitment device that restrains impulsivity during the working years with the result of increasing consumption all throughout retirement. ■

Proof of Corollary 2. Solve the inequality in (A10) for ψ, which yields $\psi > 0$. This indicates that initial consumption is restrained by social security if impulsivity exists at any degree. Now, solve the inequality in (A15) for ψ, which also gives $\psi > 0$. This means that consumption will be everywhere higher during retirement in a state of the world with social security if the individual possesses any degree of impulsivity. ■

Proof of Corollary 3. Set $t_0 = 0$ in (8) and (11), which gives the optimal consumption path in a state of the world where mandatory savings in unnecessary due to the absence of impulsivity (i.e., $\mu = 0$),

$$c^*(t)|_{\theta=0} = wT/\bar{T}, \quad \text{for } t \in [0, \bar{T}]. \tag{A16}$$

This consumption program is time-consistent and it achieves maximal life-cycle utility. Set impulsive consumption in a world with mandatory savings equal to optimal consumption when mandatory savings in unnecessary because of no impulsivity, meaning set $c_i(t)|_{\theta>0} = c^*(t)|_{\theta=0}$, or

$$\frac{1}{\bar{T}-t}k(t) + \frac{1}{\bar{T}-t}q(t) + \psi(1-\theta)w = wT/\bar{T}. \tag{A17}$$

Make the appropriate substitutions into (A17) and solve for the tax rate that holds at this equality,

$$\theta^* = 1. \tag{A18}$$

Note from (A18) that this tax rate eliminates the ability for a consumer to be impulsive, since the government in the model holds all of his disposable income. The individual consequently finances his consumption during the working years by borrowing. Therefore, a social security (forced savings) program can be theoretically parameterized to replicate optimal consumption and to realize maximal life-cycle utility (the level of utility attained by a non-impulsive individual who consumes optimally) for impulsive consumers. ■

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Table 1. Consumption Impulse: $\mathcal{I}(t) = \psi\left[(1-\theta)w - c^*(t;t)\right]$ and $\rho = r$.

Optimal Tax Rate and Compensating Variation in Fully-Funded Program.

r	ψ 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0%	27.3% 0.1%	27.3% 0.3%	27.3% 0.7%	27.3% 1.5%	27.3% 2.8%	27.3% 5.0%	27.3% 8.8%	27.3% 16.1%	27.3% 33.7%
0.5%	24.6% 0.1%	24.6% 0.2%	24.6% 0.6%	24.6% 1.2%	24.6% 2.3%	24.6% 4.1%	24.6% 7.3%	24.6% 13.6%	24.6% 28.6%
1.0%	22.1% 0.0%‡	22.1% 0.2%	22.1% 0.5%	22.1% 1.0%	22.1% 1.8%	22.1% 3.4%	22.1% 6.1%	22.1% 11.4%	22.1% 24.2%
1.5%	19.7% 0.0%‡	19.7% 0.1%	19.7% 0.4%	19.7% 0.7%	19.7% 1.5%	19.7% 2.7%	19.7% 5.0%	19.7% 9.5%	19.7% 20.3%
2.0%	17.5% 0.0%‡	17.5% 0.1%	17.5% 0.3%	17.5% 0.6%	17.5% 1.2%	17.5% 2.2%	17.5% 4.1%	17.5% 7.8%	17.5% 17.0%

Table 1. Consumption Impulse: $\mathcal{I}(t) = \psi\left[(1-\theta)w - c^*(t;t)\right]$ and $\rho = r$.

Optimal Tax Rate and Compensating Variation in Unfunded Program.

r	ψ 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0%	27.3% 0.1%	27.3% 0.3%	27.3% 0.7%	27.3% 1.5%	27.3% 2.8%	27.3% 5.0%	27.3% 8.8%	27.3% 16.1%	27.3% 33.7%
0.5%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	1.5% 0.1%	14.1% 1.3%	18.8% 4.2%	21.6% 10.0%	23.4% 24.4%
1.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	1.5% 0.0%‡	10.7% 1.3%	16.2% 5.5%	19.7% 17.1%
1.5%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	2.7% 0.1%	11.0% 2.5%	16.2% 11.4%
2.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	6.0% 0.7%	12.9% 7.2%

Note: The top number is the optimal tax rate and the bottom number is the compensating variation (percentage increase in period consumption required to equalize the lifetime utility of an impulsive consumer without social security to that of an identical consumer who participates in social security at the optimal tax rate).

‡ Strictly greater than zero at higher decimal places.

Table 2a. Consumption Impulse: $\mathcal{I}(t) = \mu(1-\theta)w$ and $\rho = r$.

Optimal Tax Rate and Compensating Variation in Fully-Funded Program.

r	μ 0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45
0.0%	100% 0.2%	100% 0.7%	100% 1.7%	100% 3.3%	100% 5.5%	100% 8.6%	100% 13.1%	100% 19.7%	100% 30.5%
0.5%	100% 0.2%	100% 0.7%	100% 1.7%	100% 3.2%	100% 5.4%	100% 8.6%	100% 13.1%	100% 20.2%	100% 32.3%
1.0%	100% 0.2%	100% 0.8%	100% 1.7%	100% 3.2%	100% 5.4%	100% 8.6%	100% 13.3%	100% 20.9%	100% 35.2%
1.5%	100% 0.3%	100% 0.8%	100% 1.7%	100% 3.2%	100% 5.4%	100% 8.6%	100% 13.6%	100% 22.0%	100% 40.1%
2.0%	100% 0.3%	100% 0.8%	100% 1.7%	100% 3.2%	100% 5.4%	100% 8.7%	100% 14.0%	100% 23.5%	100% 50.4%

Table 2a. Consumption Impulse: $\mathcal{I}(t) = \mu(1-\theta)w$ and $\rho = r$.

