„About being accurate for the best estimate:

Product designs for participating life annuities"

Be sure you know the condition of your flocks, give careful attention to your herds.

Proverbs 27:23

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Abstract

The buyers of traditional participating life annuities with a year-to-year guarantee seem to have different preferences for participating life annuity (PLA) product design than product providers. We investigate into this phenomenon from both demand and supply side using a full-fledged, stochastic company model. Our focus lies on the detailed modelling of surplus appropriation modes. We distinguish between the requirement to use local accounting principles for estimating the absolute surplus amount and the market-consistent valuation requirements for a risk-based solvency framework. Taking both views of beneficiary and annuity provider, we analyse the impact of surpluses using utility equivalent comparisons and study the effects on the economic balance sheet as well as ruin probabilities. Besides investigating long-term product design effects for PLA, we also take the possibility of fixed life annuities into account. We demonstrate a crucial role of surpluses and successively tackle the problem of their estimation for the entire annuity contract lifetime in a stochastic solvency framework. We show that methods and strategies of surplus distribution as well as their estimation can have big effects for annuity providers and beneficiaries, thus explaining the empirically observed discrepancies.

Contents

1. Introduction

Have you ever been in the situation, where you searched for certain product characteristics, and learned that, for some reason, they were scarce in the market? Not that you have searched for something really unusual, such as generous family domicile with spacious garden and swimming pool in the midst of a leading global city. You would expect, for example, a party caterer to offer its services also on weekends and in the afternoon rather than only in the early morning hours, and would be surely very surprised to learn that only a few of them are really doing so.

This paper explores participating or with-profit life annuities (PLAs), which is the standard product offered in the German life insurance market. PLA is a financial product offered by life insurance companies where, in exchange for a non-refundable premium, annuitants receive guaranteed minimum lifelong benefits and additional yearly surpluses. These non-guaranteed surpluses depend on the life insurer's asset returns and mortality trajectories in the annuitant pool. Realised surpluses are distributed to policyholders in two different participation schemes: surplus annuitisation and direct payment of surpluses. In case of direct payment, the policyholder receives yearly lump-sum payments. In case of surplus annuitisation, it becomes part of the guaranteed benefits in subsequent years (see Maurer et al. 2013).

We find some intriguing empirical indications that for PLA the participation scheme predominantly offered by life insurance companies may not be the most purchased by annuitants. On the supply side, we analyse the current and historical quotes provided for PLA with annuitisation and direct payment of the surpluses. Using data on historical quotes for PLA with annuitisation and direct surplus payment, representing the vast majority of the tariffs offered by annuity providers in the German insurance market, we discover that the number of quotes for the option "surplus annuitisation" is twice as high as the number of quotes for the option "Direct payment of surpluses"[1]. This is true not only for current, but also for historical quotes, which are available since 1996. Obviously, companies are

[1] Data provided by Morgen&Morgen, a comparison platform for brokers. For current quotes, we retrieve quotes for an immediate participating life annuity at a cost of a one-off contribution of €100,000 for males and females aged 67 in 2017.

more interested to offer products with surplus annuitisation. On the demand side, we see, on the contrary, that the majority of the customers prefer products with direct payment of surpluses. We use this empirical fact as a starting point for our detailed investigation on possible reasons for preference differences on the demand and supply side. On the demand side, the reasons can primarily lie in the payout differences for analysed annuity types, on the supply side, the reasons can stem from the means to account for long-term liabilities affected by both the biometric and the capital market risk.

In risk-based solvency frameworks, market-consistent valuation of insurer's assets and liabilities, especially the valuation of expected future surpluses is an important challenge for the annuity provider. Yet, this challenge opens the door for a detailed analysis of both the annuity provider's position in terms of own funds and ruin probabilities, as well as the influence of surplus participation on the utility of the annuitant. This is the starting point of our paper.

A participating life annuity gives the beneficiary a contractual right to receive lifelong discretional additional benefits based on such factors as mortality and investment performance as a supplement to the guaranteed fixed minimum benefits. The participating life annuity is traditionally the main product in the German market (see GDV 2016). In many Far East countries such as Hong Kong, Malaysia and Singapore, there is also a market for participating annuities. For the US, the majority of top ten writers of annuity business according to Insurance Information Institute offer participating annuity or life insurance products, although no publicly available statistics is available on their importance. Participating annuities are also offered for occupational pension plans such as for example TIAA-CREF. Profit participating products are also part of the product range in Canada and UK. The rise in the popularity of participating life annuities stems from their risk-sharing ability between the annuitant and the annuity provider by simultaneously offering guaranteed level of income, which is extremely valuable in the environment of rising longevity, declining interest rates and sophisticated regulation. Statistically, for the US market, PLAs fall into the category of variable annuities, which experienced considerable growth in the recent years (see IRI 2016), and already have a big share in the portfolios of American retirees: approximately 75% of annuities held are variable annuities, while only 25% of annuities held are fixed annuities (Gallup 2013 and 2009).

There is a growing literature investigating the use of annuities as retirement income instruments. For the fixed annuities, these are studies by Milevsky and Young (2007), Horneff, Maurer, and Rogalla (2010), for variable annuities, studies by Richter and Weber (2011), Maurer, Mitchell, Rogalla, and Kartashov (2013) to name only the recent ones. These studies focused, however, only on the demand side, exploring the welfare implications for different types of variable or fixed life annuities and purchase timing.

Only a few take a viewpoint of annuity supplier and surveyed the incentives or perils to offer a certain product type. Most recently, Koijen and Yogo (2015) investigate the impact of financial and regulatory frictions on the supply and pricing of life insurance. Charupat et al. (2015) demonstrate deferred and asymmetric responses of annuity providers to changes in interest rates. Kojen and Yogo (2016) model and quantify the effects of tightened and complex regulation on shifting life insurance and annuity liabilities to reinsurers. Al-Darwish et al. (2014) stress the unintended consequences of complex regulations for cost of capital and risk migration.

In our paper, we look at the least surveyed type of annuities - participating life annuities (PLA) – because of their very special income streams. Over the lifetime of the PLA contract the importance of the discretional surpluses increases in the light of risk-based solvency frameworks, such as Solvency II or the Swiss Solvency Test, which model the development of own funds and the ruin probability based on the market valuation of assets and liabilities. In addition, the surplus cash flows can be considerably steered by choosing the legally permitted methods to assign and pay out the surpluses to the policyholder – the influence on both the annuity provider and the beneficiary has not been examined in details, yet.

We follow the approach chosen by Maurer et al. (2016) and examine both the demand and the supply side of PLA annuity contract. On the demand side, we consider different PLA designs such as different surplus attribution methods and surplus participation strategies and determine their influence on annuitant's utility comparing utility equivalent fixed life annuities (UE FLAs). We also take into account different kinds of annuitants, based on their risk aversion and subjective discounting factors.

Lately, smoothing of the value of assets came under fire stemming from difficulties in assessing a real financial status due to lack of transparency, which becomes especially relevant in the present low interest rate environment. Maurer et al. (2016) show that smoothing really provides positive economic effects on annuitants, as opposed to findings by Guillen, Jorgensen and Nielsen (2006) and Jorgensen (2004). In our paper, we enable an analysis of smoothing-incurred the economic effects for annuitant and annuity provider.

On the supply side, we look at own funds and the ruin probability as key indicators. In our paper, we also address the challenges posed by a market-consistent valuation of asset and liabilities and the necessity to value future discretionary benefits for the PLA.

Our aim is to find out whether the preferences of the annuity provider in choosing the type of annuity and surplus participation characteristics are the same as for the beneficiary using a risk-based solvency framework, i.e. market-consistent valuation of insurer's assets and liabilities as well as the detailed valuation of expected future surpluses. We also aim to back our findings against our preliminary empirical evidence.

2. Surplus Participation Systems and their Role in Lifetime Benefits

In our analysis, we focus on PLAs as they are offered in the German market, where this annuity type is the main product. It consists of fixed guaranteed lifelong benefits and a variable nonguaranteed surplus. The fixed benefits depend on the guaranteed interest rate, which is set at the time the policy is issued and remains unchanged during the lifetime of the contract. This actuarial interest rate employed for pricing is usually limited by the respective maximum technical interest rate prescribed by the supervisory authorities (see §65 (old) and § 88 (new) VAG). In 1994, the maximum technical interest rate for all life and annuity insurers was set at 4 percent per year. Afterwards, it was stepwise lowered to 0.9 percent in 2017. For deferred annuities, in the face of capital market volatility and low interest rates, more and more insurance companies include in their insurance conditions the right to reset the guaranteed interest rate at the beginning of the payout phase based on the market conditions.

Non-guaranteed surpluses in the annuity business result from the regulatory requirement to choose the calculation basis prudently and depend on insurer's experience with mortality and expenses as well as on the performance of the investment portfolios. This is why, according to German regulation, life insurers have to distribute the bulk of non-guaranteed surpluses to policyholders.

Table 1 shows the total surplus development of German life insurers for the years 2005-2012 and the share of the total surpluses allocated to the policyholders. Despite the considerable decline in the absolute amount of industry-wide earned surplus (from 14 bn in 2005 to 9 bn), the share of surpluses distributed to the policyholders remained stable in the area of 90%. This means that the absolute amount available to the insurance company and thus the ability to strengthen the risk-bearing capital, becomes smaller. Due to a progressive reduction of guarantees during the low interest rate environment the role of surpluses comes into focus and thus issues connected to surplus distribution, projection and estimation are contemporary very relevant.

Table 1 here

The main surplus sources of an annuity provider, backing both the guaranteed and non-guaranteed surplus part for the policyholders, stem from the so-called mortality return and the asset return. Mortality return stems from the difference of anticipated and observed mortality in the pool of insureds while asset return arises from the difference between the net investment returns and the interest rate used to calculate guaranteed benefits (GIR).

The environment of low interest rates, the pressure to lower guarantees and the introduction of risk-based solvency frameworks challenge traditional product design in the German market. Especially the sustainability of the participating life annuity products in their current form to the insurer and the corresponding annuitant's lifetime utility are adressed. In our paper, we investigate the influence of different profit participation modes on the insurer and beneficiary in a general setting.

Figure 1 depicts the split of surpluses in risk and net investment return for 2009-2015 as a percentage of the gross premium earned. During these five years, the surpluses by life insurers declined by more than two percentage points, from more than 14% in 2009 to approximately 10% in 2015. The risk return remained with approximately 7-8 % relatively stable, while the net investment return declined considerably from levels comparable to risk return in 2009-2010 to approximately 3% in 2015. In the current low interest rates environment, the net investment returns of insurance companies are expected to decline further, as the bond investments bearing high interest are ending and not offered to the same long lasting conditions, thus lowering the surplus even further.

Figure 1 here

In 2015, the Life Insurance Reform Act (LIRA/LVRG) came into force. Since then, new regulations apply to profit distribution as well as to the insured's participation in the unrealised reserves, distribution of dividends, specification of a profitability indicator, accounting of acquisition costs and reduction of the guaranteed interest rate. The Act specifies the sources of allocable policyholder surpluses - asset returns, mortality returns and other returns. At least 90% of each the mortality and net asset returns have to be distributed to the policyholders according to the latest regulation, with the possibility of offsetting negative asset returns by positive mortality or other returns, however. Policyholders can choose the way of participation in the distributed surpluses. Several surplus participation modes are available.

Surpluses are not guaranteed, but German life insurers traditionally preferred to keep surplus rates stable over time, not least to maintain the additional sense of stability their customers were seeking. This is achieved by using accounting and actuarial techniques permissible under accounting standards accepted in Germany. The goal hereby is to employ surpluses from good years to cover for total benefit payouts in bad years. For discussion on return smoothing see Maurer et al. (2016).

An additional big challenge for insurers is to estimate future surplus payments according to different surplus participating strategies and their influence on the company's insolvency risk. This issue becomes crucial especially with the introduction of Solvency II, which requires to cover for long-term guarantees and future distributed surpluses to the policyholders on the one hand, but allows to account for future surpluses assigned to shareholders in the calculation of own funds on the other hand.

