Factor Mobility and Non-Harmonized Public Pension Systems in Europe

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Abstract

The open economy literature on the macroeconomic effects of PAYG systems and demographic change has so far focused on capital as the only mobile factor. This paper adds labor mobility as a second dimension of factor mobility to the existing literature. The starting point of the analysis is the observation that the generosity of public pension systems differs greatly between European countries. Within a two-country model, I study how both capital and migration flows respond to differences in the public pension systems of Germany and Austria and discuss the resulting effects on prices, aggregates and welfare. For empirically plausible values of moving costs, the model predicts a large outflow of workers from Austria which runs the more generous PAYG system. The interplay of capital and labor mobility induces significant welfare gains in Austria, while it leads to small welfare losses in Germany. Population aging is shown to increase the reallocation of labor, which can, however, be weakened by suited policy reforms.

JEL Classification: H55, F22, C68, J11.

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1 Introduction

The creation of a single market involving a free movement of goods, services, capital and people is the long-term political goal of the European Union. Since the treaty of Maastricht in 1992, many steps towards the achievement of this goal have been taken. However, a complete market integration has not been reached yet. This is partly due to the fact that social security arrangements have remained mainly under the domain of national policies. Among the different social security systems, one observes a wide heterogeneity regarding both their institutional structures and their generosity. With free movement of capital and labor on the one side, and the non-harmonization of social security systems on the other, Europe finds itself in a situation of incompleteness. The following study aims at analyzing the consequences of this incompleteness while concentrating on the role of public pensions that make up the largest share of social security expenditures. In particular, I investigate how migration decisions are influenced by differences in public pension systems and what macroeconomic effects follow from the migration flows arising. I further explore the role of capital mobility, firstly to compare it to the effects of labor mobility and secondly to analyze the interaction between the two dimensions of factor movements. Besides studying the effects on prices and aggregates, I conduct a welfare analysis in order to determine how the utility of individuals in one country is affected by the design of the public pension system in the other. All model results are recomputed for the demographic scenario of the year 2050 as aging will put severe pressure on public pension systems. To address the research question, I set up a two-country large scale overlapping generation model whereas the two regions are calibrated to resemble Austria and Germany.

Concerning the effects of capital mobility, the model predicts capital inflows into the economy with the more generous public pension system (Austria) thereby increasing domestic wages and decreasing the interest rate. Free movement of labor allows individuals to choose under which public pension system they want to live. In the model economy, individuals prefer living in the country with the less generous public pension system (Germany). How strong migration responses are depends highly on the level of moving costs. However, compared to empirical estimates, moving costs must be extremely large to make migration too costly. For reasonable estimates (up to 100% of annual GDP per capita), the model predicts a significant reallocation of labor. Labor mobility increases wages in Austria due to the outflow of workers and the increase in the capital to worker ratio. With regard to the welfare effects, both capital and labor mobility are shown to reduce utility costs stemming from the public pension system in Austria, whereas the opposite holds for Germany. Overall, aging is predicted to increase the migration pressure considerably. Nevertheless, the model also suggests that with appropriate policy reforms, the additional pressure could be significantly mitigated.

My paper builds on a large field of literature that has analyzed the effects of public pension systems on key economic variables. Building on the seminal work by Auerbach and Kotlikoff (1987), Imrohoroglu, Imrohoroglu, and Joines (1995) are the first to study the welfare effects of PAYG systems in the context of a large-scale OLG model. While early work has concentrated mostly on closed economy models, several studies in the last decade has shifted the attention to open economy settings. Further, this specific literature not only focuses on public pensions but rather on how the interaction of demographic change and PAYG systems affects the economy. Krueger and Ludwig (2007) demonstrate that capital mobility induces significant spill-over effects of the faster aging process and the more generous public pension systems in Europe on the U.S. economy. Moreover, Börsch-Supan, Ludwig, and Winter (2002) analyze, inter alia, intra-European capital flows, whereas Attanasio, Kitao, and Violante (2006) investigate the effects of the demographic transition in the industrialized world on developing countries. All these models have in common that capital mobility is the only dimension of factor mobility, hence they do not feature an endogenous migration decision.[1] On the contrary, Klein and Ventura (2009) who study the long-term welfare affects from abolishing all barriers to labor mobility, set up a two-country model that treats migration as a life-cycle decision influenced by individual preferences, resource costs of moving and skill losses when working abroad. By taking their model as the basic building block, my contribution consists of introducing the dimension of labor mobility to the open economy literature on public pensions and demographic change.[2]

The paper is organized as follows. In section 2, I provide an overview about the heterogeneity in costs and benefits of public pension systems in Europe. Thereafter, I describe the theoretical model (section 3). The calibration is outlined in section 4 and section 5 presents the numerical results. Finally, section 6 concludes.

[1]Within these models, migration is (implicitly) exogenously given since it is contained in the demographic projections.

[2]In the class of two period OLG models, firstly Homburg and Richter (1993) and afterwards Breyer and Kolmar (2002) discuss the implication of non-harmonized public pension systems in Europe w.r.t the efficiency of labor allocation.

2 Pension Systems in Europe

Public pensions account for the largest share of total expenditure on social security in Europe. According to Eurostat, the EU-28 countries devoted 39.1% of their social protection expenditure to old-age benefits in 2012. The second largest category was that of sickness and health-care, with a share of 28.5%. Public pension benefits are further large compared to total economic activity: In the same year, the EU-28 countries spent on average about 12.5% of GDP on old-age benefits. Even though this figure is large in every single member state, there are still significant differences w.r.t. how much of its income a country spends on public pensions. Table 1 shows the differences among selected member states.

Table 1: Public expenditures on public pensions

Country	Level % of GDP (2009)
Austria	**13.5**
Belgium	10
France	13.7
Germany	**11.3**
Italy	15.4
Portugal	12.3
Spain	9.3
United Kingdom	6.2

Source: OECD, Pensions at a Glance (2013).

The heterogeneity displayed by table 1 is striking. The costs of the Austrian pension system exceed the German ones by about 2 percentage points (costs expressed relative to GDP). Almost the same holds true for France. In 2009, Italy run the most expensive public pension system in Europe. In contrast, the Spanish one is relatively modest, which is further undercut by the UK.

Whereas the previous table reflected the cost side, table 2 reflects the spending side and displays the net replacement rates in the respective country. The net replacement rate is defined as "the individual net pension entitlement divided by net pre-retirement earnings, taking account of personal income taxes and social security contributions paid by workers and pensioners" (OECD, 2013, p. 140). Net replacement rates are shown for different earning classes, whereas "1" corresponds to the average earner, "0.5" to 50% of average earnings and "1.5" to 150% of average earnings, respectively. As it is the case with expenditures on social security, we

Table 2: Net replacement rates

Country	0.5	1	1.5
Austria	**91.2**	**90.2**	**86.2**
Belgium	72.9	50.1	39.9
France	75.9	71.4	60.9
Germany	**55.9**	**55.3**	**54.4**
Italy	78	78	77.9
Portugal	77.7	67.8	68.4
Spain	79.5	80.1	79.8
United Kingdom	61.7	38	27.2

Source: OECD, Pensions at a Glance (2013).

see a wide heterogeneity. In 2013, the average net replacement rate of the average earner in the EU-27 countries was 56.6%. Hence, Germany lies slightly below this average, Austria on the other hand exceeds it by far. In particular, only Hungary has a higher net replacement rate than Austria (for average earners) in Europe. For Germany and Austria, the two countries in the focus of this study, it is true that the more expensive public pension system also grants the higher replacement rates. Comparing the two tables, however, we find country pairs for which this relation does not hold. For example, in Spain, net replacement rates are slightly higher than those in Italy, wheres the expenditures on public pensions in Italy exceed those in Spain considerably. In general, it is of course not only the replacement rate that determines the actual costs of a pension system. Other factors include the demographic structure, labor force participation and regulations concerning the retirement age.

3 Model

The economic environment is described by a large-scale two-country OLG model. The modeling of the production side and the migration decision closely follows Klein and Ventura (2009).

3.1 Production

Each country $x \in \{h, f\}$ produces a single good using a CRS technology containing capital, labor and land as inputs. The latter input factor is assumed to be fixed and immobile. Its presence in the production function implies jointly diminishing returns to labor and capital which guarantees the existence and uniqueness of the equilibrium population distribution in the absence of moving costs. The profit maximization problem of the firm reads:

$$\max_{K_{x,t},L_{x,t}} \pi_{x,t} = Y_{x,t} - w_{x,t}L_{x,t} - (r_{x,t} + \delta)K_{x,t} - R_{x,t}F_x \tag{1}$$
$$s.t. \quad Y_{x,t} = A_{x,t}K_{x,t}^{\lambda}L_{x,t}^{\sigma}F_x^{1-\lambda-\sigma}$$

In equilibrium, factor prices equal their marginal products. They are given by:

$$r_{x,t} = \lambda A_{x,t}K_{x,t}^{\lambda-1}L_{x,t}^{\sigma}F_x^{1-\lambda-\sigma} - \delta \tag{2}$$

$$w_{x,t} = \sigma A_{x,t}K_{x,t}^{\lambda}L_{x,t}^{\sigma-1}F_x^{1-\lambda-\sigma} \tag{3}$$

$$R_{x,t} = (1 - \lambda - \sigma)A_{x,t}K_{x,t}^{\lambda}L_{x,t}^{\sigma}F_x^{-\lambda-\sigma} \tag{4}$$

TFP ($A_{x,t}$) is assumed to grow over time at the constant rate ρ_x.

3.2 Households

3.2.1 Demographics

In each period, a new generation of households is born in both countries. Populations grow at the rate n_x. Agents may live up to a maximum age of J and retire at age R. In each country x, they face an idiosyncratic mortality risk and survive from age j to age $j + 1$ with probability $\psi_{x,t,j}$ (in a given period t), where $\psi_{x,t,0} = 1$ and $\psi_{x,t,J} = 0$.

3.2.2 Decision Problem

Besides a standard life-cycle saving and consumption decision, households choose their location of residence. I assume that once an agent has migrated, he will not move back to his country of birth, hence there is no return migration. If agents decide to migrate in period t, they have to pay a fix costs of m and then become active in the other region in period $t + 1$. Due to the delay in the migration process, agents necessarily spend their first period of life in their country of origin. Further, they can migrate in all periods of their working life except the last.[3] Households are credit constrained throughout their whole life ($a_j \geq 0 \ \forall \ j$). Hence, they cannot borrow against future income (including pension claims) to pay the moving costs. Further, annuity markets are closed by assumption.