Optimal Tax Rate and Compensating Variation in Unfunded Program.

r	μ 0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45
0.0%	100% 0.2%	100% 0.7%	100% 1.7%	100% 3.3%	100% 5.5%	100% 8.6%	100% 13.1%	100% 19.7%	100% 30.5%
0.5%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	20.1% 0.5%	40.7% 3.1%	54.1% 8.4%	63.3% 18.5%
1.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	6.0% 0.1%	28.1% 3.1%	43.0% 12.5%
1.5%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	12.8% 0.8%	31.9% 10.4%
2.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	4.6% 0.1%	27.0% 13.1%

Note: The top number is the optimal tax rate and the bottom number is the compensating variation (percentage increase in period consumption required to equalize the lifetime utility of an impulsive consumer without social security to that of an identical consumer who participates in social security at the optimal tax rate).

‡ Strictly greater than zero at higher decimal places.

Table 2b. Consumption Impulse: $\mathcal{I}(t) = \mu(1-\theta)w$ and $\rho = 1\%$, $\phi = 1$.

Optimal Tax Rate and Compensating Variation in Fully-Funded Program.

r	μ 0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45
0.0%	100% 0.2%	100% 0.7%	100% 1.7%	100% 3.3%	100% 5.7%	100% 9.3%	100% 14.9%	100% 24.6%	100% 46.7%
0.5%	100% 0.2%	100% 0.7%	100% 1.7%	100% 3.3%	100% 5.5%	100% 8.9%	100% 14.0%	100% 22.5%	100% 40.0%
1.0%	100% 0.2%	100% 0.8%	100% 1.7%	100% 3.2%	100% 5.4%	100% 8.6%	100% 13.3%	100% 20.9%	100% 35.2%
1.5%	100% 0.3%	100% 0.8%	100% 1.7%	100% 3.2%	100% 5.3%	100% 8.3%	100% 12.7%	100% 19.6%	100% 31.7%
2.0%	100% 0.3%	100% 0.8%	100% 1.8%	100% 3.2%	100% 5.2%	100% 8.1%	100% 12.3%	100% 18.5%	100% 29.0%

Table 2b. Consumption Impulse: $\mathcal{I}(t) = \mu(1-\theta)w$ and $\rho = 1\%$, $\phi = 1$.

Optimal Tax Rate and Compensating Variation in Unfunded Program.

r	μ 0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45
0.0%	100% 0.2%	100% 0.7%	100% 1.7%	100% 3.3%	100% 5.7%	100% 9.3%	100% 14.9%	100% 24.6%	100% 46.7%
0.5%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	22.8% 0.7%	42.2% 3.9%	55.0% 10.5%	63.9% 25.3%
1.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	6.0% 0.1%	28.1% 3.1%	43.0% 12.5%
1.5%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	4.7% 0.1%	26.5% 4.6%
2.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	10.1% 0.6%

Note: The top number is the optimal tax rate and the bottom number is the compensating variation (percentage increase in period consumption required to equalize the lifetime utility of an impulsive consumer without social security to that of an identical consumer who participates in social security at the optimal tax rate).

‡ Strictly greater than zero at higher decimal places.

Table 2c. Consumption Impulse: $\mathcal{I}(t) = \mu(1-\theta)w$ and $\rho = 1\%$, $\phi = 2$.

Optimal Tax Rate and Compensating Variation in Fully-Funded Program.

r	μ 0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45
0.0%	100% 0.4%	100% 1.5%	100% 3.7%	100% 7.3%	100% 12.9%	100% 21.9%	100% 37.0%	100% 65.5%	100% 134.0%
0.5%	100% 0.4%	100% 1.5%	100% 3.6%	100% 7.2%	100% 12.7%	100% 21.6%	100% 36.7%	100% 65.5%	100% 137.6%
1.0%	100% 0.4%	100% 1.5%	100% 3.6%	100% 7.1%	100% 12.5%	100% 21.3%	100% 36.2%	100% 65.3%	100% 141.1%
1.5%	100% 0.4%	100% 1.5%	100% 3.6%	100% 7.0%	100% 12.3%	100% 20.9%	100% 35.8%	100% 65.0%	100% 144.4%
2.0%	100% 0.5%	100% 1.5%	100% 3.6%	100% 6.9%	100% 12.1%	100% 20.6%	100% 35.3%	100% 64.6%	100% 147.4%

Optimal Tax Rate and Compensating Variation in Unfunded Program.

r	μ 0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45
0.0%	100% 0.4%	100% 1.5%	100% 3.7%	100% 7.3%	100% 12.9%	100% 21.9%	100% 37.0%	100% 65.5%	100% 134.0%
0.5%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	7.3% 0.0%‡	40.6% 2.7%	58.4% 9.3%	69.2% 21.8%	76.2% 46.7%	81.0% 109.8%
1.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	30.1% 2.5%	49.5% 11.8%	61.5% 33.1%	69.5% 91.5%
1.5%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	2.5% 0.0%‡	33.3% 5.1%	50.7% 22.7%	61.7% 76.9%
2.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	0.0% 0.0%	17.0% 1.2%	41.6% 14.6%	55.9% 64.6%

Note: The top number is the optimal tax rate and the bottom number is the compensating variation (percentage increase in period consumption required to equalize the lifetime utility of an impulsive consumer without social security to that of an identical consumer who participates in social security at the optimal tax rate).

‡ Strictly greater than zero at higher decimal places.