For policyholders, surpluses are essential as well, as they determine the income stream from annuities considerably, especially in times of low and cautiously chosen guarantees.

We take the view of both, insurance company and the policyholder, and analyse the influence of different surplus participation designs on the stability of the insurer and on the utility of the policyholder. We note that at the end of the day that both points of view are connected. PLAs were not analysed very detailed so far and we close this gap. Maurer et al. (2016) is, to our latest knowledge, the latest paper, comprehensively explaining the function of the PLA and analysing it from both the viewpoint of annuity provider and the beneficiary.

Any PLA has a series of embedded guarantees, such as an interest rate guarantee and a mortality rate guarantee, which at the end of the day determine the surpluses: These guarantees are already analysed in the literature predominantly taking only the view of either insurer or the beneficiary. Kling et al. (2007a) and Kling et al. (2007 b) investigate the insurer's reliability influence of interest rate guarantees. Bauer et al. (2006) and Zaglauer and Bauer (2008) examine the interdependencies between the guarantees and interest rate level as well as Gatzert and Kling (2007) and Eling and Holder (2013) and come the similar result. Gatzert et al. (2012) looks at the guarantee problem from both the policyholder and the insurer's perspective, highlights the default risk as a concern for both investigated parties and stresses the willingness of the beneficiary to sacrifice some guarantees in order to lower the default risk for the insurance company. Schmeisser and Wagner (2013) consider the effect of regulating the maximum interest rate.

Only a view researchers look into the choice of products: While the bulk of research concentrates on life insurance products in their saving phase, there are only a view contributions examining the product choice. Bohnert et al. (2015), for example, focus on endowment contracts and temporary annuities.

Our paper fills the literature gap and focuses on annuities in the payout phase with an ex-ante unknown duration and thus very high potential influence of surpluses and guarantees. We introduce the stochasticity of the capital markets and the mortality within a fully fledged asset-liability-model with interdependent balance sheet positions for both the local GAAP and the economic balance sheet. Our model allows for annually changing own funds position, which constitutes an important difference to comparable studies such as Gatzert et al. (2012). The later study focuses on the deferral phase of an endowment insurance with surpluses either increasing the death and survival benefits, or reducing the contract duration with no surplus smoothing.

In our paper, we analyse two ways to assign surpluses to the policyholder, which we call surplus participation methods, and two ways to actually disburse the assigned surpluses, which we call surplus participation strategies.

First participation method is the annuitisation of surpluses using the same calculation inputs as at the contract signing. By this method, in case of positive surpluses, policyholder's guaranteed annuity is increasing. The second method is the direct payment, where the guaranteed part of the annuity remains unchanged during the whole lifetime of the contract and serves as the lower limit. Annual surpluses added on top of the guaranteed annuity payment without annuitisation vary annually and may be zero as a minimum.

The considered participation strategies are smoothed and unsmoothed surpluses. For the first strategy, the funds are transferred to special balance sheet positions – the profit participation reserves. There are two types of profit participation reserves (PPR) – the committed (CPPR) and the uncommitted (UCPPR). Once the funds are in the CPPR, they have to be disbursed to the policyholder within the following business year, which means the already existing guaranteed annuity is topped up. This augmentation is guaranteed and permanent for the surplus annuitisation method, and only one-time and not guaranteed for future periods for the direct surplus distribution method.

The funds in the UCPPR serve as a buffer, as their payout to the policyholder can be deferred for a couple of years. There is no outright time limit for funds in UCPPR, but the maximum amount of allocation to this reserve type is restricted and depends on business volume, the amount of funds in CPPR and the net average asset return.[2] The UCPPR enable the insurance company to offer stable profit participation rates over a long period of time, that is, the so-called smoothing of payouts. Smoothing is popular with the customers, as it gives them additional impression of security, but recently was criticized for the lack of transparency, see Maurer (2016) for details. If surpluses are left unsmoothed, there is no buffer account and the whole amount of allocated surpluses to the policyholder are put into the CPPR.

Although the surplus pot, available for the beneficiaries is the same for both surplus participation methods and surplus strategies, handling of the surpluses results in

[2] see MindZV§11, Bundesgesetzblatt Jahrgang 2016, Teil I, Nr. 18, issued in Bonn on 21st April 2016 for details.

considerable differences both from the standpoint of the policyholder and the insurance company.

For the annuity providers, the choice of participation mode defines the amount of guarantees: If the surpluses are annuitised, the newly annuitised part adds each year on top of the previous year's guarantees with all resulting implications for the company's liabilities. The corresponding increased reserve is built using the guaranteed interest rate, and thus the choice of the guaranteed interest rate is of a crucial importance for the amount of liabilities. The choice of direct payment method results only in the initial guarantees on the part of the insurer. Once established, the surpluses are transferred to the CPPR and paid out to the policyholders entirely within the next business year while the guarantee remains restricted to the level agreed upon at the time of contract signing.

We investigate both surplus participation methods (annuitisation and direct payment) as well as both possible strategies (with smoothing and without smoothing) for their effect on insurer's stability and policyholder's utility.

3. Economic Balance Sheet

With the Framework Directive on Solvency II by the EU Parliament in 2009 the foundation was laid for a Europe-wide harmonized, principle-oriented insurance supervisory system. In the following we are considering Solvency II as our representative risk-based solvency framework. The valuation is based on the idea of calculating a transfer value and thus quantifying insurer's obligations in a market consistent framework. These obligations consist of the Best Estimate of Liabilities (BEL) and a risk margin for non-hedgeable risks. There are several proposals for calculating the risk margin, one of them is a cost of capital approach. The BEL states the expected present value of all future cash flows concerning the insurance obligations, justifying the need of a stochastic simulation model. In case of a participating payout life annuity future discretionary benefits (FDB) are an important part of the cash flows. Existing profit sharing mechanisms for determination, allocation and distribution of surpluses depend on the respective local GAAP book values, which means that for a consistent projection of future surpluses it is inevitable to apply the local GAAP within the market consistent valuation of liabilities. In order to derive the key figure of the economic balance sheet own funds (OF) of an insurance company the market value of assets is compared to the technical provisions. All further calculations are based on this indicator, e.g. the solvency capital requirement (SCR) which shall cover for unexpected losses with respect to existing business. The SCR corresponds to the value-at-risk of own funds with a confidence level of 99.5% over a one-year period.

In this paper, we want to offer a detailed analysis of the basic economic balance sheet items used for a lot of further calculations under risk-based solvency frameworks. This is why we consciously avoid the analysis of the SCR as compounded figure, but covering the detailed developments over time. Being aware of different calculation approaches for the SCR, for example the standard formula or an (partial) internal model, we ensure transferability by providing a general, simplified economic balance sheet without risk margin[3].

Our contributions consists, among others, in setting up an on-going, consistent balance sheet, linking the annual changes in cash flows profit and losses, assets and liabilities to each other. Due to the fact, that we analyse the participating life annuities in a setting typical for the German market, we have to work with two types of balance sheet for the same company. The balance sheet set up according to the German accounting principles (HGB, referred to as local GAAP) is used to determine the surpluses, whereas the economic balance sheet is used to determine the influence of the surplus distribution methods and strategies on insurer's own funds and beneficiary's utility. We allow for stochasticity in the financial markets, mortality and therefore, our asset liability model fully reflects the main risk sources of an annuity provider enabling research focus on the effects of different surplus methods and strategies.

4. Stochastic Modelling

In order to model the effects stemming from local GAAP accounting interacting with economic valuation under Solvency II exactly, our analysis is twofold: As long as annuitants are alive in the respective cohort we annually set up a stochastic local GAAP balance sheet. The model is based on the work by Maurer et al. (2014). From this point we base the evaluation of the economic balance sheet on a stochastic, future cash flow

[3] E.g. in the widely spread standard formula the SCR within the life-relevant risk modules is calculated as the difference between unstressed and stressed own funds based on the assumption that the risk margin is constant, i.e. the risk margin is neglected.

projection using Monte Carlo simulations. Again, future cash flows are determined consistently by projecting the German GAAP balance sheet for every year.

4.1. Asset Model

Our insurance company invests in dividend paying stocks and coupon bonds following a constant mix strategy. That means the portfolio must be adjusted every year to maintain the target asset mix by selling the assets outside of the limits set by the target share. The reallocation takes place when the insurance company sells assets to meet its annuity benefit payment obligations. Annuity benefits are paid both from the resolution of hidden reserves, i.e. the asset sale at market prices, and asset income consisting of dividends and coupons. The one-factor CIR term structure model determines the development of bond prices. The short rate r^P under the real-world P-measure in a frictionless and continuous market follows a square root diffusion process:

$$dr_t^P = \alpha_P(\mu_P^{CIR} - r_t^P)dt + \sigma^{CIR}\sqrt{r_t^P}dW_t^{1^P}, \quad (1)$$

where α_P, μ_P^{CIR} are positive constants, σ^{CIR} is the volatility parameter, and $r_t^{CIR} > 0$, if $2\alpha_P\mu_P^{CIR} > (\sigma^{CIR})^2$ and $W_t^{1^P}$ being a standard Wiener process.

Under risk-based solvency frameworks insurance companies must provide own funds for their future long-term liabilities which are often due in more than 40 years. Therefore, they need cash flows projections under the risk-neutral measure under which these payment streams are assessed as expected discounted values. Thus, we assume the existence of a risk-neutral measure Q equivalent to P. In case of the standard assumption for the market price of risk $q(t, r_t) = \lambda\sqrt{r_t}/\sigma^{CIR}$ and, consequently, the following relations $\alpha_P = \alpha_Q - \lambda$ and $\mu_P^{CIR} = \mu_Q^{CIR}\frac{\alpha_Q}{\alpha_P}$, the dynamics under the risk-neutral Q evolve as follows:

$$dr_t^Q = \alpha_Q(\mu_Q^{CIR} - r_t^Q)dt + \sigma^{CIR}\sqrt{r_t^Q}dW_t^{1^Q}, \quad (2)$$

where $W_t^{1^Q}$ is a standard Wiener process under Q-measure. The term structure of interest rates is affine. For a detailed derivation of the zero bond prices within the CIR model see Maurer et al. (2013).

We assume investment only in coupon-paying par bonds, because our insurance company must earn the guaranteed interest rate each year and is thus interested in sources of a stable income. The bond price B_t^T at time t with fixed maturity T is therefore calculated as

$$B_t^T = B_0 \cdot \left[\sum_{k=t+1}^{T} c_t^T \cdot \exp\big(-R(t, k-t)\big) + \exp\big(-R(t, T-t)\big) \right], \tag{3}$$

with B_0 the face value, c_t^T the constant coupon rate over T and $R(t,\tau)$ the τ-period spot rate at time t The coupon rate is reliant on the current term structure and is determined at the issuance of the bond:

$$c_t^T = \frac{1 - \exp\big(-R(t, T-t)\big)}{\sum_{k=t+1}^{T} \exp\big(-R(t, k-t)\big)}, \tag{4}$$

with $B_0^T = B_T^T = B_0$.

We follow the assumptions from EIOPA for a yield curve under the risk-neutral Q-measure for very long durations without liquid information from the capital market. The ultimate forward rate (UFR), currently 4.2%, is a value that reflects the interest rates of the past decades and is supplemented by forecasts for the economic development. The convergence period to the UFR varies, depending on the currency between 10-50 years. According to the European insurance supervisory EIOPA companies should gradually approach this rate from year 20 (assumed to be the last liquid point) to 40 reaching 4.2 %. Based on the simulated CIR-model under the risk-neutral Q-measure, in a second step we extrapolate the yield curve starting in year 20 finalizing in a forward rate of 4.2% after 60 years. This yield curve is used for discounting to determine the expected present value of future cash flows.

The calibration of the term structure model is based on historical spot rates of German Federal Securities with 1 to 15 year maturity. We use data provided by Deutsche Bundesbank over the period March 1993 to June 2009.