The model features heterogeneous agents. Households differ w.r.t psychic costs they face when living abroad (μ_s). Preference types $s \in S$ are realized at birth and fixed over the life-cycle. The distribution of preference types is described by the density $\alpha(s)$.

For any given time period and in each country, households maximize lifetime utility in the beginning of age 1:

$$\max \sum_{j=1}^{J} \beta^{j-1} (\prod_{k=1}^{j} \psi_{k-1}) \left[\frac{c_j^{1-\gamma}}{1-\gamma} - \mu_s \mathbb{1}_{x_j \neq y} \right] \tag{5}$$

where x_j is destination at age j and y denotes the agent's birthplace. The indicator function implies that individuals only suffer from psychic costs when they reside in the foreign destination ($x_j \neq y$). These costs are constant and do not vanish over time. From the perspective of the agents, both consumption goods are perfect substitutes. γ is the standard CRRA parameter governing the inter-temporal elasticity of substitution. Following Klein and Ventura (2009), I use $-x$ to denote the *other* location. Hence, if an individual is born in x and moves abroad, his new location is given by $-x$. The budget constraint in period t for the working period of an individual of age $j \in [1, R]$ residing in either home or foreign reads:

$$\begin{cases} (1+r_t)a_t(j) + w_{x,t}(1-\tau_{x,t})\bar{h}\epsilon(j) + tr_t = a_{t+1}(j+1) + c_t(j) + \varphi_t(j)m_{x,t} & \text{if} \quad x_t(j) = y \\ (1+r_t)a_t(j) + w_{-x,t}(1-\tau_{-x,t})\bar{h}\epsilon(j)(1-\theta) + tr_t = a_{t+1}(j+1) + c_t(j) & \text{if} \quad x_t(j) \neq y \end{cases} \tag{6}$$

I will now explain the different income sources of the households. Firstly, agents derive income from wealth which may consist of two assets, capital and land. Both asset types are divisible and individuals can invest abroad. In the open economy, two no-arbitrage conditions have to hold. The first is an intra-regional one, demanding

[3] Note that this assumption simplifies the problem without affecting the results since there is no gain from migrating in the periods excluded.

the equalization of returns on capital and land. The second is an inter-regional one and requires equal returns on investment at home and abroad:

$$1 + r_{x,t} = \frac{p_{x,t} + R_{x,t}}{p_{x,t-1}} \tag{7}$$

$$r_t = r_{x,t} \quad \forall x \in \{h, f\}. \tag{8}$$

This makes both assets identical from the individual perspective and justifies why wealth can be summarized in one single variable: $a_t(j) = k_t(j) + \sum_{x=h,f} p_{x,t-1} f_x(j)$.

The second income source is labor. Supply of labor ($\bar{h}$) is exogenous and does not differ between the regions. Wage income is taxed at the contribution rate of the pension system in each country. Labor income varies over the life-cycle due to an age-dependent efficiency profile (ϵ) and exogenous TFP growth. If agents have decided to migrate, they earn the foreign wage and pay the foreign contribution rate. Potentially, they experience efficiency losses when working abroad (θ). $\varphi_t(j)$ is equal to one if the agent migrates in t. Lastly, individuals receive a lump-sum transfer tr_t from a supranational authority.

The budget constraints for the retirement period is defined as:

$$(1 + r_t)a_t(j) + \pi_t(j_m) + tr_t = a_{t+1}(j + 1) + c_t(j). \tag{9}$$

In the retirement period, individuals save and consume and receive benefits π which - if the individual has moved abroad - are a function of the period of migration as explained in the next section.

3.2.3 Pension Benefits

In each country $x \in \{h, f\}$ there is a PAYG system in place that collects contributions from the currently working and distributes it to the retirees. The retirement systems are organized according to a *place of residence* principle, i.e. workers acquire pension claims in each country they work. Individual pension claims are set by the following rule:

$$\pi_t(j_m) = \frac{j_m b_{x,t} + (R - j_m) b_{-x,t}}{R} \quad \text{for} \quad 0 < j_m \leq R, \tag{10}$$

where b_x (b_{-x}) are the pension payments in the home (foreign) country and j_m is defined as the highest age at which the individual still works in his country of origin (equal to the period of migration if the agent migrates). R is identical in both countries. For an individual who does not move, $j_m = R$ holds so that his pension claims are equal to those paid in his country of birth ($\pi_t(R) = b_{x,t}$). If an individual has migrated, the function $\pi_t(j_m)$ basically forms a weighted average of the pension benefits paid in both countries whereas the weights are determined by how much time has been spent in either destination. Due to the dependence of pension benefits on the point of time of migration, j_m enters the household optimization problem as a state variable.

3.3 Supranational Authority

Due to the idiosyncratic mortality risk and the absence of annuity markets, a certain fraction of individuals in every period dies with positive asset holdings. I assume that there is a supranational authority that collects the bequests of the deceased and redistributes it in a lump-sum fashion to the survivors (tr_t). The assumption of a supranational authority - instead of country-specific authorities - is necessary to avoid that the transfer payments influence the migration decision.

3.3.1 Recursive Formulation

The household problem can be represented in a recursive way. Define the vector of state variables as $z = (a, s, j, j_m, x, y)$. To depict the value function we have to distinguish between different cases. If the individual has migrated in the past, the value function $V_t(z)$ is obtained by:

$$V_t(a, s, j, j_m, -x, x) = \max_{c,a'} \Big[U(c) + \beta \psi_{x,t,j} V_{t+1}(a', s, j+1, j_m, -x, x)\Big] \tag{11}$$

$$s.t. \quad c + a' = \begin{cases} (1+r_t)a + w_{-x,t}(1-\tau_{-x,t})\bar{h}\epsilon(j)(1-\theta) + tr_t & \text{if } j \leq R \\ (1+r_t)a + \pi_t(j_m) + tr_t & \text{if } j > R \end{cases}$$

$$c, a' > 0, V_t(a, s, J, j_m, x, y) = 0.$$

If migration has not taken place yet, the value function for a working agent with

$j < R$ reads:

$$V_t(a,s,j,j,x,x) = \max_{c,a'}\Big[U(c) + \beta\psi_{x,t,j}\Big\{\varphi V_{t+1}(a',s,j+1,j,-x,x) \\ + (1-\varphi)V_{t+1}(a',s,j+1,j+1,x,x)\Big\}\Big] \\ s.t. \quad c + a' = (1+r_t)a + w_{x,t}(1-\tau_{x,t})\bar{h}\epsilon(j) + tr_t \\ c, a' > 0, \varphi \in \{0,1\}. \tag{12}$$

And for an agent of age $j \geq R$:

$$V_t(a,s,j,R,x,x) = \max_{c,a'}\Big[U(c) + \beta\psi_{x,t,j}V_{t+1}(a',s,j+1,R,x,x)\Big] \\ s.t. \quad c + a' = \begin{cases} (1+r_t)a + w_{x,t}(1-\tau_{x,t})\bar{h}\epsilon(j) + tr_t & \text{if } j = R \\ (1+r_t)a + \pi_t(R) + tr_t & \text{if } j > R \end{cases} \\ c, a' > 0, V_t(a,s,J,R,x,y) = 0. \tag{13}$$

3.4 Equilibrium

I define $\Phi_t(a,s,j,j_m,x,y)$ as the mass of people with asset stock $a \in A$, type $s \in S$, age $j \in [1,J]$, last period of working in country of birth $j_m \in [1,R]$, residence $x \in \{h,f\}$ and place of birth $y \in \{h,f\}$ in period t. The measure Φ_t is defined for all A in $\mathcal{A}$, the class of borel subsets of $\mathbb{R}$, all borel subsets $S \subset \mathcal{S}$, all $j \in [1,J]$, all $j_m \in [1,R]$ and finally for $x \in \{h,f\}$ and $y \in \{h,f\}$.

Definition 1. A competitive equilibrium consists of sequences of individual functions for the household, $\{V_t(\cdot), c_t(\cdot), a'_t(\cdot), \varphi(\cdot)\}_{t=0}^{\infty}$, sequences of production plans for the firms $\{K_{x,t}, L_{x,t}\}_{t=0,x\in\{h,f\}}^{\infty}$, policies $\{\tau_{x,t}, b_{x,t}\}_{t=0,x\in\{h,f\}}^{\infty}$, transfers $\{tr_t\}_{t=0}^{\infty}$, prices $\{w_{x,t}, r_{x,t}, p_{x,t}, R_{x,t}\}_{t=0,x\in\{h,f\}}^{\infty}$ and measures $\{\Phi_t\}_{t=0}^{\infty}$ such that

1. Given prices and transfers, $c_t(\cdot), a'_t(\cdot), \varphi(\cdot)$ solve the individual's dynamic problem and $V_t(\cdot)$ are the associated value functions.

2. Factor prices satisfy (2),(3),(4).

3. Transfers are given by:

$$tr_{t+1} = \sum_{y\in\{h,f\}}\sum_{x\in\{h,f\}}\sum_{j=1}^{J}\sum_{j_m=1}^{R}\int_{\mathbb{R}\times\mathcal{S}} a'(a,s;j,j_m,x,y)(1-\psi_{y,t,s})(1+r_{t+1}) \\ d\Phi_t(a,s;j,j_m,x,y). \tag{14}$$

4. The social security budget clears in both countries:

$$\tau_{x,t} w_{x,t} L_{x,t} = Pen_{x,t}, \tag{15}$$

whereas pension payments in country x are given by:

$$\begin{aligned} Pen_{x,t} = & \sum_{j=R+1}^{J} b_{x,t} \Phi_t(\mathbb{R}, \mathcal{S}, j, R, x, x) \\ & + \sum_{j=R+1}^{J} \sum_{j_m=1}^{R} \frac{R - j_m}{R} b_{x,t} \Phi_t(\mathbb{R}, \mathcal{S}, j, j_m, x, -x). \end{aligned} \tag{16}$$

5. Markets clear in all t and x

$$L_{x,t} = \sum_{j=1}^{R} \bar{h}\epsilon(j)\Phi_t(\mathbb{R}, \mathcal{S}, j, j, x, x) + \sum_{j=2}^{R} \sum_{j_m=1}^{j-1} \bar{h}\epsilon(j)(1-\theta)\Phi_t(\mathbb{R}, \mathcal{S}, j, j_m, x, -x). \tag{17}$$

$$A^w_{t+1} = \sum_{y \in \{h,f\}} \sum_{x \in \{h,f\}} \sum_{j=1}^{J} \sum_{j_m=1}^{R} \int_{\mathbb{R}\times\mathcal{S}} a' d\Phi_t(a, s; j, j_m, x, y), \tag{18}$$

whereas total assets have to be distributed among capital and land:

$$A^w_{t+1} = K^w_{t+1} + \sum_{x \in \{h,f\}} p_{x,t} F_x. \tag{19}$$

The aggregate resource constraint is given by:

$$\begin{aligned} & \sum_{x \in \{h,f\}} Y_{x,t} + (1-\delta)K^w_t = \sum_{y \in \{h,f\}} \sum_{x \in \{h,f\}} \sum_{j=1}^{J} \sum_{j_m=1}^{R} \int_{\mathbb{R}\times\mathcal{S}} c(a, s; j, j_m, x, y) \\ & d\Phi_t(a, s; j, j_m, x, y) + K^w_{t+1} + \sum_{x \in \{h,f\}} \sum_{j=1}^{J} \sum_{j_m=1}^{R} \int_{\mathbb{R}\times\mathcal{S}} \varphi(a, s; j, j_m, x, x) m_{x,t} \\ & d\Phi_t(a, s; j, j_m, x, x). \end{aligned} \tag{20}$$

6. There are no arbitrage-opportunities as expressed by (7) and (8).

7. The cross-sectional measure is generated as explained in the appendix.

Definition 2. A stationary equilibrium is a competitive equilibrium in which all individual functions are constant over time and all aggregate variables grow at a constant rate.

