"DARM and DALM, generalized portfolio insurance for asset only and asset-liability approaches"

Preliminary Draft

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The literature around portfolio insurance is quite prolific. From the CPPI to the TIPP, the TPPI and the Dynamic Core Satellite, research has been done in different directions, to solve the pitfalls of the initial CPPI.

This paper brings together the different research fields in the domain and proposes a generalized form of portfolio insurance that we call DARM (Dynamic Asset and Risk Management). In this generalized form, we go from two to n asset classes by embedding several TPPI structures. This new form of portfolio insurance that accounts for diversification, allows for a severe decrease of the monetization probability, along with an improved expected return.

After an initial application to the asset only world, we have extended the research to the presence of liabilities, in a Defined Contributions setting. In this generalized form of portfolio insurance that takes into account liabilities, that we call DALM (Dynamic Asset and Liability management), the risk measure takes into account the funding ratio and the risk free asset is a liability-hedging portfolio.

The paper is organized as follows: in the first part, we review the advances made in portfolio insurance. The second part presents a generalized form of portfolio insurance, called DARM and its extension to the presence of liabilities. In the third part, we present a case study of DALM construction for a DC pension fund. The last part concludes.

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[2] Grigoriu, Johny, Merelle and Paris (2015) http://active-asset-allocation.com/wp-

1- Literature review

CPPI (Constant Proportion portfolio insurance) techniques have been introduced by Merton (1971), Black and Jones (1987), Perold (1986) and Perold and Sharpe (1988). It is a two assets world only: the risk free asset and the risky asset. The restriction of the investable universe to two assets, mainly dictated by the simplifying mathematical hypothesis made about the market completeness, takes away all hope for diversification. The main problem encountered by the CPPI is monetization. Once the cushion is consumed, there is no hope of performance until the end date. This feature comes from the perfect correlation between the floor and the risk-free asset. The other drawback of the initial CPPI comes from the way that the multiplier was calibrated (on historical data) and the fact that it was supposed to be fixed throughout the life of the product (which implies sub optimal risk-taking as the probability of breaching the floor is not stable through time).

In an attempt to offer an easy to implement approach to portfolio insurance, Estep & Kritzman (1988), propose the TIPP (Time Invariant Portfolio Protection, calling it "a simple modification of the CPPI". The main point is to make the approach independent from the market conditions on the starting date by constructing a floor that would protect the portfolio from declining below a pre-set value. This maximum drawdown floor is also adjusted to the highest values reached by the portfolio such that it protects a higher and higher level of wealth. The main achievement however is to de-correlate the floor from the risk free asset, giving the approach the opportunity to re-generate cushion in case the portfolio level reaches the floor.

While the CPPI and the TIPP approaches aim at controlling the absolute performance of the portfolio, Amenc, Malaise, Martellini (2004) extend the portfolio insurance principles to the relative performance amongst two assets, the core and the satellite. The method leads to increasing the allocation to the satellite when its performance exceeds that of the core and vice-versa.

Amenc, Goltz and Grigoriu (2009) develop the Dynamic Core Satellite idea to construct applications to absolute return funds and tactical asset allocation with ETFs. They show that this approach enables for a dissymmetrical returns by truncating the relative return distribution below a certain level towards a greater potential outperformance.

The interesting contribution that we see in these two papers is the idea of using CPPI like principles in the presence of two risky assets, as opposed to a risk-free and a risky asset. It opens the way to diversification, both in the use of assets and the underlying investment vehicles (i.e. active versus passive).