Table 2 here

We estimate the parameters of the interest rate model with the data until 2009, since a significant interest rate reduction was introduced by the ECB as a part of its quantitative easing policy. Including data after 2009, would lead to a long-term level of the short rate of about 0%. We assume, however, that the extremely low interest rate environment is not permanent in the long term. In order to provide a generalized model we refrain from market data since 2010 for the calibration of our CIR model.

Stochastic market prices of stocks follow a geometric random walk. Stock prices S_t evolve according to

$$S_t = S_{t-1} \cdot e^{r_t^P + r_t^{Ex_P}} = S_{t-1} \cdot e^{r_t^P + \mu - \frac{1}{2}\sigma^{Ex^2} + \sigma^{Ex} W_t^{2^P}} \quad under\ P\ ,$$
$$S_t = S_{t-1} \cdot e^{r_t^Q + r_t^{Ex_Q}} = S_{t-1} \cdot e^{r_t^Q - \frac{1}{2}\sigma^{Ex^2} + \sigma^{Ex} W_t^{2^Q}} \quad under\ Q, \qquad (5)$$

where r_t^P and $r_t^Q$is again the short rate, and $r_t^{Ex_P} \sim N\left(\mu - \frac{1}{2}\sigma^{Ex^2}, \sigma^{Ex}\right)$ is the log excess return under *P*-measure and $r_t^{Ex_Q} \sim N\left(-\frac{1}{2}\sigma^{Ex^2}, \sigma^{Ex}\right)$ is the log excess return under *Q*-measure. As the insurance company must finance periodic annuity payments, it relies on a regular asset income stream. That is why we explicitly model dividends D_t paid on stock holdings under the real-world *P*-measure. There are no dividend payments in a risk-neutral framework. The annual dividend D_t based on a fixed dividend yield μ^Devolves per

$$D_t = S_{t-1} \cdot \left(e^{\mu^D} - 1\right), \qquad (6)$$

The development of the stock prices and dividend rates relies on DAX Total Return Index and DAX Price Index over the same time interval as the term structure calibration. This results in the following estimates: the risk premium/drift ($\mu - \frac{1}{2}{\sigma^{Ex}}^2$) is equal to 1.1%, the volatility parameter (σ^{Ex}) is equal to 30%, and the fixed dividend (e^{μ^D}) is 2.1%.

Besides assets and bonds our company also has a company cash account C_t which bears the one-year spot rate given by the CIR-model.

4.2. Mortality Model

As usual in the German insurance business for pricing we use non-stochastic modeling derived by the DAV (German Actuarial Society) for a mortality table for annuities (DAV 2004 R). These pose tables for the expected mortalities in a certain year for a certain age: on the one hand estimated realistically (estimates of first order), on the other hand with security loadings (estimates of second order). It might be possible to use mortality of first order to forecast the development of our insurance company, but as we want to analyse the detailed, pathwise evolution we need a stochastic mortality model. Focusing on immediate life annuities in order to predict systematic longevity risk we follow a stochastic, extrapolative, two-factor model - the Cairns-Blake-Dowd model, see Cairns et al. (2006).

This model is designed for modeling longevity risk in pensions and annuities as it provides a good fit for higher ages by exploiting the near log-linearity of the mortality curve[4]. The stochastic dynamics of the annuitants' actual mortality rates $q_x^P := q(x,t)$ for a person aged x at time t are set to

$$\text{logit}\,(q_x^P) = \log\left(\frac{q_x^P}{1-q_x^P}\right) = A_{o,t} + A_{1,t} \cdot (x - \bar{x}), \qquad (7)$$

where $\bar{x}$ is the average age of the considered age range, $A_t = (A_{o,t}, A_{1,t})$ is assumed to be a two-dimensional random walk. More precisely, the stochastic factor $A_{o,t}$, which can be interpreted as the 'level' of mortality, is reflecting generally improving mortality rates for

[4] By choosing the CBD-model we neglect the minor effect in our context of the possibility for trend assuming in mortality. For details see Börger (2013).

all ages. The factor $A_{1,t}$, also called the ‘slope’ coefficient, reflects age-dependent mortality shocks.

For the purpose of forecasting mortality rates we use a bivariate random walk model with drift, which is characterized by

$$A_{t+1} = \tau + A_t + VZ_{t+1}, \quad (8)$$

with drift τ of A_t, V, the lower triangular Cholesky matrix of the covariance $\sum$ of A_t (i.e., $\sum=VV^T$) and Z_{t+1}, a two-dimensional standard normal random variable.

We calibrate the CBD model to data from Human Mortality Database (1990-2013 for German males and females) by using OLS regression[5]. Hence, we receive the subsequent parameter estimates for our model:

Table 3 here

To forecast longevity risk precisely we not only have to incorporate systematic longevity risk, which represents the uncertainty about the variation of mortality rates over time but also idiosyncratic longevity risk, which displays the uncertainty about individual lifetimes.

Thus, the idiosyncratic longevity risk is incorporated into the number of individual I_t at time t by consisting of indicator variables $I_{t,j}^i$ for every male or female $j = \{m, f\}$ in the cohort showing their life status:

$$I_t = \sum_{i=1}^{n} \sum_{j} I_{t,j}^i, \quad (9)$$

[5] Specifically, we use the German Life Tables (period 1×1) for Males and Females; last modified: April 10, 2016, version MPv5 for the period 1990–2013. See http://www.mortality.org.

where the variable $I_{t,j}^{i}$ is equal to 1 if the annuitant i $(i = 1, \dots, n;\ n = I_0)$ is alive at time t and it is 0 if the annuitant is dead. The sequence of these variables states a Markov chain for each annuitant i with

$$P\left(I_{t+1,j}^{i} = 1 \middle| I_{t,j}^{i} = 1\right) = 1 - q_{x+t,j}^{P} = p_{x+t,j}^{P},$$

$$P\left(I_{t+1,j}^{i} = 0 \middle| I_{t,j}^{i} = 1\right) = q_{x+t,j}^{P}, \tag{10}$$

$$P\left(I_{t+1,j}^{i} = 0 \middle| I_{t,j}^{i} = 0\right) = 1\,,$$

with $q_{x+t,j}^{P}$, $j = \{m, f\}$ being the actual mortality rate of an annuitant aged x at time t with

4.3. Liability Model

Our insurance portfolio is made up exclusively of immediate PLAs which consists of lifelong guaranteed payments as well as additional non-guaranteed surpluses. Every annuitant in the cohort pays a one-off contribution. The fundamental principle in actuarial mathematics with its applications in the insurance practice, particularly in life insurance, is the actuarial principle of equivalence of contributions and benefits. It says that, if the pool of annuitants is sufficiently large, the present value of premium payments by the policyholder and the present value of benefit payments by the insurance company have to be equivalent. The initially guaranteed benefit payments BP_0 of an annuitant of age x is calculated with the premium P and an annuity factor (expression in parentheses), that is given by

$$BP_0 = P \cdot \frac{1}{\left(\sum_{k=0}^{\omega-(x+1)} \frac{{}_{k}p_{x}^{A}}{(1+i)^{k}}\right)}, \tag{11}$$

where ${}_{k}p_{x}^{A} = \prod_{j=0}^{k-1}\left(1 - q_{x+j}^{A}\right)$ is the k-period survival probability at age x, q_{x}^{A} are the actuarial mortality rates taken from “DAV 2004 R” tables recommended by the German

Actuarial Society, ω is the terminal age of this mortality table and i_j is the interest rate guaranteed at year j. We do not consider explicit costs in terms of loading in our model.

4.3.1. Local GAAP Balance Sheet

After having showed the dynamics of the asset side we now turn to the liability side and set up a simplified GAAP balance sheet at time t with positions which we will explain in the following section

Assets (Book Values)	**Liabilities**
Company Cash C_t	Equity Capital E_t
	Uncommitted PPR $UCPPR_t$
Stocks S_t	Committed PPR $CPPR_t$
Bonds B_t	Actuarial Reserve V_t

The asset side consists of a cash account called company cash, as well the book values of the bonds, which coincides with the nominal amount under the German GAAP as we assume them to be considered as to held to maturity. As we suggest that the insurance company aims to generate stable book value returns affecting annuitant's total benefit payments stocks are also valuated at historical costs. If market and book value of bonds or stocks at selling time does not coincide a gain or loss is realised by the dissolution of so-called hidden reserves. The biggest position at the liability side in an insurer's balance sheet is the actuarial reserve. As we use the same interest rate for pricing and reserving and we are considering immediate annuities for I_0 annuitants of the same age at the same date of purchase the initial actuarial reserve equals the total premium income $V_0 = P \cdot I_0$. Afterwards, the development of the actuarial reserve depends on the guaranteed benefit BP_t, the annuitant factor (formula in brackets) as well as the number of annuitants I_t:

$$V_t = I_t \cdot BP_t \cdot \left(\sum_{k=0}^{\omega-(x+1+t)} \frac{{}_kp_{x+t}^A}{(1+i)^k} \right). \tag{12}$$

The actuarial reserve plays an important role as it determines the amount of assets the insurer is required to maintain in order to ensure the fulfillment of contract obligations. Besides, for participating payout life annuities and their different kinds of surplus participation modes two more balance sheet items ($CPPR_t, UCPPR_t$) have to be taken into account, which depend on the respective allocation of the total annual surplus of the insurance company. The threefold annual surplus appropriation can be traced in Figure 2:

Figure 2

Step 1: Surpluses arise due to cautious calculation assumptions. To decide about the further application of these surpluses the total amount of annual surplus TS_t of the insurance company has to be determined primarily. As we neglect costs, the remaining sources of surplus are the mortality return MR_t and the asset return AR_t less the interest on actuarial reserve IR_t building the total annual surplus

$$TS_t = MR_t + AR_t - IR_t. \tag{13}$$

The annual mortality return stems from the difference between actual mortality and mortality assumed for pricing. The annual asset return results from the interest i_t^{AR} on the remaining reserve after annuity payments and evolves according to

$$i_t^{AR} = \frac{\alpha_{S,t} D_t + \beta_{S,t}(S_t - S_0) + \alpha_{B,t} c_t^T B_0 + \beta_{B,t}(B_t^T - B_0)}{\alpha_{S,t}\, S_0 + \alpha_{B,t}\, B_0}. \tag{14}$$

This interest consists of the number of stocks (bonds) held in year t $\alpha_{S,t}$ ($\alpha_{B,t}$), D_t ($c_t^T B_0$) is the dividend (coupon) payment for each stock (bond). Additional, we have realised gains or losses compared to the price at purchase S_0 of $\beta_{S,t}(S_t - S_0)$ from selling $\beta_{S,t}$ number of stocks at market price S_t. Accordingly, we have a realised gain or loss from selling $\beta_{B,t}$

bonds at market price B_t^T relative to a book value of B_0 is given by $\beta_{B,t}(B_t^T - B_0)$. The sum of dividends, coupons and realised gains is divided by the book value of the assets at the beginning of the year. If stocks or bonds with different book values are sold it is made use of the FIFO rule.

Besides these two sources of return the insurance company has to pay annual interests IR_t on the actuarial reserve less this year's benefit payments amounting to the promised guaranteed interest rate.

Step 2: Subsequently, the total annual surplus has to be allocated between policyholder and company. The allocation is subject to regulatory requirements depending on equity and solvency capital as well as the sources of return. The maximum amount ap is allocated if the solvency requirement is fulfilled. Otherwise, policyholders receive at least 90% of mortality and asset return less interest on the actuarial reserve. Additionally, there is the possibility of offsetting negative results from the asset return minus interest on actuarial reserve with other sources of surplus, in our case with a positive mortality surplus as we ignore costs. If the insurer is not able to meet the solvency requirements but still has equity capital a regulatory minimum surplus is allocated. The complete surplus is kept by the annuity provider if it is out of equity capital. Policyholders are not involved in a negative sum of surplus.[6]

After allocation to the policyholder the remaining profit $TS_t - AS_t$ is kept in the company and increases or decreases the equity capital. As long as solvency requirements are met a fixed dividend rate μ^D of the current equity capital is paid to the shareholders at the end of each period.