In the stationary equilibrium, both the population growth rate and the TFP growth rate are constant and identical in both regions. Assume that there exists a common balanced growth path along which the capital to output ratios are constant. Then, the growth rate of aggregate output in both regions is given by: $g = \Big[(1+\rho)(1+n)^{\sigma}\Big]^{\frac{1}{1-\lambda}}$.[4] With the growth rate of output at hand, the price of land in the stationary equilibrium can be derived as follows. The return on land (4), can also be written as:

$$R_{x,t} = \frac{(1-\lambda-\sigma)Y_{x,t}}{F_x}.$$

From (7), it then follows that the price of land in period t can be rearranged in the following way[5]

$$P_{x,t} = \frac{\frac{(1-\lambda-\sigma)Y_{x,t+1}}{F_x} + P_{x,t+1}}{1+r}.$$

One can now substitute recursively for future land prices thereby expressing the current price of land as the discounted presented value of all future output per unit of land multiplied by the land share. To obtain a finite solution for the current land price, the interest rate must be larger than the output growth rate:

$$P_{x,t} = \frac{(1-\lambda-\sigma)}{r-g}\frac{Y_{x,t+1}}{F_x} \tag{21}$$

This is exactly the case when the economy is on a dynamically efficient BGP.[6]

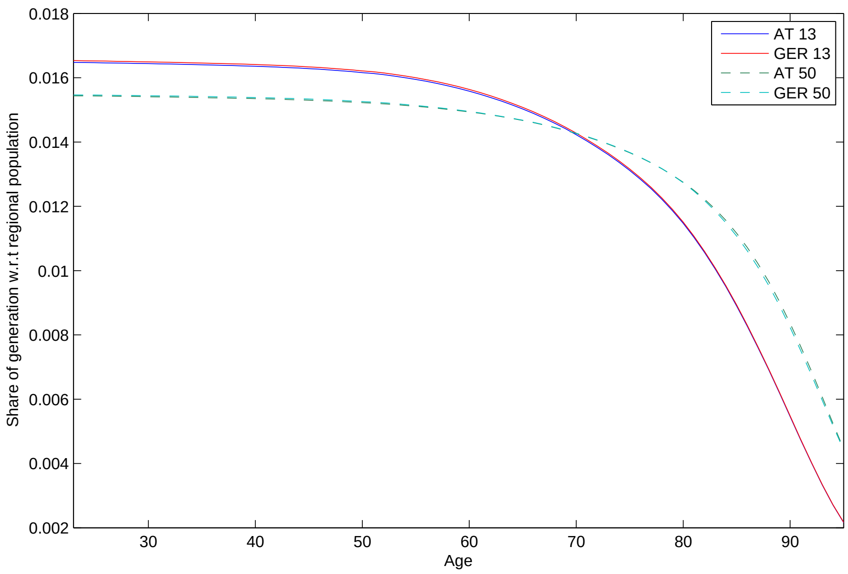
[4]Derivation is given in the appendix.
[5]Note that the interest rate loses its time index since it is constant in the stationary equilibrium.
[6]Imrohoroglu, Imrohoroglu, and Joines (1999) show that including a fix factor in an OLG model (in the same fashion as in my model) can have important implications for the welfare effects of pension systems. Basically, the presence of the fix factor rules out the possibility of dynamic inefficiency, i.e. an overaccumulation of capital. The possibility of eliminating dynamic efficiency, however, is an important feature of PAYG systems and might lead to significant welfare gains. In an economy that is always dynamically efficient, PAYG systems have much less scope for being welfare improving.

4 Calibration

4.1 Demographics

I choose the parameter values such that the two regions resemble Germany and Austria. I compute results for two demographic scenarios, one referring to the year 2013 and the other to the year 2050. For both scenarios I set the region-wide population growth rate to zero.[7] The differences in the demographic scenarios are then described by differences in the idiosyncratic survival probabilities. The corresponding data on age-specific and country-specific mortality risk, including the forecast for 2050, is taken from Eurostat. I set the maximum age equal to 95, whereas agents enter the model at age 23. I further assume agents in both regions to retire at age 65. By plotting the population distribution corresponding to the years 2013 and 2050, figure 1 documents the significant change in demographic structures in both countries.

Figure 1: Invariant Population Distribution



[7]Note that population growth is actually already negative in both countries in 2013, whereas it is only slightly negative in Austria. In 2050, population growth rates are projected to fall even further. However, forecasts suggest that the demographic trend stabilizes towards the end of the century. Since I do not want to make projections too far into the future, I assume the demographic transition to be completed in 2050 and hence set the corresponding growth rate to zero. To capture the effect of aging, I also set the growth rate of 2013 equal to zero, otherwise the change in the demographic structure from 2013 to 2050 would seem less significant than it actually is.

4.2 Productivity and Preference Parameters

Table 3: Producitvity and Preference Parameters

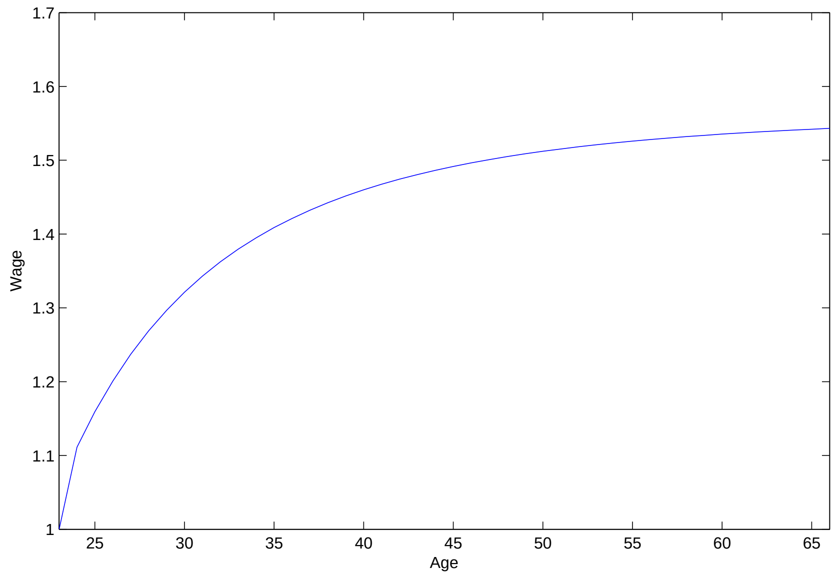
Parameters	Value	Explanation
β	0.978	Discount factor
γ	2	Intertemporal Substitution
θ	0	Efficiency loss
λ	0.317	Capital share
σ	0.632	Labor share
δ	0.081	Depreciation rate
f	1	Land per woker
ω_{AT}	0.1045	Relative population share Austria
A_{GER}, A_{AT}	1	TFP factor
$\tilde{g}_{GER}, \tilde{g}_{AT}$	0.01	Per capita output growth

The productivity and preference parameters are mainly set in accordance with Klein and Ventura (2009) who use US data to pin down the values for the capital, labor and land share in the production as well as for the depreciation rate. They are summarized in table 3. The capital share targeted is consistent with a capital to annual output ratio of 2.18. I calibrate the discount factor such that the ratio is matched. The stock of land per worker is assumed to be equal in both regions and normalized to one thereby ensuring that - in the absence of labor mobility - wage gaps do not result from differences in endowments of land. The parameter γ governing the intertemporal substitution in the CRRA utility function is set equal to two as common in the literature. For the benchmark calibration I assume the efficiency loss to be equal to zero which is based on the assumption that cultural differences between Germany and Austria are too small to actually affect productivity. In the appendix, I present a sensitivity analysis for a small value of efficiency loss. ω_{AT} denotes the relative population share of Austria in the economy without labor mobility. I calibrate this parameter w.r.t. the relative population sizes in 2013. Lastly, I set the annual per capita output growth rate ($\tilde{g}$) to 1% which equals the average of the empirical counterpart in the last 10 years. $\tilde{g}$ is identical in both 2013 and 2050.

4.3 Wage Profile

The empirical wage profile is taken from Rupert and Zanella (2015) who analyze life-cycle profiles for hours, wages and earnings. This constitutes a deviation from the related literature which mostly refers to an efficiency index constructed by Hansen

Figure 2: Empirical wage profile



(1993) to obtain the shape of the life-cycle wage profile. Hansen (1993) estimates life-cycle efficiency by computing relative average hourly earnings for different age-sex groups over the years 1979 to 1987 on the basis of CPS and BLS data. Rupert and Zanella (2015) use much more up to date data to conduct their analysis. One of their data sources is also the CPS, the other is the PSID. After the full release of the 2011 wave, the PSID data set now covers individual life-cycle profiles over 43 years. Whereas the Hansen efficiency index implies a hump-shaped wage profile, the PSID indicates rising wages over the life-cycle for cohorts who have entered the labor market during the 1960s.[8] Note that even though wages do not fall, the findings by Rupert and Zanella (2015) are not in conflict with the usual wisdom that earning profiles are falling over the life-cycle. However, earnings result from both wages and hours worked. As shown by the authors, labor supply is indeed falling towards the end of the working life. I will use the wage profile of the youngest cohort in the in the PSID data set (born between 1942 and 1946). It is restricted to men. As pointed out by Rupert and Zanella (2015), this restriction is reasonable since female labor supply exhibited significant changes in the period considered. The profile is depicted in figure 2. The wage at age 23 is normalized to one.