The surplus allocation depends on previous year's surplus distribution which will be explained in the next section.

Step 3: Surplus distribution is necessary if policyholders' annual surplus payout shall be smoothed over time as non-smoothing of surplus does not require the position of a contingency reserve called $UCPPR_t$. Smoothing of surpluses, however, means the assignment of allocated surpluses to the two items of the balance sheet $CPPR_t$ and

[6] The above-mentioned regulatory requirements are valid since introduction of the Life Insurance Reform Act (LVRG) in 2014. Before that, inter alia policyholder where eligible to 75% of mortality surplus and offsetting negative returns with positive returns from other risk categories was prohibited.

$UCPPR_t$. While surpluses associated to the committed PPR are distributed directly to the policyholder in the following year, the uncommitted PPR operates as a collective buffer account that stores and releases surpluses over time in order to even the total annuity payouts to the policyholders. The annual division of allocated surpluses to these two items is performed by an optimization algorithm (see Maurer et al. (2013)).

In such kinds of local GAAP book value balance sheets smoothing happens twice, directly and indirectly. The latter is an asset smoothing performed by denoting the book value and thus gaining some distance from market fluctuations. The direct smoothing is elaborated by the use of the provision for premium refund items.

Benefit payments BP_t consist of guaranteed benefits plus surplus participation. Depending on the surplus participation method the guaranteed benefit is increased by the additional surplus SP_t or kept at its initial level. We have to differentiate between two cases: direct payment and surplus annuitisation. In the first case the guaranteed benefit is kept constant at the level of the initially guaranteed benefit BP_0 and surpluses assigned to the committed PPR are paid out singularly to the annuitant, i.e.

$$BP_t = BP_0 + SP_{t-1}, \tag{15}$$

$$SP_t = CPPR_t, \tag{16}$$

for t>0. In the second case of surplus annuitisation, surpluses become part of the guaranteed benefit and annually increase its level by the annuitised committed PPR, which is

$$BP_t = BP_{t-1} + SP_{t-1}, \tag{17}$$

$$SP_t = \frac{CPPR_t}{I_t \cdot \left(\sum_{k=0}^{\omega-(x+1+t)} \frac{{}_k p_{x+t}^A}{(1+i)^k} \right)}. \tag{18}$$

As it is usual in the insurance practice to give surpluses to the annuitants in advance, we assume a positive initial surplus at t=0.

Finally, we have all necessary items to set up the balance sheet. The corresponding asset position to the equity capital and committed PPR is the company cash. Stocks and bonds in turn add up to the uncommitted PPR and the actuarial reserve. The local GAAP balance sheet is set up in each year. As mentioned above, these balance sheet items are the reference values for surplus appropriation and thus needed for an accurate, in the sense of consistent, determination of surplus development.

4.3.2. Economic Balance Sheet

The basic idea of the Solvency II balance sheet is a market-consistent valuation of all assets and liabilities. Taking market values at the asset side the determination of the market value for liabilities is quite challenging due to high dependency on financial market and mortality developments. Hence, these positions have to be determined in a stochastic cash flow projection model. We assume a the existence of risk-neutral probability measure Q equivalent to P under which payment streams can be valuated as expected cash flows discounted at the risk-free rate. Since especially the value of future payments stemming from surpluses is path dependent und cannot presented in a closed form, we use Monte Carlo simulation.

Figure 3 here

For every year t we set up the full local GAAP balance sheet for the remaining lifetime of the considered cohort of annuitants with stochastic mortality as well as stochastic bond and stock evolution under the real-world measure P. This is the basic structure for the next steps to derive an economic balance sheet for every point in time where people in the cohort are alive. To this end we have to determine the cash flows of the insurance company originating from the status quo under the real world measure at each point of time. In practice, the stochastic cash flows on the liability side are usually estimated deterministically by historic experience and determined separately from the asset side. Often, a reason is limited computational capacity. But the consistent determination of cash flows can only be derived with an interacting ALM-model. Especially, the future surplus determination depending on asset as well as liability figures needs a detailed forecast. Consequently, in our model we again set up the GAAP balance sheet, but now under the risk-neutral measure Q, for every point in time to derive stochastic cash flow projections.

With the help of the projected future cash flows we are able to determine the best estimate liabilities BEL_t . They correspond to the expected present value of all future payouts including guaranteed benefit payments BEL_t^{gar} as well as future surplus payments FDB_t depending on the respective smoothing and surplus mechanism. The own funds OF_t are the residual item from the market value of assets and best estimate liabilities. Eventually, we are able to set up a simplified economic balance sheet including the mentioned items:

Assets (Market Values)	**Liabilities**
Company Cash C_t	Own funds OF_t
Stocks S_t	Future Discretionary Benefits FDB_t
Bonds B_t	Best estimate guaranteed liabilities BEL_t^{gar}

with

$$BEL_t^{gar} = \frac{1}{N}\sum_{n=1}^{N}\sum_{j=t}^{T}\frac{BP_t^{[n]}}{(1+\mathrm{R}(\mathrm{j},1))^{\mathrm{j}}} \tag{19}$$

$$BEL_t = \frac{1}{N}\sum_{n=1}^{N}\sum_{j=t}^{T}\frac{BP_j^{[n]}}{(1+\mathrm{R}(\mathrm{j},1))^{\mathrm{j}}} \tag{20}$$

$$FDB_t = BEL_t - BEL_t^{gar} = \frac{1}{N}\sum_{n=1}^{N}\sum_{j=t+1}^{T}\frac{X_j^{[n]}}{(1+\mathrm{R}(\mathrm{j},1))^{\mathrm{j}}} \tag{21}$$

$$OF_t = C_t + S_t + B_t - BEL_t^{gar} - FDB_t, \tag{22}$$

with $\mathrm{R}(\mathrm{j}, 1)$ the spot rate deducted analogously to EIOPA at time j, N the number of simulations, $BP_j^{[n]}$ the benefit payment on path n at time j, $X_j^{[n]}$ the surplus payments beyond the up to time t guaranteed benefit on path n at time j.

As we consider single premium contracts the best estimate liabilities BEL_t consist of the expected present value of all future annuity payments considering guaranteed payments and surpluses depending on capital market development and the dying process for every considered cohort on different paths. The best estimate guaranteed liabilities BEL_t^{gar} assumes a constant payment of the achieved guaranteed benefit at time t, i.e. in case of surplus annuitisation the guaranteed benefit includes the up to time t distributed surplus in the committed PPR. In case of direct payment the guaranteed benefit stays at the initial level. The difference between those two items constitutes the expected present value of the future discretionary benefits FDB_t, i.e. the part of surplus that is distributed to the annuitants. Summarizing the market value of assets and subtracting today's best estimate liabilities we receive the residual value of own funds OF_t.

In practice usually these items are calculated as "best estimates", e.g. with estimations concerning the number of survivors in the cohort or the amount of surpluses. As these values form the basis of all further calculations under risk-based solvency frameworks, in this paper we concentrate on the accurate calculation of these figures (in terms of path-dependent stochastics in mortality and capital market developments) by projecting the entire local GAAP balance sheet for each point of time. More precisely, we develop all items of the balance sheets for different surplus mechanisms and the inter-relating accounting principles.

4.4. Utility Equivalent Fixed Life Annuity

The fixed annuity is alongside with the participating payout life annuity also a very popular product in the international insurance market, for example, for the defined benefit plans. For that reason, we take the fixed annuity as means of comparative measure and search for the fixed annuity offering the same expected lifetime utility for the annuitant U_j as the participating annuity

$$U_j = E\left(\sum_{t=0}^{\omega-(x+1)} \beta^t \; {}_tp_{x,j}^P \; \frac{{BP_t}^{(1-\gamma)}}{1-\gamma}\right), \tag{23}$$

by using a time additive CRRA utility function, ${}_tp_{x,j}^P, j = \{m, f\}$ the gender-specific survival probability of an individual aged x, γ the coefficient of relative risk aversion and $\beta < 1$ is the individuals' subjective time preference. In order to define the amount of a corresponding the annuity we calculate the utility equivalent fixed life annuity (FLA) by transforming the annuitant's utility:

$$FLA = E\left[\frac{U_j\,(1-\gamma)}{\sum_{t=0}^{\omega-(x+1)} \beta^t \; {}_tp_{x,j}^P}\right]^{\frac{1}{(1-\gamma)}}. \tag{24}$$

In our case, the FLA shows which lifelong guaranteed and constant income stream the annuitant is willing to exchange for a PLA with potentially high uncertain surpluses, while maintaining the same utility.

5. Results

5.1. Setup

The insurance company sells only single premium immediate participating life annuities for a premium of €100,000, which is also the initial reserve and initially has an insured pool of 5,000 women and 5,000 men both aged 67. Due to differences in male and female mortalities, the gender composition of the pool changes over time. For calculation of benefits a guaranteed interest rate of 2.25% and life table DAV 2004 R are used, resulting in an initially guaranteed annual benefit of €5,392. The premium is invested in coupon paying par bonds with an initial maturity of 10 years and dividend paying stocks according to a constant mix strategy with 10% / 90% ratio for stocks/bonds, as well as in a cash account. We yearly set up a local GAAP balance sheet including stocks, bonds and a cash account on the asset side, while the liability side consists of equity capital, actuarial reserve and two surplus accounts CPPR and UCPPR. The initial equity capital equipment amounts to 1.9% of the actuarial reserve, which is in line with the market. We focus on analysing the long-term effects of different profit participation methods (annuitisation vs direct payment) and strategies (smoothing vs non-smoothing of surpluses) depending on the capital market and mortality experience. The nested simulation is performed for 5,000 real-world scenarios and 100 risk-neutral scenarios. Results are stable up to 1% variation.

Our model follows the usual practice of immediately guaranteeing the surpluses for the first benefit payment year, which slightly increases the amount of the guarantee for the first year as compared to €5,392. The initial CPPR / UCPPR is set to 2%/ 3.25%, which are also usual figures in practice. This introduces differences between the actually guaranteed amount for the first year depending on surplus participation methods and strategies, and thus the variations of liabilities for the first year. Surpluses in all other years are at not known and thus not guaranteed at contract signing.

In case that annuitisation of surpluses is chosen as profit participation method, these farther surpluses become guaranteed and increase the lifelong guarantee liabilities once they are declared and transferred to CPPR or UCPPR. For surplus participation in form of direct payment, the allocated surpluses are transferred to the CPPR and payed out completely without delay to the annuitant.

The surplus participation strategy allows for surpluses to be smoothed, that is kept relatively stable over many periods. This is achieved by transferring some part of annual surplus to the UCPPR and using a surplus algorithm to determine the amount and time of disbursement to the annuitant in the following years. The surpluses can also be left unsmoothed, that is transferred in whole to the annuitant after determination.

5.2. Policyholder’s View

5.2.1. Participating Payout Life Annuity

We take the view of a policyholder first and investigate the influence of different surplus participation modes on the benefit development and the respective utility. Figure 4 represents the simulated development of the total annuity benefit payments for a representative annuitant from age 67 to age 95 for both profit participation schemes with and without smoothing. Darker color represents higher probability mass.

Figure 4 here

First, let us look at annuitisation and direct surplus payment method: From the first sight, it becomes obvious, that during the first ten annuitisation years the total benefits for the direct surplus payout method are higher than for the surplus annuitisation method. The possible benefit range for the direct payment method is also considerably higher in these years. After the age of 80, the benefits for the annuitisation method remarkably increase, as well as the range of the possible outcomes, so that at the age of 95, the lowest possible outcome for the annuitisation method is approximately at the same level as the best possible outcome for the direct payment method for any surplus development scenario. The range of attainable payouts at the end of the payout phase is quite big, and depends on the capital market and mortality development: When the annuitisation of surpluses is applied, the policyholder can be certain that an annuity level reached once cannot be lowered, as new surplus comes on top of already existing guarantees from the previous years. Especially in the first decades, the direct payment method may lead to higher total benefits in a particular year as compared to the annuitisation method, but this level is not guaranteed and can change depending on the surplus situation in the following years up to the guaranteed part. Surpluses are paid out one year after they are earned. Thus, for this payout method, mortality in the insured cohort and asset development have a direct influence on the development of benefits. This explains the visually observable higher total benefit variability of the direct payment method as compared to the even outcomes of the surplus annuitisation method.

Next, let us look at the differences between smoothed and unsmoothed surpluses: For surplus annuitisation, especially during the last annuitisation years, the range of attainable outcomes is slightly higher for smoothed surpluses. For direct payment, after the filling the buffer account during the first annuitisation years, a higher variability of annual outcomes can be observed during the whole lifetime of the cohort for unsmoothed surpluses for reasons already mentioned in the comparison between the surplus annuitisation and direct payment, namely the direct influence of mortality and asset development.

Higher benefits from the beginning of the payout phase, offered by the direct payment method can be a desirable quality for many customers.

Profit participation and distribution methods have different effects on both the beneficiary and the insurance company. For annuitants, over the lifetime of the annuity contract, the timing and amount of arriving surpluses crucially determines the utility. For annuity providers, surpluses are partly accounted for as own funds and partly as additional liability. The lion's share of surpluses is allocated to annuitants thus increasing insurer's liabilities. The risk of such liabilities should be assessed properly, especially in the risk-based solvency frameworks.

5.2.2. Fixed Life Annuity

The fixed life annuity (FLA) is in many countries a dominant old age product, which exhibits the advantage of clarity about the expected payouts: At contract signing, a lifelong payment of a fixed amount is guaranteed, resulting in no further financial obligations on the side of the annuity provider. This guarantee makes a FLA comparable to the guaranteed part of the PLA. The unknown distribution of surpluses at contract signing, however, is missing for FLA.

Using the FLA with it's guaranteed fixed payouts as a reference product, we produce an easy to handle indicator for measuring the differences between different types of PLAs and different annuitant characteristics. We calculate the utility equivalent fixed life annuity (UE FLA): It transforms a partly variable, due to surplus distribution, income stream of a PLA into an annuity with fixed benefit payments but the same utility for the annuitant. We distinguish between annuitants with different risk aversions and subjective discounting factors using the relative risk aversion coefficients of γ =2/5/10 for low/medium/high risk averse annuitants and subjective discount factors of β =0.98/0.96/0.94 for patient/normal/impatient annuitants. The UE FLAs for different profit participation methods and strategies as well as for different annuitant characteristics are displayed in table 4:

Table 4 here

In our analysis, we calculated the respective UE FLA for a participating life annuity with an initially guaranteed annual benefit of €5,392. For all surplus participation methods and surplus distribution strategies, the increase in risk aversion decreases the attractiveness for a PLA. Let us look, for example, at the PLA with smoothed surplus annuitisation: For a subjective discount factor of 0.96 and a coefficient of a relative risk aversion of 10 the UE FLA is €6,054, while for the same subjective discount factor and a risk aversion of 2 it is approximately €800 higher and amounts to €6,843. The influence is comparable for all surplus participation methods and strategies. This is explained by the fact that for a risk-averse individual participation in uncertain, possibly high, surpluses in case of a favorable business development is not very valuable.

For any risk aversion, the decrease in the subjective discount factor also reduces the attractiveness of a PLA. For example, for the same relative risk aversion of 5, the UE FLA is €6,520 for the subjective discount factor of 0.98, and €6,225 for the subjective discount factor of 0.94. That means for patient individuals the PLA with annuitisation and smoothing is more valuable than for an impatient annuitant, as total benefits increase significantly in later periods. In absolute terms, the subjective discount factor has slightly lower influence than the risk aversion.

For the analysis of the surplus' influence we consider an investor with risk aversion coefficient of 5 and the subjective discount factor of 0.96. For such an investor, the most desirable PLA has unsmoothed direct payment of surpluses: The UE FLA is €7,374. In general, annuitisation or smoothing causes a lower and later payout of surpluses. The least valuable PLA has surplus annuitisation with smoothing and UE FLA of €6,359, approximately €1000 less than the most valuable.

One reasons is, that the time preference for direct payment has little influence on the UE FLA compared to annuitisation as in this case the variation of total benefits over time is much lower. At the same time, the table shows that the effect for different risk aversion is more pronounced in case of unsmoothed surpluses. E.g. for direct payment the difference between the UE FLA of a low and high risk individual with smoothed surpluses is €766 in contrast to €866 for unsmoothed surpluses. This shows the more uniform distributions of surpluses with smoothing. The outlined ranking holds for different risk aversion and subjective discount factors.

Independently of the surplus payout methods and strategies, from our analysis follows logically, that for any analysed annuitant type, a much higher FLA is needed to maintain the same utility as compared to the initially guaranteed part of PLA, independently of the surplus participation methods and strategies.

5.3. Insurer’s View – Economic Balance Sheet Analysis

5.3.1. Participating Life Annuity

The different participating modes remarkably influence the insurer’s own funds. To obtain the amount of own funds, we have to set up the economic balance sheet. Figure 5 shows the basis for drawing up the economic balance sheet at t=0; namely the expected, projected annual cash flows under Q starting with the items from the initial local GAAP balance sheet. The year-by-year development during more than fifty years is shown for the guaranteed part of the annuity payments to the whole insured cohort, as well as cash flows stemming from surpluses. We differentiate between two surplus participation methods and strategies (annuitisation of the surpluses smoothed/unsmoothed; direct payment of the surpluses smoothed/unsmoothed). The expected cash flows for the best estimate of the guaranteed benefits are smoothly decreasing from being a lions’ share of the liabilities at the time of contract signing to almost negligible after forty years because the cohort has only a few survivors.

Figure 5 here

The development of surplus cash flows exhibits pairwise similar developments: There is one distinctive pattern for annuitisation of surpluses – both smoothed and unsmoothed, and another pattern for direct payment – also both smoothed and unsmoothed. We suggest, therefore, that surplus payment method – that is annuitisation or direct payment - plays a more crucial role than choice of surplus distribution strategies – smoothing or no smoothing. The differences between smoothing and non-smoothing are bigger for direct payment than for surplus annuitisation.

Surplus annuitisation cash flows start at considerably lower levels than for direct payment of surpluses and exhibit a lower annual volatility especially in the first twenty benefit payment years because the surpluses are annuitised using the same prudent pricing conditions as at contract signing. Smoothing results in slightly lower cash flows for the first twenty annuitisation years, and slightly higher cash flows thereafter. This is due to the use of surpluses for building the buffer account UCPPR in the first years, whereas afterwards negative capital market or mortality effects can be compensated for. After first fifteen annuitisation years, a slight increase in cash flows can be observed. This increase is induced by asset returns, is more pronounced for direct payment and is commented in more details there. After peaking twenty five years after the benefit that period, the gradual decrease starts due to shrinkage of the insured cohort and asset depletion for financing benefit payments. At the end of cohort's lifetime it reaches the level of zero.

Both smoothed and non-smoothed cash flows for direct payment are considerably higher during the first two benefit payment decades than for surplus annuitisation because yearly generated (and committed - in case of smoothing) surpluses are paid out in a single payment. These high total benefits cause a more intense asset sales for purposes of financing the benefit payments if asset returns are not sufficient. At advanced age of the insured cohort this leads to less assets in comparison to the surplus annuitisation.

After the first twenty years the direct payment surplus cash flows begin to rapidly decline and reach the zero level approximately ten years earlier than the surplus annuitisation cash flows, thus illustrating the positive role of surplus annuitisation in securing higher level of income in advanced ages. For surplus annuitisation, the majority of cash flows consists of surpluses, cumulated since the contract signing, while for direct payment, the guaranteed benefits constitute the bulk of the liabilities. Lower level of benefits payments at the first years of contract's lifetime can be financed predominantly by investment earnings without much asset sales.

After 10 benefit payment years, for both smoothed and unsmoothed surpluses, the increase in cash flows can be observed. For unsmoothed benefits it is more pronounced but more short-lived than for smoothed benefits, which start the increase later at slightly lower level, but last for a couple of years longer. The reason lies in the portfolio-regrouping after first ten years, where all bonds initially bought for the insured cohort are maturing. In the event that newly bought bonds have to be sold again immediately for benefit payment purposes, the selling return is zero. The sale of the new bonds one year later results in asset returns and thus higher surpluses due to different book and market prices, especially for direct payment without smoothing. Similar tendency in attenuated form can be seen every ten years later. The effect is more and more watered, however, by the fact that our portfolio has more mixed maturities, because in the course of years, bonds are sold and new bought for the purpose of financing benefit payments.

The year-by-year representation in figure 5 helps to explain results for the adjoining comprehensive analysis, which looks at today's present value of the expected liability cash flows and its relation to company's assets and own funds.

We measure the present value of company's liabilities at time point t=0 (at the beginning of an annuity contract) as a percentage of insurance company's assets. Liabilities consist of the best estimates of the annually payable guaranteed benefits (BEL^{gar}) and surpluses - the future discretionary benefits (FDB). To estimate today's values, we discount future annually expected payment obligations as they are exemplified in figure 5. Own funds (OF) are calculated as the difference between the assets and total best estimate liabilities. For our calculations, we assume the guaranteed interest rate of 2.25% and the interest rate level based on historic date before 2009.

Our results for PLA and FLA are shown in figure 6 (Panel A for PLA and Panel B for FLA)

Figure 6 here

For the description of this figure, we use the direct payment of surpluses without smoothing as a benchmark because it is the most straightforward product with no additional guarantees in form of annuitised surpluses or smoothing obligations.

From today's viewpoint our benchmark - shown in the figure 6, panel A, column 4 - results in less own funds than any other surplus participation method. The own funds lie in the range of 5% of total assets for direct payment without smoothing and 11% for annuitisation with smoothing. Surplus smoothing positively influences insurer's own funds for both surplus payment methods as it aims to keep benefit payments constant. To achieve this, especially in the first benefit payment years a capital buffer is built from surpluses.

When comparing the smoothed annuitisation of surpluses with smoothed direct payment, we see that annuitisation results in more own funds than direct payment. For comparing the annuitisation with direct payment for unsmoothed benefits, this also holds true. The reason is, that for direct payment, during the first two benefit payment decades, the benefits are much higher, which has a direct positive effect on the present value of benefit payments and thus causes a smaller present value of own funds as a percentage of the market value of assets. As own funds are calculated as the difference between assets and liabilities, the described differences in own funds can be explained by differences in liabilities.

Today's value of the BEL^{gar} is about 70% of the assets and it's absolute value is the same across all analysed constellations as we don't vary the initial guaranteed annuity payment, the insured cohort and actuarial assumptions. Its relative value in case of non-smoothing of surpluses is slightly higher than for smoothing as the item UCPPR is missing in the insurer's the balance sheet in case of non-smoothing: This shortens the balance sheet and increases the relative importance of unchanged BEL^{gar}.

The FDBs show today's value of surpluses and lie in the range of 21% of total assets for direct payment without smoothing and 18% for annuitisation with smoothing. Unsmoothed FDBs for both annuitisation and direct distribution of surpluses are slightly higher than smoothed for the same reason as own funds: The surplus cash flows for the insured cohort are higher during the first two benefit payment decades as surpluses especially in the first years are not used to build a buffer account, but paid out directly. Figure 5 illustrates this on year-by-year development.

5.3.2. Fixed Life Annuity

FLA is an alternative to PLA for securing retirement income. For this reason, we investigate the balance sheet influence of a UE FLA as compared to PLA. Assuming an insurance company with only fixed life annuities, we conduct the same analysis as in figure 6 Panel A and look at expected liabilities for the whole cohort lifetime in relation to the assets at the time of annuity contract is signing.