[8] Using a pseudo panel constructed with CPS data, Rupert and Zanella (2015) find that wages decrease slightly, however, 10-15 years later than implied by the Hansen efficiency index. Otherwise, the life-cycle profiles obtained from both data sets are almost identical.

4.4 Pension System

The data used to calibrate the pension system are provided by OECD (2013). In particular, I choose the social security contribution taxes to match the net replacement rates of average earners in each country in 2013. The net replacement rate (ζ_x) has to fulfill the following equation:

$$b_{x,t} = \zeta_x \frac{1}{R} \sum_{j=1}^{R} w_{x,t-(R-1)+j} \bar{h} \epsilon(j)(1 - \tau_{x,t-(R-1)+j}) \tag{22}$$

Pensions benefits adjust to ensure budget clearing. Hence, $t_{x,t}$ and $b_{x,t}$ can be solved from (15),(16) and (22). Note that this calibration procedure implies that a migrant's replacement rate differs from that of a stayer since he has to work at least the first period in his country of origin. The net replacement rates of Germany and Austria are given by $\zeta_{GER} = 0.553$ and $\zeta_{AT} = 0.902$, respectively.

5 Results

I solve the model in the following way: First of all, the analysis is restricted to stationary equilibria. Further, I aim at identifying the effects of each dimension of factor mobility. Therefore, I first solve the model for a closed economy scenario, in which both countries coexist in autarky. In the second step, I compute the stationary equilibrium associated with capital mobility alone. In the third step, I additionally allow for labor mobility. Besides analyzing the effects of factor mobility on aggregates and prices, I further conduct a welfare analysis. Next, I repeat the procedure outlined for the demographic scenario of the year 2050.

5.1 Capital Mobility

Table 4 compares the stationary equilibrium for two specific model variants: The closed economy and the one with mobile capital and immobile labor. I start with explaining the results of the first. Due its size, aggregate variables of Austria are much smaller than the German ones.[9] Further, net foreign asset positions are necessarily zero. Since both countries solely differ w.r.t. their public pension arrangement (besides the idiosyncratic mortality risk, whereas the differences are very small), the much more generous pension system in Austria decreases saving incentives relative

[9] See table 3 for the relative population share in the non-migration case.

to the German economy resulting in higher interest rates. The lower saving incentives are also expressed in the lower per capita capital stock. Since the marginal productivity of labor increases in the capital stock, wages need to be lower, too. The difference in land prices is explained by (21): The output per unit of land is smaller in Austria, and hence, so is the land price. In the closed economy, household income equals domestic production. Due to lower investment, production possibilities in Austria are suppressed which pushes down per capita consumption. It is important to note that the contribution rates calibrated to match the net replacement rates deviate from actual ones: The statutory German contribution rate is equal to 19,6% and hence about three percentage points higher than the model counterpart. In contrast, the model contribution rate for Austria is about two percentage points lower than the actual one (22,8%). A deviation from the actual rates is not surprising since - besides the abstraction from detailed regulations - the model misses the redistributive elements of the pension system.

Introducing capital mobility leads to an equalization of both interest rates and wages: Given the calibration of the land share, (8) reduces to the equalization of the capital to land ratios: $\frac{K^{GER}}{F^{GER}} = \frac{K^{AT}}{F^{AT}}$. If this holds, (3) then additionally implies $w_{AT} = w_{GER}$. After the introduction of capital mobility, the lower saving incentives in Austria prevail. Therefore, an equalization of interest rates can only be achieved through capital flows from Germany to Austria. In the new stationary equilibrium, Austria exhibits a strongly negative net foreign asset position (expressed relative to domestic GDP), while the German one is positive, but relatively modest.[10] Capital flows from Germany to Austria lead to an increase in the Austrian capital stock and in its production (both in aggregate as well in per capita terms). As a result of the inflow of capital, the Austrian interest rate decreases while the wage increases. Exactly the opposite is true for Germany. Pension benefits in both countries respond proportionally to wages thereby leaving the contribution rates unchanged. Price changes necessarily effect life-cycle consumption. A priori, the effect on per capita consumption is unclear since interests and wages respond in different directions. In total, per capita consumption in Austria decreases significantly, while there is a slight increase in per capita consumption in Germany. It remains to mention that from a region-wide perspective, the effect of capital mobility is only redistributive in nature: Neither the world capital stock nor world production are significantly affected.

[10]Note that the difference in the size of the effect in both regions stems from the fact that Austria is relatively small compared to Germany.

Table 4: Effects of Capital Mobility

	Closed	Capital Mobility	
	Abs. value	Abs. value	% Change
Aggregates			
K^w	0.9978	0.9966	-0.12%
K_{AT}	0.095	0.1038	8.91%
K_{GER}	0.9025	0.8928	-1.07%
Y^w	0.4568	0.4567	-0.03%
Y_{AT}	0.0463	0.0476	2.74%
Y_{GER}	0.4105	0.4091	-0.34%
NFA_{AT}	0	-1.2025	
NFA_{GER}	0	0.1393	
Prices			
r_{AT}	0.0730	0.0643	-11.94%
r_{GER}	0.0632	0.0643	1.69%
w_{AT}	0.9413	0.9671	2.74%
w_{GER}	0.9704	0.9671	-0.34%
b_{AT}	0.3447	0.3542	2.74%
b_{GER}	0.2413	0.2404	-0.34%
p_{AT}	0.5228	0.6234	19.24%
p_{GER}	0.6380	0.6233	-2.30%
τ_{AT}	0.2460	0.2460	0.00%
τ_{GER}	0.1651	0.1651	0.00%
Per capita			
k_{AT}	0.9123	0.9935	8.91%
k_{GER}	1.0078	0.9970	-1.07%
c_{AT}	0.3627	0.3450	-4.90%
c_{GER}	0.3664	0.3684	0.56%

Note: NFA denotes the net foreign asset position, here expressed relative to domestic GDP. Per capita units refer to region-specific averages.

5.2 Labor Mobility

The question of how migration decisions are influenced by the architecture of public pension systems connects to a field of literature that studies the welfare implications of pension systems to find the optimal replacement rate. The connection consists in the following way: Individuals deciding whether to move to another country with a different social security arrangement basically ask which system grants them the higher lifetime utility. Hence, before turning to the quantitative results, one can refer to the literature on optimal pensions to provide a motivation for the qualitative results of the model. In a recent study, Heer (2015) finds that the optimal replacement rate in the US economy amounts to approximately 5%. In an earlier study, Imrohoroglu, Imrohoroglu, and Joines (1999) claim that the optimal replacement rate is equal to zero. These results lead to the conclusion that the models commonly used to analyze the effects of pension schemes can not rationalize the high replacement rates observed in European countries. Therefore, - ceteris paribus - individuals in the model presented here prefer living in the economy with the lower replacement rate, i.e. Austrians would prefer living in the German economy. There are three mechanisms in the model that prevent all Austrian citizens from migrating to Germany. Firstly, the disutility of living abroad to which a certain fraction of individuals is exposed. Secondly, migration comes at a cost. Hence, even though the German pension system is preferable, the migration process itself might be too costly. Thirdly, migration outflows in one country lead to a rise in the domestic wage relative to the foreign one. Using (2), (3) and (8), the relative wage of Austria can be written as:

$$\frac{w_{AT}}{w_{GER}} = \Big(\frac{L_{AT}/F_{AT}}{L_{GER}/F_{GER}}\Big)^{\frac{1-\lambda-\sigma}{\lambda-1}} \tag{23}$$

Since the exponent is smaller than zero, a decrease in the Austrian labor input accompanied by an increase in the German one, rises the Austrian wage. Hence, at some point migration incentives diminish since the higher wage in Austria overcompensates the welfare loss stemming from the more generous pension system.

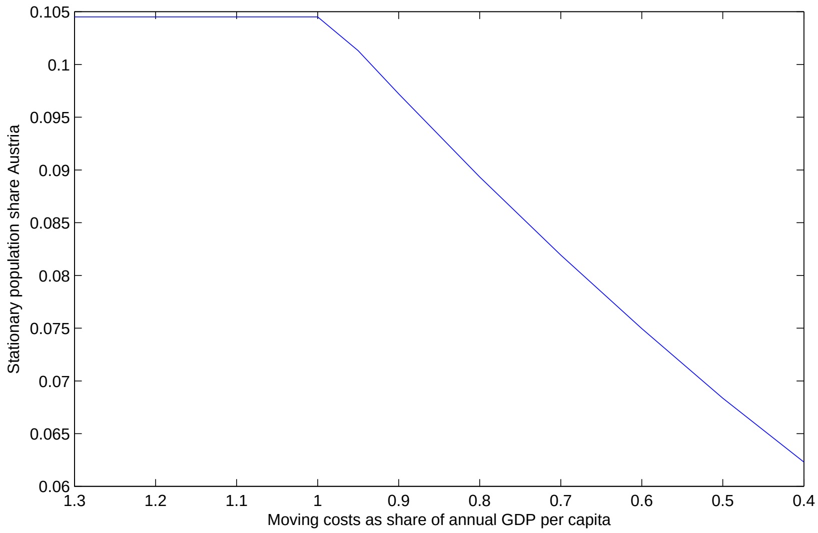
Before describing the effects of labor mobility, some further remarks concerning the parameter choice have to be made. The distribution of preference types as well as the parameter of moving costs have been left unspecified so far. Regarding the former, it is important to recall that I restrict the analysis to stationary equilibria. Hence, I focus on states in which migration might have taken place in the past, but migration incentives do not exist anymore.[11] In this respect, an important property outlined in Klein and Ventura (2009) carries over to my model: The stationary equilibria are independent of the distribution of utility costs as long as zero is in the support of the distribution, i.e. as long as there are agents facing no disutility from migrating. Those agents will always migrate if it is economic beneficial. The exact distribution of preference types then only controls the speed of convergence from one stationary equilibrium to the other. Since I do not analysis the transitional dynamics, it is sufficient to assume that there is a certain fraction (possibly small) with zero utility costs from living abroad, while the remaining part faces positive costs. Regarding the costs of moving, the parameter m could in principle be taken from empirical studies. Bayer and Juessen (2012) estimate migration costs from US interstate migration data using a structural model that explicitly takes self selection problems into account. This proves to be important because a model neglecting self selection might lead to upward-biased estimates of migration costs.[12] The authors come up with a cost estimate of about two-third of average annual household income. However, despite the fact that Bayer and Juessen (2012) provide the most reliable estimate of moving costs, the pattern of migration are known to differ greatly between Europe and the US. This impedes a direct transferability, but the estimate might still serve as a benchmark. Further, results in my model are highly sensitive w.r.t the choice of m. Hence, I will report the results for a given range of moving costs.

Figure 3 plots the population share of Austria in the new stationary equilibrium as a function of moving costs, which are themselves expressed as a share of annual GDP per capita. Every point on the curve corresponds to one stationary equilibrium associated with the specific level of moving costs. Basically, figure 3 answers the following question: After barriers to labor mobility have been removed, by how much does the relative population share of Austria have to be reduced such that the effect on wages is strong enough to switch off migration incentives. The range of migration costs comprises the values between 130% and 40% of annual GDP per capita. Between 1.3 and 1, the curve is flat, i.e. in this range moving costs are too high to induce migration. In the remaining interval, the curve is linearly declining

[11] If migration still took place, the population distribution could not be invariant which would contradict the definition of a stationary equilibrium.

[12] The reason is that the joint distribution of income and location of individuals results partly from past migration choices since migrants might have moved to the region where they are most productive. Therefore, if one does not take self selection into account, low migration rates might be attributed to high migration costs, whereas they just might be low because individuals have already selected themselves in their preferred region.

Figure 3: Stationary Population Distribution

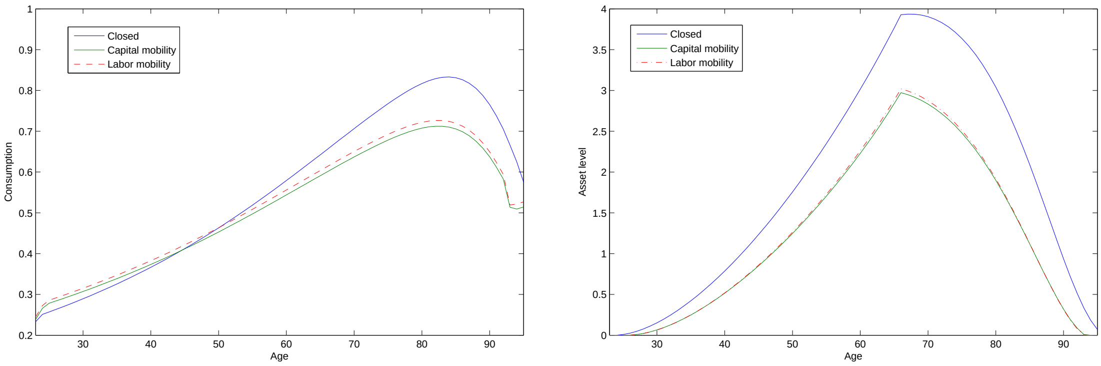


thereby showing a significant reallocation of labor. For the lowest point in the interval, the Austrian population is decreased by more than 30%. To set the results in relation to the estimate by Bayer and Juessen (2012), note that due to the negative net foreign asset position, annual average household income in Austria is lower than annual GDP per capita. More precisely, in the economy without labor mobility, the estimate of Bayer and Juessen (2012) corresponds to $m = 0.62\%$ which would involve a significant reallocation of labor.

Table 5 summarizes the effects of introducing labor mobility to the model. Results are displayed for two values of moving costs, one equal to 90% and the other equal to 60% of annual GDP per capita. The first row in the category *Aggregates* refers to the relative population share of Austria (Ω_{AT}). For a value of 90% one already sees a significant migration response. For 60%, the outflow of Austrian workers is extremely large. Moreover, the wage effect discussed beforehand becomes quantitatively visible. Austrian wages increase, while German wages decline. Interest rates fall in both countries. This is due to the fact that now more people live in Germany and therefore more people have a higher saving propensity. Again, pension benefits adjust proportionally to wages which leaves contribution rates unchanged. On the aggregate level, Austrian figures decline because of the decrease in its relative population share. As in the case with capital mobility alone, the no arbitrage condition requires capital flows from Germany to Austria. In absolute terms, the net foreign asset position becomes less negative in Austria (+5.9%) and less positive in Germany (−5.1%). One channel to explain this observation is the following. Due the inflow of workers, it requires higher investment in the German economy. However, lower wages in this region imply lower savings which reduces the foreign claims. Recall that the NFA figures in table 5 refer to the net foreign asset position relative to domestic GDP. Therefore, the change in Austrian net foreign assets is negative because the absolute NFA position increases by less than GDP decreases. While the introduction of capital mobility led to a decline in average consumption in Austria, labor mobility rises the figure. Note that despite this increase, average consumption is still below the closed economy level.

5.3 Welfare Analysis

Figure 4: Life Cycle Profiles



(a) Consumption Profile **(b)** Asset Profile

Note: The graphs refer to the Austrian economy. The blue line corresponds to the closed economy case. The green one to the case with just capital mobility. The dashed red line refers to case with labor mobility. Moving costs are equal to 60% of annual GDP per capita.

This section aims at uncovering the welfare implications of the interaction of factor mobility and non-harmonized public pension systems. In the preceding part of the section I showed that capital mobility decreases average consumption in Austria whereas labor mobility reverses the effect without leading it back to the closed economy level. However, lifetime utility is not determined by average consumption but by its allocation over the life-cycle. Figure 4 depicts the Austrian life-cycle profiles of both consumption and assets for different model variants. Due to the decline in the interest rate and the increase in wages, agents in the economy with capital mobility shift consumption to the earlier stages of life. Further, the overall asset holdings decline. By introducing labor mobility, wages increase further and the consumption profile shifts upwards.[13] How do these adjustments affect utility?

[13]Note that the kink in the consumption profiles of the open economies arises because agents are credit constrained in these late stages of life as also visible in the figure 4 b.

Table 5: Effects of Labor Mobility

	Capital Mobility	Capital and Labor Mobility			
	Abs. value	Abs. value	% Change	Abs. value	% Change
Aggregates		mc=0.9		mc=0.6	
Ω_{AT}	0.1045	0.0971	-7.09%	0.0749	-28.29%
K^w	0.9966	0.9972	0.06%	0.9989	0.23%
K_{AT}	0.1038	0.0971	-6.53%	0.0765	-26.30%
K_{GER}	0.8928	0.9002	0.83%	0.9224	3.31%
Y^w	0.4567	0.4568	0.02%	0.4569	0.05%
Y_{AT}	0.0476	0.0445	-6.56%	0.0350	-26.43%
Y_{GER}	0.4091	0.4123	0.78%	0.4219	3.13%
NFA_{AT}	-1.2025	-1.2107	-0.68%	-1.2372	-2.89%
NFA_{GER}	0.1393	0.1310	-5.92%	0.1042	-25.21%
Prices					
r_{AT}	0.0643	0.0642	-0.09%	0.0640	-0.39%
r_{GER}	0.0643	0.0642	-0.09%	0.0640	-0.39%
w_{AT}	0.9671	0.9726	0.57%	0.9922	2.60%
w_{GER}	0.9671	0.9667	-0.04%	0.9655	-0.16%
b_{AT}	0.3542	0.3562	0.57%	0.3634	2.60%
b_{GER}	0.2404	0.2403	-0.04%	0.2400	-0.16%
p_{AT}	0.6234	0.5831	-6.46%	0.4607	-26.09%
p_{GER}	0.6233	0.6289	0.89%	0.6458	3.61%
τ_{AT}	0.2460	0.2460	0.00%	0.2460	0.00%
τ_{GER}	0.1651	0.1651	0.00%	0.1651	0.00%
Per capita					
k_{AT}	0.9935	0.9996	0.61%	1.0211	2.77%
k_{GER}	0.9970	0.9970	0.00%	0.9971	0.01%
c_{AT}	0.3450	0.3467	0.50%	0.3527	2.26%
c_{GER}	0.3684	0.3681	-0.08%	0.3671	-0.35%

Note: The first column contains the outcomes of the stationary equilibrium shown in table 4. The second and fourth columns contain values of stationary equilibria associated with labor mobility. The second corresponds to migration costs equal to 90% of annual GDP per capita, the fourth to 60% of annual GDP per capita. Columns three and five display the percentage changes relative to the stationary equilibrium with immobile labor.

To address this question I compute the consumption equivalent measure (Δ) to display the change in welfare. In particular, Δ denotes the percentage change in consumption necessary to make the individual indifferent between living in the closed and open economy. Δ is calculated from:

$$(1+\Delta)^{1-\gamma} V_{closed} = V_{open}, \tag{24}$$

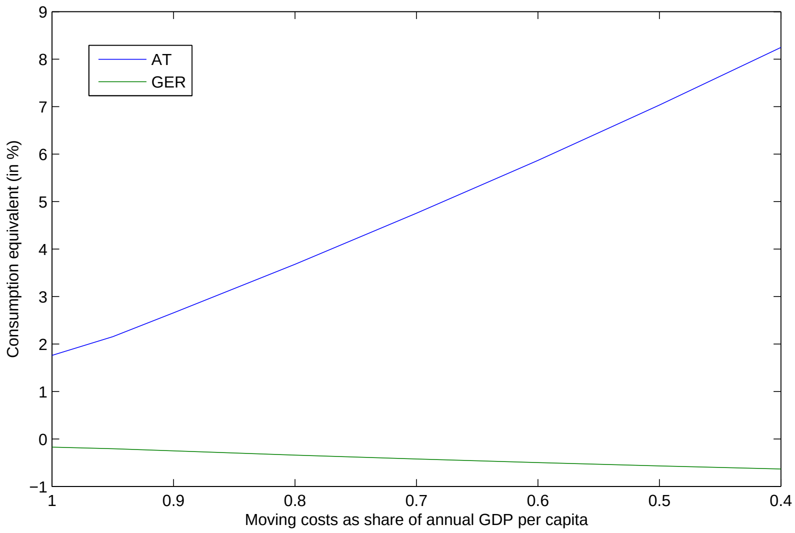
where V is stationary lifetime utility. The welfare effects are depicted in figure 5. Again, results are plotted for a certain range of moving costs. The highest level of moving costs considered amounts to 100% of annual GDP per capita which is exactly the lowest value for which migration is too costly. Hence, at the most left point of the x-axis one sees the pure welfare effect of capital mobility. From there onward, migration takes place and contributes to the welfare change. One can see that the introduction of capital mobility rises welfare in Austria and decreases it in Germany. Further, the lower migration costs and the stronger the reallocation of labor, the more pronounced are the welfare effects (while not changing the direction). So why does the different shape of the consumption profile in figure 4 induce a welfare gain? First of all, due to discounting the higher consumption at younger ages implies a positive utility change. Moreover, consumption is reduced where it is already relatively high. Hence, the concavity of the utility function makes the positive welfare effect even stronger. For residents in Germany, welfare changes are much less intense since the effects on prices are smaller.