We retain our four cases: surplus annuitisation smoothed/unsmoothed, direct surplus distribution smoothed/unsmoothed. Each surplus participation method and participation strategy results in a different utility equivalent FLA (see table 4). First, we conduct the analysis of balance sheet impact for the outlined list of utility equivalent FLAs and then compare the results to the corresponding PLAs.

For our analysis we consider the utility equivalent FLAs of a normal individual with medium risk aversion, i.e. €6,359/€6,471 for PLA with annuitised and smoothed/unsmoothed surpluses, and €7,222/€7,374 for PLA with direct payment of surpluses with smoothing/without unsmoothing. Our previously analysed PLA offers the same utility with an originally guaranteed amount of €5,392 and on the top of that, the non-guaranteed surplus participation in four settings.

Naturally, the main difference to Panel A is that the represented liabilities contain no FDB, because there is no profit participation. The liabilities solely consist of the BEL^{gar}. It can be easily seen, that the share of own funds for FLA equivalent to annuitisation is twice higher than the share for FLA equivalent to direct payment. For FLAs equivalent to unsmoothed surplus payment the share of own funds is slightly lower than for the smoothed counterpart. Both observations can be explained by the amount of UE FLA: in case of annuitisation UE FLAs are lower than in case of direct payment, and for both annuitisation and direct payment, the smoothed payment UE FLAs are lower than unsmoothed payments. Thus, while for the annuitant high surpluses in the end of the lifetime (in case of annuitisation) are less valuable than in the beginning (in case of direct payment), which can be seen by the UE FLA, for the insurance company it is valid the other way around.

The first observation when comparing the balance sheet composition of UE FLA and the corresponding PLA is the increased share of $BEL^{gar}$for all UE FLAs. This is due to a clearly higher level of guaranteed benefits for UE FLAs, which are paid out constantly from the contract signing for the remaining lifetime of the cohort. The share of $BEL^{gar}$for FLA is fifteen to twenty five percentage points higher than for corresponding PLA. If we compare the BEL, that is in case of the PLA the BEL^{gar}plus FDB, we observe an amount of about 90-93% of the assets for PLA, whereas for FLA the BEL amount to about 80%-90% of the balance sheet, which is lower than for PLA. The relative big difference of 10% for annuitisation shows the diverting meaning of the distribution of surpluses for annuitant and insurer. High surpluses in the end of lifetime are taken more into account in terms of company measurement (BEL^{gar}plus FDB for PLA) than according to the UE FLA reflected in the BEL. THUS the BEL-differences between PLA and FLA for direct payment are smaller, as there is no extreme difference of total benefit payments over time.

The own funds for UE FLA are on average seven percentage points higher than for the corresponding PLA and the difference lies between three percentage points for unsmoothed direct payment and twelve percentage points for unsmoothed annuitisation.

5.3.3. Development of the Economic Balance Sheet Items over time

As a next step, we carry forward the economic balance sheet items for the liabilities, shown in figure 6, Panel A for the time of policy issuance, until the end of the cohort's lifetime.

Figure 7 here

Figure 7 shows the development of absolute values of the respective economic balance sheet items. Additionally, we will comment on the development of the items relative to the market value of assets without using an extra figure.

Starting with the last illustration, BEL^{gar} position decreases gradually during the cohort's lifetime as people in the cohort a dying. The year-by-year development shows that annuitisation of surpluses results in higher BEL^{gar} as direct surplus distribution. This is due to the fact that surplus annuitisation adds over years additional guarantees on top of the original guarantees given at contract signing. In the first two decades, we can observe slightly higher BEL^{gar} for unsmoothed surpluses for the respective participation methods. The reasons for this and the following facts can roughly be anticipated from figure 5. For the relative values, we can observe that BEL^{gar} for direct payment smoothed is smaller than unsmoothed, which is again smaller than annuitisation smoothed and smaller than unsmoothed due to the different amounts of total benefits followed by the necessity to sell assets in order to finance obligations. Overall, also the relative values decrease as the liability portfolio declines faster than the asset portfolio because benefit payments are partly, as far as possible, financed from financial returns without the need of selling.

FDBs shows similar patterns as BEL^{gar}, also here, the annuitisation of surpluses results in slightly higher discounted cash flows than direct payment after the first annuitisation years. Thus, there is a reversal from the highest FDBs for direct payment in the first years to annuitisation. In the beginning, the unsmoothed participation methods show higher FDBs than the smoothed ones, which changes after about 5 years where the role of the buffer account comes into effect. Smoothed and unsmoothed annuitisation grows over direct payment over time, because the level of benefit payments in the first decades in case of annuitisation is lower. That means fewer assets has to be sold to pay for benefit payments which in turn induces higher potential of investment returns and thus higher surplus payments. In relative terms the order of surplus participation methods and strategies develops the other way around after 20 years, i.e. annuitisation smoothed is smaller than unsmoothed, which is smaller than direct payment smoothed, which once more is smaller than direct payment unsmoothed. Again, this is due to asset selling over time awarding the highest surplus payments the greatest relative role.

The development of own funds is of course driven by the BEL^{gar} and FDB development resulting in an inverted u-shape: The own funds peak after 10-15 years and then decrease. Annuitisation of surpluses results in higher own funds than direct payment, the difference being especially pronounced after 20 contract years. Direct payment of surpluses results in a OF discounted cash flows peak five years earlier as for surplus annuitisation. The decrease is more quick and pronounced. During the first 5 years the positive effect of smoothing can be observed which afterwards is superimposed by the amount of remaining assets in the portfolio resulting in the highest own funds for smoothed and unsmoothed annuitisation. For the relative values the highest own funds in the beginning can also be seen with smoothing ending up after about 30 years with the highest own funds for direct payment, unsmoothed and smoothed almost equal. This is mainly due to the low BEL^{gar} because of low annuitisation guarantees.

5.4. Ruin probability

In the preceding analysis, we observed that the choice of annuity type (fixed vs. participating) and, for participating annuity, the choice of participation methods and strategies, considerably influences the economic balance sheet of the annuity provider, the amount of own funds and as a result the ruin probabilities of an insurance company. For that reason, we explicitly analyse the ruin probabilities as a figure of interest for both annuitant and annuitant provider. We define ruin as an event, when the liabilities of an insurance company are higher than assets, leaving the company with negative own funds, or when there are no assets left. To calculate ruin probabilities, we relate the number of events with the maximum number of negative own funds and no assets to the total number of realisation paths with surviving cohort members in the analysed period. We consider both total ruin probabilities embracing the whole lifetime of the cohort, as well as annual ruin probabilities for the respective point of time based on market-consistent valuation:

Table 5 here

First, we look at the total PLA ruin probabilities for the whole lifetime of the cohort. The ruin probabilities do not exceed 1% for the entire surveyed time span of over more than 50 years. For surplus annuitisation the ruin probability is higher for smoothed benefits. The opposite is true for the direct payment of surpluses: here, the surplus smoothing results in lower ruin probabilities. Direct payment without smoothing has the highest ruin probabilities of all constellations, annuitisation without smoothing – the lowest. We can explain these numbers by looking at the ruin probabilities at specific points in time.

For the year-by-year ruin probabilities, we see that there are no cases of ruin for the first twenty five benefit payment years. The ruin for direct payment starts earlier than for annuitisation (in year 30). Annuitisation delivers about a third of the ruin probabilities of direct payment in the year 35 due to high total benefit cash flows in the beginning and the necessity to sell assets in order to finance the insurance obligations. Up to year 40 the development of the ruin numbers is mainly driven by negative own funds and afterwards by the lack of assets. In the year 45, the rise of the smoothed annuitisation case has begun and already records the highest ruin probability followed by direct payment unsmoothed, smoothed and annuitisation unsmoothed. Year 50 is characterised by the great difference of annuitisation smoothed - reaching a ruin probability of 3.58% - and the other participation modes of about 2%. The reason for this is, that despite the highest absolute amount of own funds, in case of annuitised smoothed surpluses the highest benefit payments are produced in the last years resulting from annuitisation combined with smoothing of surpluses. Even in case of bad capital and mortality market developments a saved-up buffer account causes regular increases of the guaranteed annuities which have to be payed out. This growth explains the highest total ruin probability for annuitisation smoothed. Here again, although at a low level, the danger of high guarantees becomes obvious. In contrast, at the first glance the lowest ruin probability for unsmoothed annuitisation seems surprising. But annuitisation yields relative low benefit payments in the first years, thus few assets have to be sold to finance the benefit payments, in combination with unsmoothed surpluses, which means surpluses have only to be payed out if they arise.

For FLA (or more accurate UE FLA), the numbers we observe, heavily depend on the absolute amount of benefit as shown by the ruin probabilities. The FLA ruin probabilities differ considerably from the PLA ruin probabilities. It is noteworthy that the total ruin probabilities are zero for annuitisation equivalents, but about 2% and 4% for smoothed and unsmoothed direct payment equivalents, respectively. The increased ruin probabilities in case of direct payment equivalents stem from the fact, that utility–neutral substitution of PLA with direct payment of surpluses (smoothed and unsmoothed) results in FLAs with considerably higher guaranteed payments, than the substitution of PLA with surplus annuitisation. These differences amount up to €1,000 in contrast to guaranteed benefit of a PLA of €5,392 (see table 4).

The year-by-year analysis shows that in case of direct payment the ruin probabilities become positive earlier than for PLA and rise more quickly to higher levels. For example, in year 35 they are 4.10% and 9.24% for smoothed and non-smoothed direct payment UE FLA, and under one percent for PLA. For the year 50, the UE FLA ruin probabilities are 9.68% and 17.30%, while for PLA with smoothed and non-smoothed direct surplus payment the ruin probabilities are about 2%. It is noteworthy, however, that ruin probability of UE FLA for surplus annuitisation is zero for the whole lifetime of insured cohort due to the low unchangeable guarantee level, while for the PLA the ruin probabilities become positive after year 35. Considering table 4, we recognise that the ruin probability for a utility equivalent FLA is also greatly depending on the type of annuitant.

6. Conclusion

Traditional participating life annuities (PLA) with year-to-year guarantees are challenged due to new market-consistent valuation frameworks such as Solvency II because of their complex surplus participation mechanisms and possibility of inherited guarantees created by surpluses. The requirement to represent a market-consistent value for liabilities results in the need to project surplus cash flows accurately according to the respective product design. In this paper, we consider two methods of profit participation - annuitisation and direct payment – and for each of them two surplus payment strategies – with and without smoothing.

Methodically, we develop a full-fledged, stochastic, year-by-year company model within the framework of an economic balance sheet. For the purpose of surplus determination, we also enable the valuation of assets and liabilities according to local GAAP within this model. In contrast to previous research, we are able to explicitly model year-by-year development of company’s cash flows depending on stochastic development of the capital market and cohort’s mortality. This allows us to show and to quantify the importance of different surplus appropriation systems for both the beneficiary and the annuity provider in a detailed, consistent way. Against the background of the guarantees' and surpluses' crucial role in risk-based solvency frameworks, we examine the schemes from two different perspectives.

From the policyholder's view, we quantify utility differences for different kinds of beneficiaries and PLAs with different profit participation methods and strategies. We compare utilities using an innovative technique of calculating the utility equivalent fixed life annuity (UE FLA) delivering the same utility as the respective PLA. The results show that, considering annuitants' whole lifetime, for all kind of annuitants direct payment of surpluses leads to a higher utility equivalent than surplus annuitisation, and non-smoothing to a higher utility equivalent than smoothing.

From the insurer's view we look at the economic balance sheet positions, especially the on own funds as a residual term. We find that at time of issuance of the contract the favored surplus participation mode is just the opposite of the annuitant, i.e. annuitisation and smoothing delivers the highest own funds, direct payment with unsmoothed surpluses the lowest. The order of preferences for unsmoothed annuitisation and smoothed direct payment changes during the lifetime of the cohort.