The results imply an important pattern: The economy with the low replacement rate partly takes over the negative effects of the more generous public pension in the foreign country. Firstly, capital mobility triggers an outflow of capital and decreases wages. Secondly, the inflow of foreign workers reinforces the decrease in wages thereby leading to an overall decline in welfare. On the other hand, the high replacement economy benefits from factor mobility since it attenuates the negative effect of its pension system.

5.4 The Effects of Aging

The results presented so far show that the responses of capital and migration flows to differences in public pension systems can be large. An obvious question is how these results change under the influence of population aging which is predicted to put a severe pressure on public pension systems. Due to its enormous impact, the

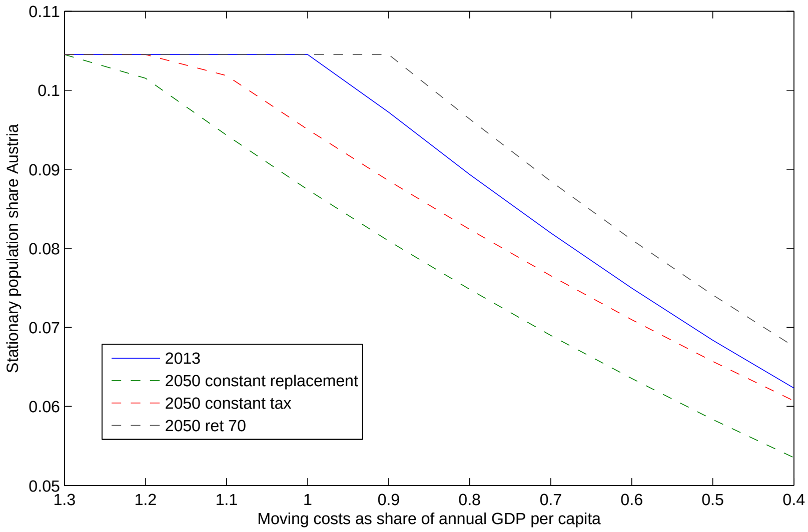
Figure 5: Welfare Effects



demographic change will demand serious reforms of the social security systems to maintain their sustainability. For this reason, I follow Krueger and Ludwig (2007) and analyze the consequences of demographic change for three different policy scenarios. Firstly, I consider an adjustment of the social security tax, while keeping the replacement rate constant, i.e. on the level of 2013. Secondly, I adjust the replacement rate, while keeping the contribution tax constant. Lastly, I increase the statutory retirement age to 70 (with a constant replacement rate).

Figure 6 extends figure 3 by additionally showing the relative population of Austria in 2050 for each policy scenario. The following observations can be made: Keeping the replacement rate constant in both countries shifts the curve to the left. In this scenario, moving costs need to amount to 130% of GDP per capita to make migration too costly. For the lowest level of moving costs depicted, the Austrian population is reduced by almost 50%. The second policy scenario under consideration leaves the contribution rates unchanged. In that case, the model responses are less pronounced. A striking result is that increasing the retirement age to 70 weakens the migration responses relative to the year 2013. The significant differences in the migration responses arise because welfare costs increase non-linearly in the distortionary tax. Hence, although keeping the replacement rate constant increases both contribution rates (and the German one even slightly more, see table 6), the welfare loss is considerably larger for agents in the Austrian economy which triggers a stronger migration response than in economy of the year 2013. The opposite is true for the scenario of the increase in the retirement age.

Figure 6: Stationary Population Distribution



Note: The blue line corresponds to 2013. All other curves refer to the year 2050. The dashed green line depicts results for the case when the replacement rate is kept constant, the dashed red line holds the tax rate constant. The dashed dark red line considers the case when the retirement age is increased to 70.

Table 6 displays the effect of aging on aggregates and prices, whereas the analysis is restricted to policy scenarios 1 and 3.[14] Everything else equal, population aging has three (partly opposing) effects on factor prices. Firstly, the higher life expectancy reduces the population share of the young relative to the old. Since in the life-cycle model the young save whereas the old dissave, aggregate savings decline thereby pushing up the interest rate. Secondly, the decrease in the working age to population ratio reduces labor supply and makes labor scarce relative to capital which increases wages and decreases the interest rate. These two effects constitute the *direct effects*. There is an additional *indirect* effect working through the social security system: If the contribution rate has to increase to keep replacement rates stable, this reduces private savings thereby driving up interest rates. Additionally, in the model with labor mobility, the reallocation of labor influences factor prices as described in the previous sections.

For the case of a constant replacement rate, the scarcity of labor increases wages in both countries. Relative to 2013 (for $m = 90\%$), migration responses are considerably larger and the Austrian population decreases by almost 17%. This additional outflow of workers leads to a stronger increase in the Austrian than in the German

[14] Results for policy scenario 2 can be found in the appendix.

Table 6: Demographic Change

	2013	2050			
	Abs. value	Abs. value	% Change	Abs. value	% Change
Aggregates		Constant ζ		R=70	
Ω_{AT}	0.0971	0.0809	-16.65%	0.1045	7.64%
K^w	0.9972	0.9905	-0.68%	1.0580	6.09%
K_{AT}	0.0971	0.0816	-15.96%	0.1104	13.73%
K_{GER}	0.9002	0.9089	0.97%	0.9476	5.27%
Y^w	0.4568	0.4393	-3.83%	0.4816	5.44%
Y_{AT}	0.0445	0.0362	-18.62%	0.0503	13.04%
Y_{GER}	0.4123	0.4031	-2.23%	0.4314	4.63%
NFA_{AT}	-1.2107	-1.3567	-12.07%	-1.0389	14.18%
NFA_{GER}	0.1310	0.1234	-5.87%	0.1210	-7.65%
Prices					
r_{AT}	0.0642	0.0596	-7.17%	0.0633	-1.38%
r_{GER}	0.0642	0.0596	-7.17%	0.0633	-1.38%
w_{AT}	0.9726	1.0050	3.33%	0.9660	-0.68%
w_{GER}	0.9667	0.9842	1.81%	0.9661	-0.06%
b_{AT}	0.3562	0.3515	-1.31%	0.3826	7.41%
b_{GER}	0.2403	0.2369	-1.42%	0.2559	6.47%
p_{AT}	0.5831	0.5186	-11.07%	0.6701	14.92%
p_{GER}	0.6289	0.6719	6.84%	0.6690	6.37%
τ_{AT}	0.2460	0.2799	13.78%	0.2131	-13.37%
τ_{GER}	0.1651	0.1916	16.06%	0.1416	-14.22%
Per capita					
k_{AT}	0.9996	1.0079	0.83%	1.0563	5.67%
k_{GER}	0.9970	0.9889	-0.81%	1.0582	6.14%
c_{AT}	0.3467	0.3320	-4.24%	0.3650	5.27%
c_{GER}	0.3681	0.3507	-4.73%	0.3877	5.34%

Note: The table compares the model outcomes for the years 2013 and 2050. All cases refer to a stationary equilibrium in the model variant with labor mobility. Moving costs are equal to 90% of GDP per capita. The analysis for 2050 is distinguished for the case of a constant replacement rate (columns 2 and 3) and the case of a retirement age of 70 (columns 4 and 5).

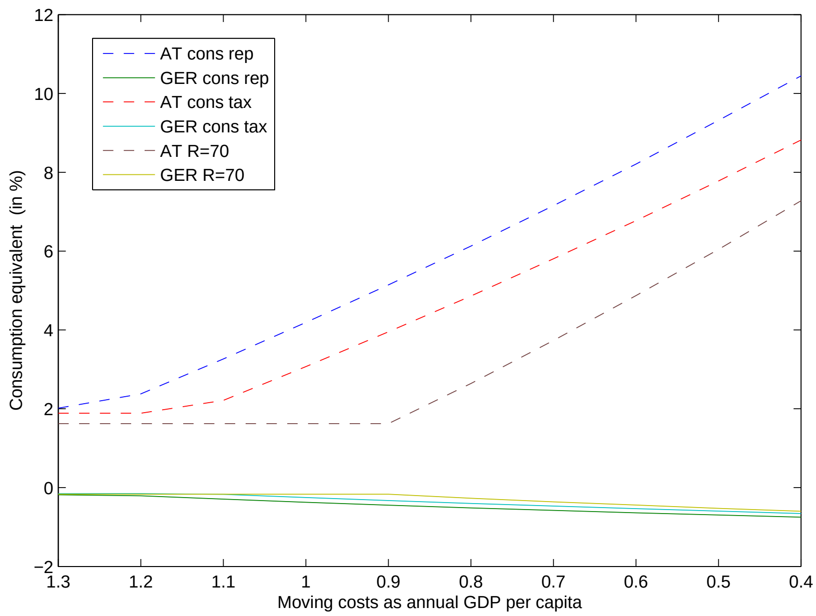
wage. Concerning the interest rate, the relative abundance of capital and the larger reallocation of labor dominate the other two effects (higher savings and higher contribution rate) and pushes down the interest rate by about 7%. Even though capital is relatively abundant, the world capital stock decreases in absolute terms. This, in conjunction with lower labor input, decreases world output and per capita consumption in both countries.

If the retirement age is increased, the model shows a different response of factor prices: The decline in the interest rate is less pronounced and wages even decrease. In this context it is crucial that - despite the population aging - the higher retirement age increases the working age to population ratio thereby partly reversing the effects from the first policy scenario. Due to the higher labor supply, wages fall. Further, figure 6 shows that in the stationary equilibrium with $R = 70$ and $m = 0.9$, compared to 2013 relatively more people reside in Austria. Hence, wages in Austria are lower. Abstracting from labor movements, the higher working age to population ratio puts downward pressure on the interest rate since it implies larger aggregate savings. This effect is strengthened as the decline in the contribution rates additionally encourages private savings. However, as the reallocation of labor is less strong then in 2013, relatively more people are exposed to the higher replacement rate and exhibit a lower saving propensity. This counteracts the additional downward pressure on the interest rate by the longer working life and the decrease turns out to be less severe. In total, the higher retirement age leads to a significant increase in world production and average consumption in both regions. The prolonging of the working life[15] therefore seems to be an effective measure to cope with the economic implications of population aging in general and the associated larger responses of factor mobility in particular.[16]

The welfare effects for the economy of the year 2050 exhibit a similar pattern to the ones presented in section 5.3. Basically, welfare changes resulting from capital mobility alone are close to the ones in figure 5. However, due to the larger reallocation of labor under policy scenario 1 and 2, welfare changes are more pronounced.