As a third aspect, we analyse a figure that links annuitant's and beneficiary's points of view: the ruin probability measured by negative own funds and lack of assets. We give a detailed view on both the aggregate numbers and the annual ruin probabilities during the lifetime of the cohort. On a very low ruin probability level of under one percent, we document for the PLA with unsmoothed surplus annuitisation the lowest ruin probability, whereas smoothed surplus annuitisation seems to challenge the insurer with a higher ruin probability resulting from a steady rise of guaranteed benefits combined with a buffer account for bad capital market or mortality experiences. Our different standpoint approach helps to clarify possible reasons for discrepancy between supply and demand for different participation schemes.

In addition, we set up a comparison between the PLA and FLA as a very popular old age provision product. We calculate the FLA annuity amount to be the UE FLA to the respective PLA and participation scheme. Setting up the economic balance sheet comparisons for both product categories results in clearly higher own funds for surplus annuitisation and almost the same level for direct payment of surpluses. This again shows the different meaning of surplus distribution for annuitant and insurer. Quantifying the ruin probabilities for UE FLA shows, naturally, a high dependence of ruin probabilities on the amount of the guaranteed payout. The UE FLA for annuitisation provide ruin probabilities of zero, while UE FLA for direct surplus payment leads up to the overall much higher overall ruin probability of 4% and the devastating ruin probability of 17% after 50 years.

Advantages of paying out a UE FLA instead of a PLA heavily dependent on the type of annuitant. Annuitant's risk aversion and time preference – though difficult to measure in practice – is essential, as it determines the UE FLA amount. As a result, there is a narrow ridge for the UE FLA being a strong option or ruin form the standpoint of an insurance company.

Our findings are important not only for policymakers, aiming to create an adequate regulatory environment for funded old age provisions, but also for annuity providers, struggling with traditional product designs in the current capital market environment, as well as for potential annuity purchasers, looking for the most suitable protection against old age poverty.

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8. Figures and Tables

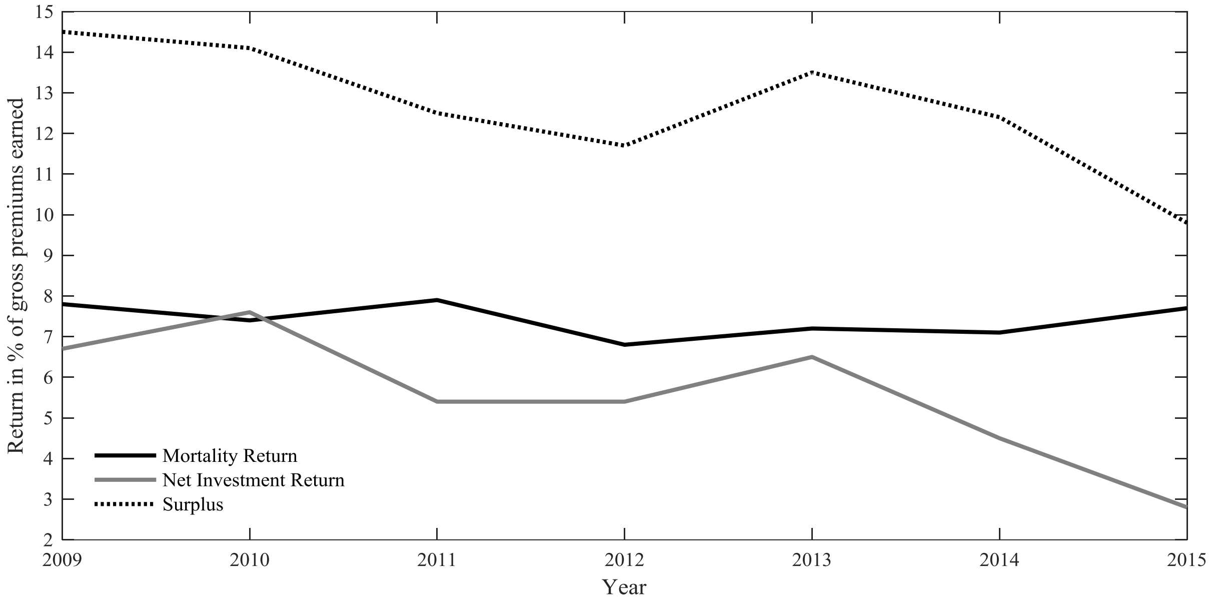


Fig. 1. Development of Mortality Return, Net Investment Return and Surplus of German Life Insurers. Notes: Average data over all German life insurers. Source: BaFin (2016).

① Surplus Determination

Mortality return

$$MR_t = V_{t+1} \cdot \left(\frac{I_t - I_{t+1}}{I_t} - q_{x+t}^{A} \right)$$

Asset return

$$AR_t = (V_t - I_t \cdot BP_t) \cdot i_t^{AR}$$

Interest on reserve

$$IR_t = (V_t - I_t \cdot BP_t) \cdot i_t$$

② Surplus Allocation

Policyholder

$$AS_t = \begin{cases} max(max(max(0.9 \cdot MR_t, 0) + max(0.9 \cdot AR_t - IR_t, min(AR_t - IR_t, 0)), 0), ap \cdot TS_t), & if\ E_t + UCPPR_t > 0.04 \cdot (V_t + CPPR_t)\ and\ E_t > 0, \\ max(max(0.9 \cdot MR_t, 0) + max(0.9 \cdot AR_t - IR_t, min(AR_t - IR_t, 0)), 0), & if\ E_t + UCPPR_t \leq 0.04 \cdot (V_t + CPPR_t)\ and\ E_t > 0, \\ 0, & if\ E_t \leq 0, \end{cases}$$

Shareholder / Company

$$E_{t+1} = \begin{cases} \left(E_t \cdot \left(1 + R(t,1)\right) + TS_t - AS_t\right) \cdot (1 - \mu^D), & \text{if } E_t + UCPPR_t > 0.04 \cdot (V_t + CPPR_t), \\ E_t \cdot \left(1 + R(t,1)\right) + TS_t - AS_t, & \text{else,} \end{cases}$$

③ Surplus Distribution

Committed PPR	Uncommitted PPR

Fig. 2. Surplus appropriation in three steps. Notes: Surplus determination according to surplus sources, surplus allocation between policyholder and shareholder / company and distribution in case of smoothed surpluses. Source: Authors' illustration

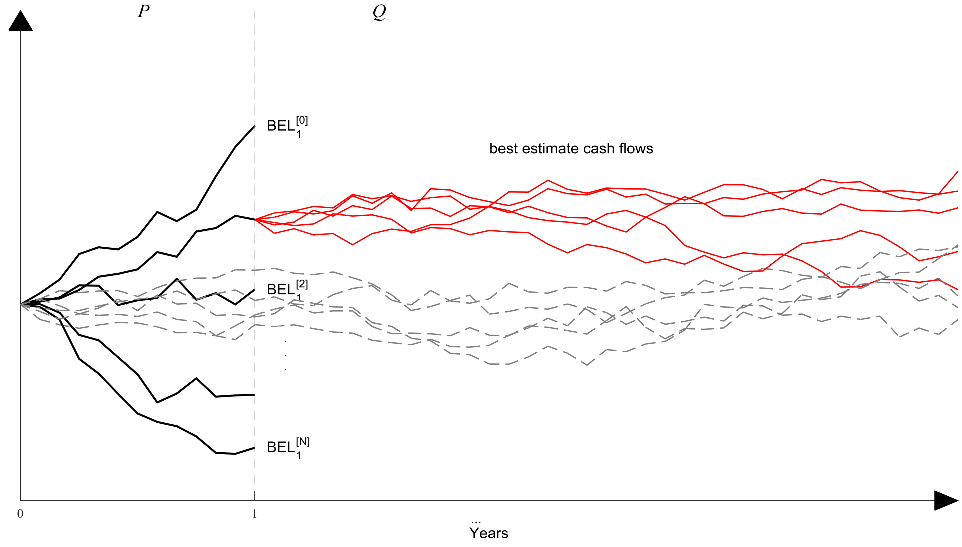


Fig. 3. Illustration of the projection of best estimate cash flows. Notes: Best estimate cash flows resulting from guaranteed and surplus benefits to policyholders for a life insurance company offering PLA. BEL (best estimate liabilities) denotes the present value of projected future cash flows. Grey and red lines display projections under the risk neutral measure and black lines under the real world measure. Source: Authors' calculation

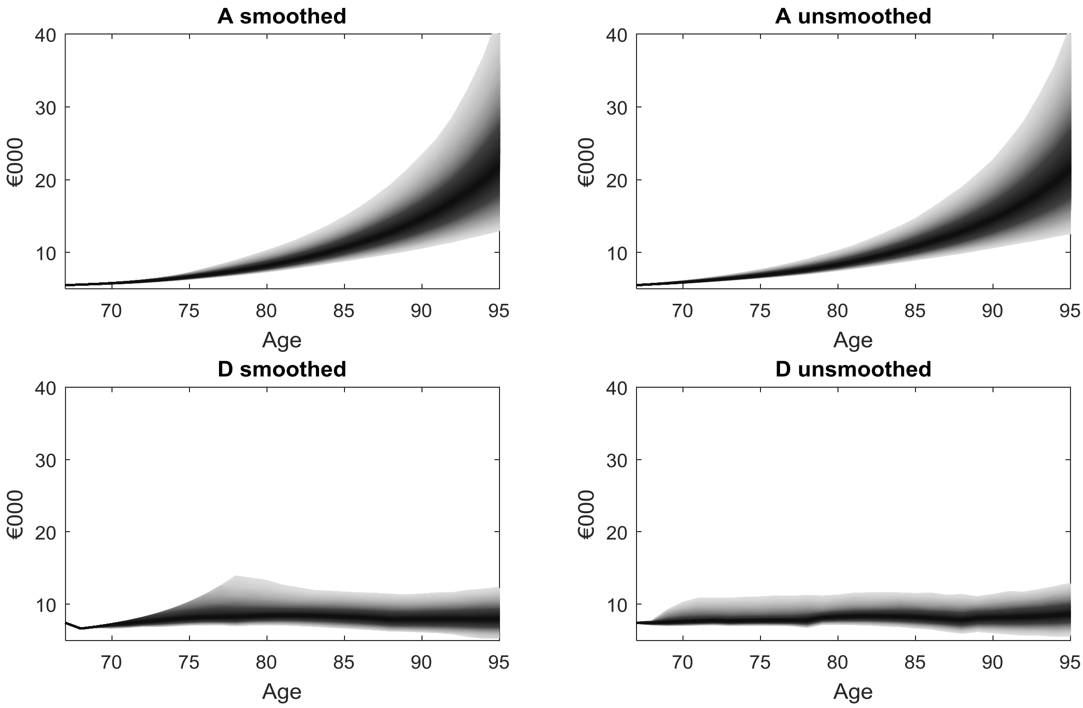


Fig. 4. Total benefit payments over lifetime. Notes: Simulated distribution of total benefit payments per contract (5%–95% quantiles) with initial guaranteed PLA benefits €10,000, guaranteed interest rate 2.25%, mortality table "DAV 2004 R", asset allocation 10% stocks / 90% bonds (with 10 years maturity). A (D) refers to PLA with annuitisation (direct payment) of surpluses, smoothed (unsmoothed) refers to surplus distribution (not) using an actuarial smoothing account. Darker areas represent higher probability density. Source: Authors' calculation