[15] In Krueger and Ludwig (2007), the increase in the retirement age is also shown to significantly mitigate the consequences of demographic change.

[16] A few words of caution are required here. As explained before, setting the common population growth rate in 2050 to zero understates the degree of the demographic change. Under negative population growth rates, the effects of the first two scenarios might be larger, whereas the increase in the retirement age might not be strong enough to overcompensate the negative effects of aging.



Note: The consumption equivalent measure is computed relative to the closed economy in 2050. The dashed lines refer to Austria, the solid lines to Germany. "Cons rep" refers to a constant replacement rate, "cons tax" to a constant contribution rate and "ret 70" to an increase of the retirement age to 70.

6 Conclusion

Factor mobility between countries that differ regarding the generosity of their pension systems entails significant consequences for prices, aggregates and welfare. The study quantifies these effects using a two-country large-scale OLG model calibrated to resemble a common economic area consisting of Austria and Germany. With a replacement rate of 90.2% for average earners, Austria runs a much more generous public pension system than Germany which grants a replacement rate of 55.3% for the average earner. Facing the higher net replacement rate, Austrian citizens in the model economy exhibit a lower saving propensity than Germans leading to a higher interest rate and a lower wage. The introduction of capital mobility to the model triggers large capital flows between the countries. In the new stationary equilibrium, Austria features a strongly negative net foreign asset position, equal to 120% of Austrian GDP. Likewise, the Austrian interest rate falls by about 1 percentage point while the wage increases by almost 3%. Extending the model by labor mobility leads to an outflow of workers from Austria whereas the strength of the reallocation effect depends highly on the level of moving costs. The threshold at which moving costs are just low enough to induce a positive migration response is equal to 100% of annual GDP per capita. For a level of 90%, the new stationary equilibrium is characterized by a reduction of the Austrian population by about 7%. Since more people reside in the low-replacement economy, the region-wide interest rate decreases slightly, while the wage in Austria increases relative to the German one (0.6%). Overall, factor mobility and the associated change in factor prices allows for a welfare improving life-cycle consumption path in Austria, while the opposite holds for Germany. Population aging will challenge the sustainability of European welfare states. The model predicts that the aging process considerably increases the economic forces resulting from capital and labor mobility. Keeping the replacement rate on the level of 2013, the economy of the year 2050 exhibits a threshold of moving costs (for inducing migration flows) of 130% of annual GDP per capita. Further, the effects on factor prices, aggregates and welfare are significantly stronger. However, the model also predicts that suited policy reforms are able to limit the economic forces released by population aging. In this regard, an increase of the retirement age to 70 is shown to be an effective measure. The results presented in this study suggest that the interplay of factor mobility and non-harmonized public pension systems - and social security arrangements in general - implies important general equilibrium effects that cannot always be foreseen by policy makers. A harmonization of social security arrangements would therefore promote a stronger connection between intention and impact of social policy measures.

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7 Appendix

7.1 Sensitivity analysis: Skill loss

Figure 7: Stationary Population Distribution

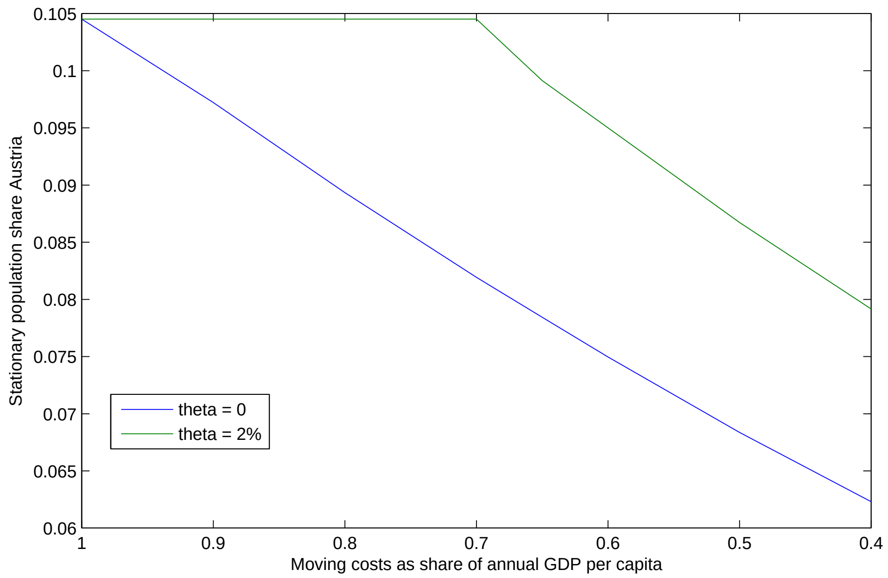

Table 7 shows the reallocation of labor in 2013 for the case without skill losses (as in figure 3) and for the case with a skill loss equal to 2%. Introducing skill losses extends the interval of moving costs for which migration is too costly (until m=70%) and therefore weakens the migration pressure significantly. In this context, two aspects should be noted. Firstly, compared to the benchmark estimate taken from Bayer and Juessen (2012), the migration response is still positive for $m = 0.62$. Secondly, in the economy of the year 2050, the curve would be further shifted to the left. Overall, their may be temporary skill losses for Austrians migrants in Germany (and vice versa) shortly after migration, however, a permanent skill loss seems to be implausible. Nevertheless, the exercise shows that the degree of labor movements depends strongly on the cultural similarity between countries.

7.2 Effects of aging - constant contribution rate

Table 8: Demographic Change II

	2013	2050	
	Abs. value	Abs. value	% Change
Aggregates		Constant τ	
Ω_{AT}	0.0971	0.0885	-8.82%
K^w	0.9972	1.0273	3.02%
K_{AT}	0.0971	0.0919	-5.28%
K_{GER}	0.9002	0.9354	3.91%
Y^w	0.4568	0.4444	-2.70%
Y_{AT}	0.0445	0.0398	-10.54%
Y_{GER}	0.4123	0.4046	-1.86%
NFA_{AT}	-1.2107	-1.2362	-2.11%
NFA_{GER}	0.1310	0.1218	-7.04%
Prices			
r_{AT}	0.0642	0.0561	-12.55%
r_{GER}	0.0642	0.0561	-12.55%
w_{AT}	0.9726	1.0099	3.83%
w_{GER}	0.9667	0.9963	3.06%
b_{AT}	0.3562	0.3104	-12.85%
b_{GER}	0.2403	0.2066	-14.02%
p_{AT}	0.5831	0.6128	5.09%
p_{GER}	0.6289	0.7251	15.29%
Per capita			
k_{AT}	0.9996	1.0384	3.88%
k_{GER}	0.9970	1.0263	2.94%
c_{AT}	0.3467	0.3352	-3.30%
c_{GER}	0.3681	0.3525	-4.25%

Note: The table compares the model outcomes for the years 2013 and 2050. All cases refer to a stationary equilibrium in the model variant with labor mobility. Moving costs are equal to 90% of GDP per capita.

7.3 Cross-sectional measure

Newborns arrive according to:

$$\Phi_{t+1}(A,S;1,1,x,y)=\begin{cases}\frac{\Omega_{x,t}(1)}{\Omega_{x,t}}\int_S \alpha(s)ds & \text{if } 0\in A \text{ and } x=y\\ 0 \text{ else}\end{cases},$$

where $\Omega_{x,t}$ is the relative population of region x in period t, and $\Omega_{x,t}(1)$ is the relative share of newborns. Hence, $\frac{\Omega_{x,t}(1)}{\Omega_{x,t}}$ denotes the birth rate.

Due to endogenous migration, one has to keep track of the movements of agents across regions over time. The policy function associated with the migration decision is used to describe the following recursion. I start with the mass of individuals located in region x who still reside in region x in the next period (who did not move). For $1<j<R$:

$$\begin{aligned}&\Phi_{t+1}(A,S;j+1,j+1,x,x)\\ &=\int_{\mathbb{R}\times\mathcal{S}}(1-\varphi(a,s;j,j,x,x))I\{a'_t(a,s;j,j,x,x)\in A\}d\Phi_t(a,s;j,j,x,x)\psi_{x,t,j},\end{aligned}$$

and for $R\le j<J$

$$\Phi_{t+1}(A,S;j+1,R,x,x)=\int_{\mathbb{R}\times\mathcal{S}}I\{a'_t(a,s;j,R,x,x)\in A\}d\Phi_t(a,s;j,R,x,x)\psi_{x,t,j}.$$

Further, agents might have migrated from region $-x$ to region x. The mass of foreign-born in region x in period $t+1$ comprises those who already have migrated in the past and those who migrate in period t. For the new arrivals of age $j\in[2,R]$:

$$\begin{aligned}\Phi_{t+1}(A,S;j+1,j,x,-x)=\int_{\mathbb{R}\times\mathcal{S}}&\varphi(a,s;j,j,-x,-x)I\{a'_t(a,s;j,j,-x,-x)\in A\}\\ &d\Phi_t(a,s;j,j,-x,-x)\psi_{-x,t,j},\end{aligned}$$

For the past arrivals of age $j\in[2,J-1]$ and all $j_m\in[1,\min\{j-1,R-1\}]$[17]:

$$\begin{aligned}\Phi_{t+1}(A,S;j+1,j_m,x,-x)=\int_{\mathbb{R}\times\mathcal{S}}&I\{a'_t(a,s;j,j_m,x,-x)\in A\}\\ &d\Phi_t(a,s;j,j_m,x,-x)\psi_{-x,t,j}.\end{aligned}$$

[17] $j=R-1$ is the last period in which an agent can migrate.

7.4 Computation

In the following section I address two features of the model that require adjustments of rather standard algorithms used to compute stationary equilibria in large-scale OLG models. Firstly, the non-concavity of the problem and secondly the steady state indeterminacy.

7.4.1 Applying the endogenous grid method to non-concave problems

To deal with the non-concavity, I follow Fella (2014) who generalizes the endogenous grid method (EGM) developed by Carroll (2006). The discrete choice contained in the migration decision and the fix costs of moving make the choice set non-convex. This, in turn, implies that the optimal policy correspondence may not be continuous and that the value function may not be differentiable. In this case, one usually has to rely on global solution methods which have the disadvantage of being notoriously slow. The basic idea behind the algorithm developed by Fella (2014) is to partition the problem into one part where the highly efficient method by Carroll (2006) can be smoothly applied and into another one where a global solution method is required.