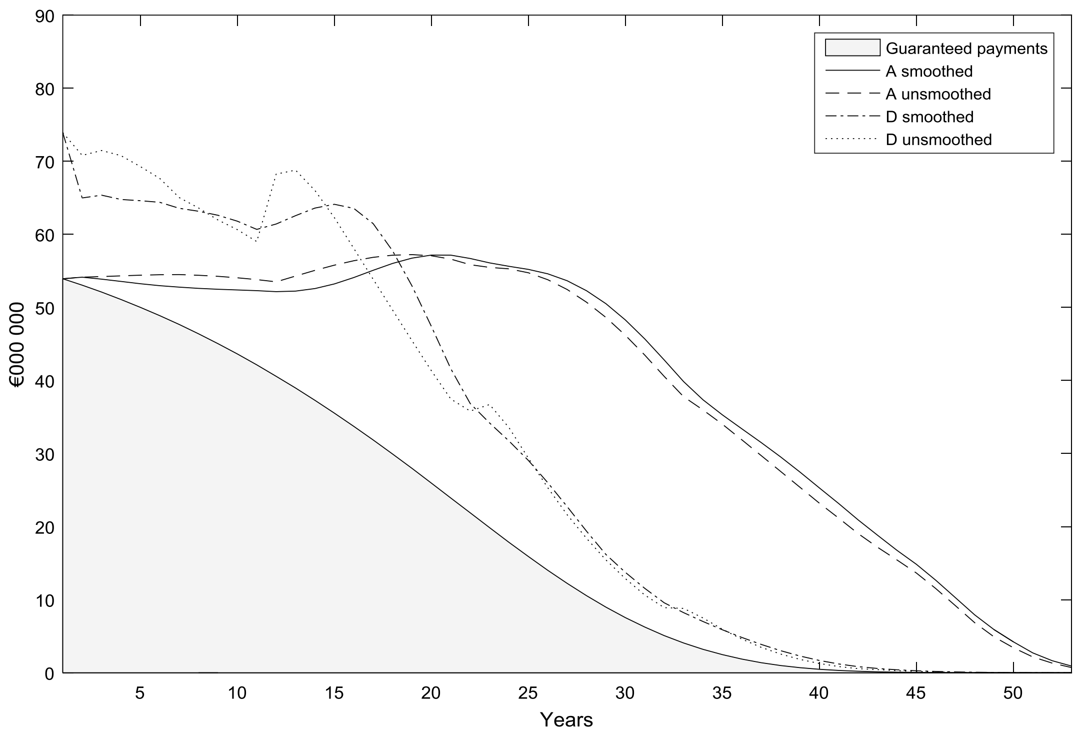
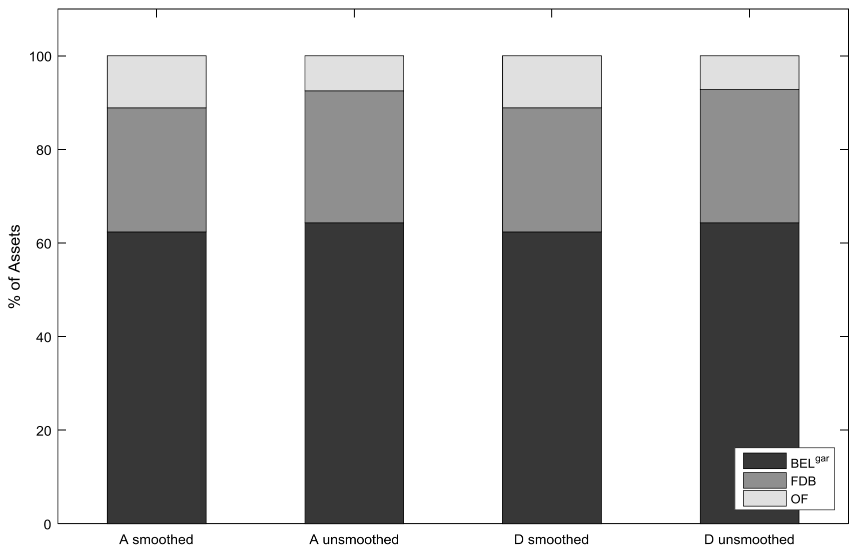


Fig. 5. Best estimate average liability cash flows. Notes: The filled part represents the guaranteed average benefit payments in the life insurance company starting at time of issuance; superimposed lines show the surplus cash flows for different participation schemes. Source: Authors' calculation

Panel A



Panel B

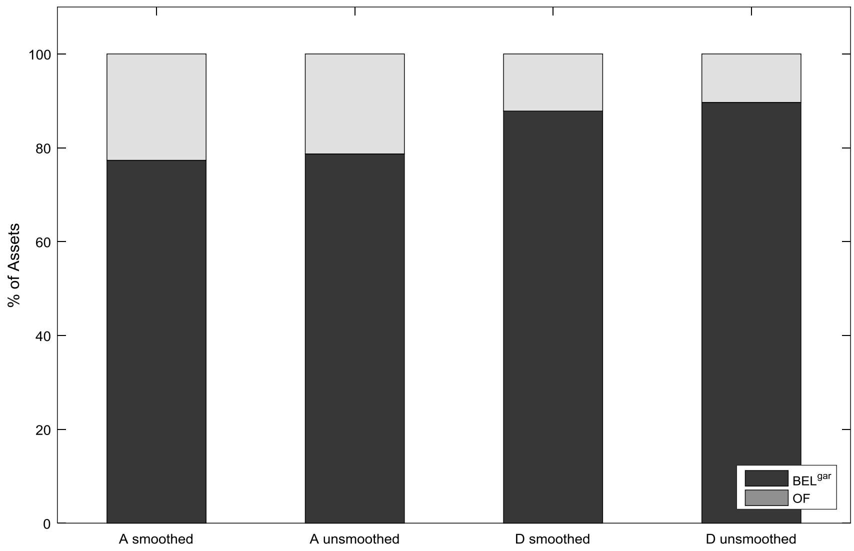


Fig. 6. Expected best estimate liabilities for different surplus participation schemes. Notes: Expected best estimate guaranteed liabilities BEL^{gar}, future discretionary benefits FDB and own funds OF in relation to the assets at the time of annuity contract's signing for a participating life annuity (Panel A) and a (utility equivalent) fixed life annuity (Panel B). Source: Authors' calculation

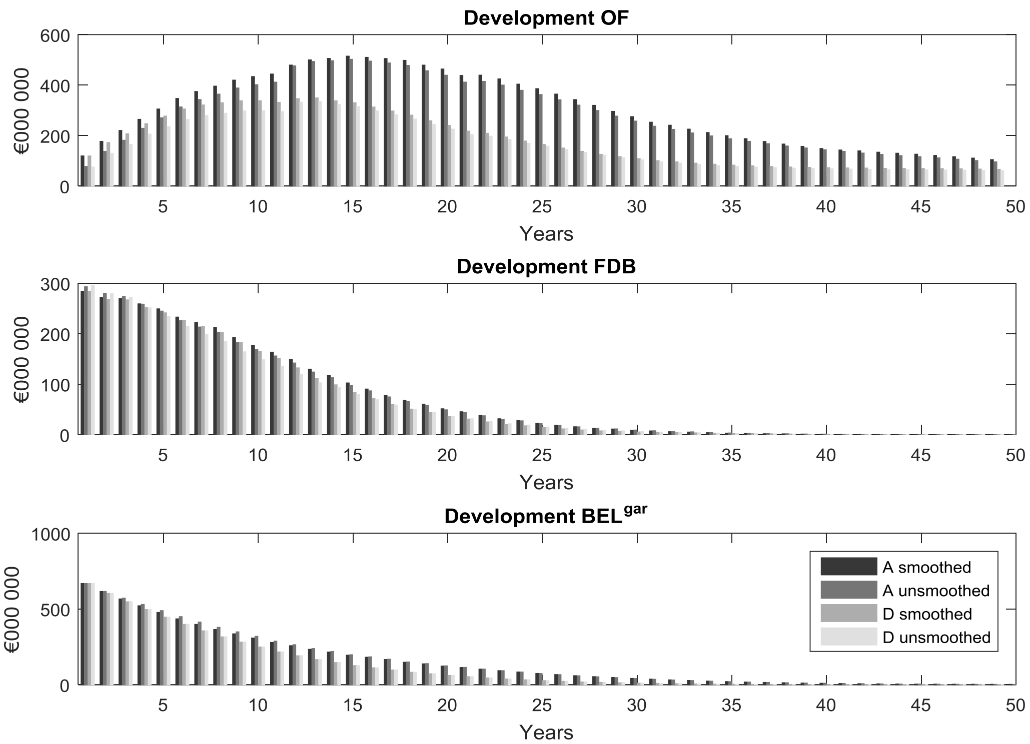


Fig. 7 Development of economic balance sheet items over time. Notes: Development of the expected best estimate liabilities over the lifetime of the cohort. Source: Authors' calculation.

Table 1

Surpluses and its allocation to policyholders.

	2015	2014	2013	2012	2011	2010	2009
Surplus in billion €	7.4	10.1	10.8	9.3	9.9	11.8	11.6
Surplus allocation in %	93.8	93.3	92.6	87.5	88.1	90	90

Notes: Generated and allocated average surpluses of German life insurers. Source: Source: BaFin (2016).

Table 2

Estimates of the 1-factor CIR Model.

μ^{CIR}	α	σ^{CIR}	λ	r_0^{CIR}
0.0196	0.2393	0.0330	-0.1924	0.0000

Note: Estimates of the 1-factor CIR model based on data provided by Datastream. Source: Authors' calculation

Table 3

Calibration of the CBD Mortality Model.

	Male				Female			
i	K_t	μ^{CBD}	Σ		K_t	μ^{CBD}	Σ	
1	-10.2340	-0.0424	0.0369	0.0000	-11.3723	-0.0370	0.0277	0.0000
2	0.0951	0.0003	- 0.0005	0.0002	0.1052	0.0003	- 0.0004	0.0002

Note: Estimated parameters of the CBD mortality model based on German mortality data for the Human mortality database. K_{t} is the period mortality index, μ^{CBD} denotes the estimated mortality and Σ is the correlation matrix. Source: Authors' calculation

Table 4

Utility equivalent fixed life annuity for different types of annuitants.

A smoothed	2	5	10	A unsmoothed	2	5	10
0.98	7,136	6,520	6,144	**0.98**	7,273	6,639	6,230
0.96	6,843	6,359	6,054	**0.96**	6,974	6,471	6,134
0.94	6,603	6,225	5,976	**0.94**	6,727	6,330	6,051
D smoothed	**2**	**5**	**10**	**D unsmoothed**	**2**	**5**	**10**
0.98	7,655	7,321	6,840	**0.98**	7,783	7,458	6,917
0.96	7,545	7,222	6,753	**0.96**	7,698	7,374	6,833
0.94	7,435	7,123	6,669	**0.94**	7,613	7,287	6,750

Note: Utility equivalent fixed life annuity for a male annuitant for different in € for a coefficient of relative risk aversion of γ =2/5/10 and a subjective discount factor of β =0.98/0.96/0.94. Source: Authors' calculation

Table 5

Ruin probabilities.

Participating Life Annuity				
	A smoothed	A unsmoothed	D smoothed	D unsmoothed
Total ruin probability (%)				
	0.33	0.09	0.13	0.35
Ruin probability at specific point in time (%)				
5	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00
15	0.00	0.00	0.00	0.00
20	0.00	0.00	0.00	0.00
25	0.00	0.00	0.00	0.00
30	0.00	0.00	0.02	0.02
35	0.04	0.06	0.34	0.66
40	0.32	0.18	0.56	1.08
45	1.82	0.46	0.66	1.21
50	3.58	0.95	1.54	1.76

Fixed Life Annuity				
	A smoothed	A unsmoothed	D smoothed	D unsmoothed
Total ruin probability (%)				
	0.00	0.00	2.01	4.23
Ruin probability at specific point in time (%)				
5	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00
15	0.00	0.00	0.00	0.00
20	0.00	0.00	0.00	0.02
25	0.00	0.00	0.22	1.18
30	0.00	0.00	1.96	5.48
35	0.00	0.00	4.10	9.24
40	0.00	0.00	4.96	10.22
45	0.00	0.00	5.66	11.17
50	0.00	0.00	9.68	17.30

Note: Total ruin probability in terms of negative own funds relative to all simulation paths with a positive number of individuals alive and (non-cumulative) ruin probability at specific points in time in terms of negative own funds relative to simulation paths with a positive number of individuals alive at the considered point in time. Source: Authors' calculation