EGM reverses the standard solution method for finding the optimal next period asset level. The standard procedure involves setting up a grid $\mathcal{G}_A$ for the initial asset level and solves the Euler equation for each point on the grid. On the contrary, the endogenous grid method defines a grid for the next period asset level ($\mathcal{G}_{A'}$) and solves the Euler equation backwards. Since the Euler equation is often linear in the initial asset level, but non-linear in the next period asset stock, the EGM avoids costly root finding and reduces computational time considerably.

In general, for non-concave problems the Euler equation is neither necessary nor sufficient for a global maximum, however, Fella (2014) argues that in the class of problems considered here the Euler equation still holds at a local maximum. Building on this property, the algorithm divides the grid for future assets ($\mathcal{G}_{A'}$) into a *concave region* ($\mathcal{G}_{A'}^{c}$) where the Euler equation is both necessary and sufficient and into a *non-concave region* ($\mathcal{G}_{A'}^{nc}$) where a global solution method is used to verify the solution obtained by EGM. If both solutions coincide, i.e. the local maximum is also a global one, the solution is saved, otherwise it is discarded. The important feature of the algorithm is that the use of the slower global method is restricted to a subset of $\mathcal{G}_{A'}$. To identify the non-concave region, one ought to take a look at the first order condition associated with the Bellmann equation:

$$U(a, a') = \beta\psi_j \frac{\partial V(a', \tilde{z})}{\partial a'},$$

where $\tilde{z}$ denotes all state variables but the next-period asset choice. The Euler equation is sufficient for $a'_i \in \mathcal{G}_{A'}$ to be a global maximum, if a'_i is the unique intersection between the upward sloping curve $U(a, a')$ and the downward sloping curve $\frac{\partial V(a', \tilde{z})}{\partial a'}$. The intersection is unique if for all $a'_j \in \mathcal{G}_{A'}$, it holds that $\frac{\partial V(a'_j, \tilde{z})}{\partial a'} > \frac{\partial V(a'_i, \tilde{z})}{\partial a'}$ for all $j < i$ and $\frac{\partial V(a'_j, \tilde{z})}{\partial a'} < \frac{\partial V(a'_i, \tilde{z})}{\partial a'}$ for all $j > i$. The regions of the value function for which this condition is not fulfilled delimits the non-concave region. Given $\tilde{z}$, the boundaries of the non-concave region (v_{min}, v_{max}) can be computed as the lowest value of $V(a'_i, \tilde{z})$ and the highest value of $V(a'_{i+1}, \tilde{z})$ such that $V(a'_{i+1}, \tilde{z}) > V(a_i, \tilde{z})$. To project the boundaries onto the grid of future assets, one can calculate $\underline{i}$ as the smallest i for which $V(a'_i, \tilde{z}) < v_{min}$ and $\bar{i}$ as the largest i for which $V(a'_i, \tilde{z}) > v_{max}$.

In the following I outline the pseudo code. I restrict the description to a preference type $s \in S$ who actually migrates.

1. In period J: For all $j_m \in [1, R-1]$, all $x \in \{h, f\}$ and all $a_i \in \mathcal{G}_{A'}$ obtain

$$\begin{aligned}
c(a_i, s, J, j_m, -x, x) &= (1+r)a_i + \pi(j_m) + tr \\
a'(a_i, s, J, j_m, -x, x) &= 0 \\
V(a_i, s, J, j_m, -x, x) &= u(c(a_i, s, J, j_m, -x, x)) \\
V_a(a_i, s, J, j_m, -x, x) &= u_c(c(a_i, s, J, j_m, -x, x)) \\
\Lambda_a(a_i, s, J, j_m, -x, x) &= V_a(a_i, s, J, j_m, -x, x)^{\frac{-1}{\eta}}.
\end{aligned}$$

   Note that I compute a transformation of the derivative of the value function ($\Lambda_a(\cdot)$). The idea is to use the transformation for the interpolation later on since it is much more linear than the derivative of the value function itself.

2. In period $j = J-1, \ldots, R+1$: The function $\Lambda'_{a'}(a', s, j+1, j_m, -x, x)$ is known from the previous step. Invert it to obtain $V_{a'}(\cdot)$. For all $j_m \in \{1, R-1\}$, all $x \in \{h, f\}$ and $a'_i \in \mathcal{G}_{A'}$, solve

$$u_c = \beta\psi_j V_{a'}(a'_i, s, j+1, j_m, -x, x),$$

   in conjunction with the budget constraint for consumption (c_i) and beginning of period assets (a_i^{beg}). One can then save the policy functions and update the value functions. Note that this involves interpolation since policy and value functions have to be defined on the grid $\mathcal{G}_{A'}$. Therefore interpolate policy and value functions on the grid $\mathcal{G}_{A^{beg}}$ for all $a_i \in \mathcal{G}_{A'}$.

$$\begin{aligned}
c(a_i, s, j, j_m, -x, x) &= c_i \\
a'(a_i, s, j, j_m, -x, x) &= a'_i \\
V(a_i, s, j, j_m, -x, x) &= u(c(a_i, s, j, j_m, -x, x)) + \beta\psi_j V(a'_i, s, j+1, j_m, -x, x) \\
V_a(a_i, s, j, j_m, -x, x) &= u_c(c(a_i, s, j, j_m, -x, x)) \\
\Lambda_a(a_i, s, j, j_m, -x, x) &= V_a(a_i, s, j, j_m, -x, x)^{\frac{-1}{\eta}}.
\end{aligned}$$

3. In period $j = R, \ldots, 1$: The individual cannot migrate in the last period of working life (R). During the remaining periods, however, migration is possible. After migration has been taken place, the problem becomes concave since he cannot migrate back. For an individual who has not migrated, I first compute the continuation value associated with migrating and then the one associated with staying. As above, for all $a_i \in \mathcal{G}_{A'}$:

$$\begin{aligned}
v(a_i, s, j, j, x, x)^{stay} &= u(c^{stay}(a_i, s, j, j, x, x)) + \beta\psi_j V(a'_i, s, j+1, j+1, x, x) \\
v(a_i, s, j, j, x, x)^{migrate} &= u(c^{migrate}(a_i, s, j, j, x, x)) + \beta\psi_j V(a'_i, s, j+1, j, -x, x).
\end{aligned}$$

Note that the problem of non-concavity arises when computing the value v^{stay} since $V(a'_i, \cdot)$ may not be differentibale due to the discrete choice as displayed in (12). Therefore, I apply the refinement of the EGM as outlined before. For all $a_i \in \mathcal{G}_{A'}$, obtain:

$$V(a_i, s, j, j, x, x) = \max\{v(a_i, s, j, j, x, x))^{stay}, v(a_i, s, j, j, x, x))^{migrate}\}$$

Further, if $v(a_i, s, j, j, x, x)^{migrate} > v(a_i, s, j, j, x, x)^{stay}$:

$$\begin{aligned}
\varphi(a_i, s, j, j, x, x) &= 1 \\
j_m &= j.
\end{aligned}$$

7.4.2 Indeterminacy of stationary equilibria

As Klein and Ventura (2009) point out, the presence of moving costs implies a indeterminacy of stationary equilibria. More precisely, lump-sum costs of moving create a continuum of population distributions and hence a continuum of pairs of wages and pensions in both countries, for which there are no migration incentives. This has the following implication for the computation. As standard, one can iterate over the the aggregate variables $\{K^w, \Omega_{AT}, Tr\}$ to obtain a solution for the stationary equilibrium.[18] However, the values computed by the algorithm describe only one solution in the entire interval of all possible stationary equilibria. This raises the question of which stationary equilibrium to choose. Starting from the non-migration stationary equilibrium, I look for the new stationary equilibrium with the population distribution that is closest to the non-migration one. At this point, migration will just have stopped. In order to find this specific stationary equilibrium, I have to apply a numerical routine that scans the interval of possible population distributions $[\Omega_1^-, \Omega_1^+]$[19], whereby the first value equals the lowest relative population size in Austria consistent with a stationary equilibrium and the second value equals the highest one (the value of interest). The routine follows Klein and Ventura (2009) and can be summarized as follows:

- Take the non-migration (Ω_1^{*nomig}) and the migration stationary equilibrium computed (Ω_1^{*mig}) as inputs. Since $\Omega_1^{*mig} \leq \Omega_1^+$ holds, the solution has to be part of the interval $[\Omega_1^{*nomig}, \Omega_1^{*mig}]$. Hence, set $[\Omega_1^-, \Omega_1^+] = [\Omega_1^{*mig}, \Omega_1^{*nomig}]$.

- Guess $\Omega_1^0 \in [\Omega_1^-, \Omega_1^+]$. Solve for a stationary equilibrium with Ω_1^0 assuming that no one moves.

- Verify whether the stationary equilibrium is stable when migration is allowed: If not, set $\Omega_1^+ = \Omega_1^0$ and return to step 2. Otherwise set $\Omega_1^- = \Omega_1^0$ and return to step 2.

- Iterate until $\frac{|\Omega_1^+ - \Omega_1^-|}{\Omega_1^+} \leq \epsilon$

7.5 Output growth rate

The object of interest is the growth rate of aggregate output for a given constant population growth rate n, and TFP growth rate ρ. I drop the index x, since growth rates are identical in both regions.

[18]Note that one only has to iterate over the relative population distribution of one country since $\Omega_{GER} = 1 - \Omega_{AT}$ follows directly.

[19]The subscript 1 indicates the first region.

$$
\begin{aligned}
\frac{Y_{t+1}}{Y_t} &= \frac{A_{t+1}K_{t+1}^{\lambda}N_{t+1}^{\sigma}F^{1-\lambda-\sigma}}{A_t K_t^{\lambda} N_t^{\sigma} F^{1-\lambda-\sigma}} \\
&\Leftrightarrow \frac{(1+\rho)A_t(1+g_k)^{\lambda}K_t^{\lambda}(1+n)^{\sigma}N_t^{\sigma}F^{1-\lambda-\sigma}}{A_t K_t^{\lambda} N_t^{\sigma} F^{1-\lambda-\sigma}} \\
&\Leftrightarrow (1+g) = (1+\rho)(1+g_k)^{\lambda}(1+n)^{\sigma}.
\end{aligned}
$$

Along the BGP, the capital-to-output ratio is constant which implies $g = g_k$. From this it follows:

$$
\begin{aligned}
(1{+}g) &= (1+\rho)(1+n)^{\sigma}(1+g)^{\lambda} \\
\Leftrightarrow g &= \Big[(1+\rho)(1+n)^{\sigma}\Big]^{\frac{1}{1-\lambda}} - 1.
\end{aligned}
$$