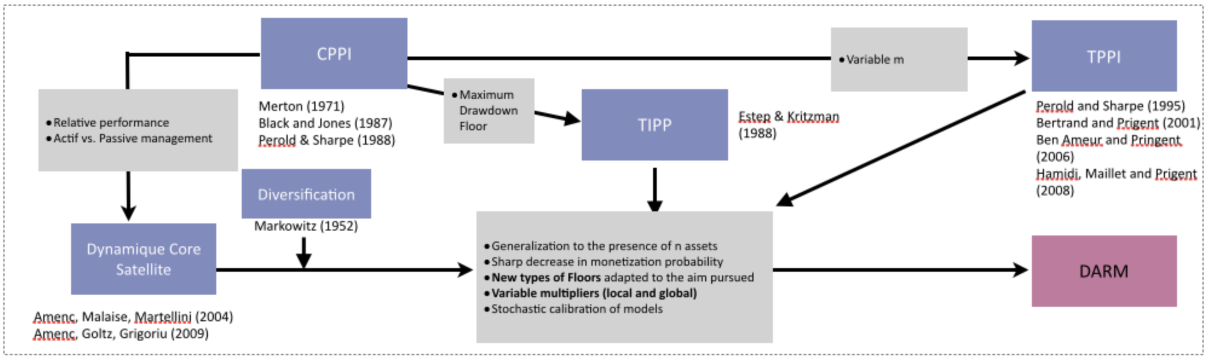
The issue of the multiplier is also very central in portfolio insurance. As stated by Perold and Sharpe (1995), "the market can fall by as much as 1/m with no rebalancing before the floor is endangered". The main reason the earliest CPPI type investments failed is because of an inappropriate choice of the multiplier m, based on historical market behavior of the risky asset or on some sort of maximum utility function. A first attempt of improving the choice of the multiplier is made by Bertrand and Prigent (2002), who introduce the extreme value approach to determine an upper bound for the multiple m.

Then Ben Hameur and Pringent (2007) examine the CPPI method where the multiple is allowed to vary over time, using a quantile approach to provide explicit values of multiple as function of the past asset returns and volatility. Finally, Hamidi, Maillet and Prigent (2008) compare various conditional multiple models in portfolio insurance strategies and introduce the Time Varying Proportion Portfolio Insurance (TPPI). The aim is to adapt the multiplier to market conditions in order to keep the probability of reaching the floor constant over time. What we find appealing in these papers is the use of stochastic simulations as opposed to historical data or utility function to determine the optimal (or maximum) value for the multiplier. Being able to keep the risk stable over time by varying the multiplier is an additional interesting feature.

2- The DARM and DALM

2.1 Presentation of the DARM technique

DARM is a generalized portfolio insurance technique, which uses concomitantly the different extensions made to the initial CPPI strategy: the idea of employing different floors (among which the TIPP), the relative return extension that allows for using 2 risky assets (as opposed to a risk free and a risky asset), and the stochastic determination of multipliers boundaries. Furthermore, we show that a two-asset DARM can be a component (core or satellite) of another DARM, with no limitations, until all available assets are employed.

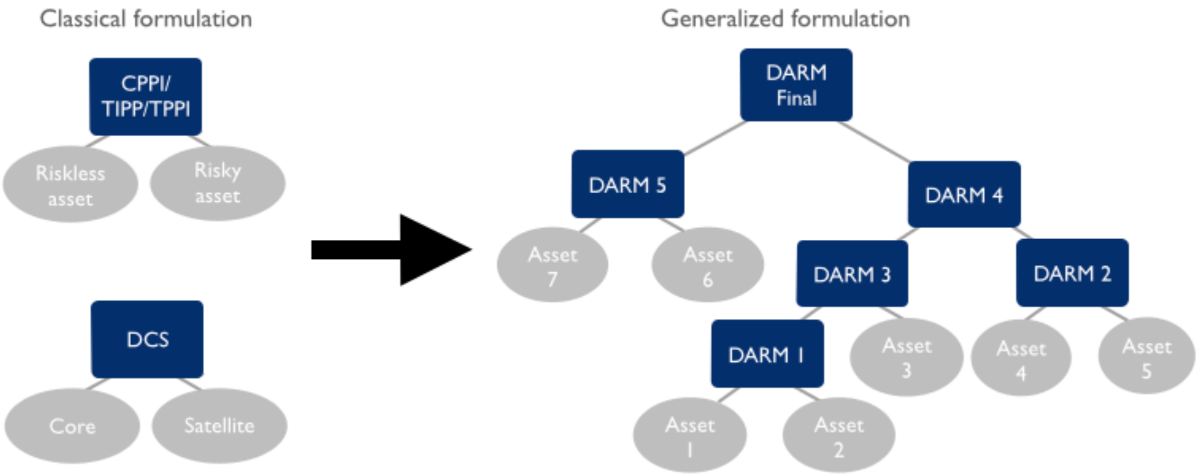


The floor is a convenient way of expressing the investor's goals: a CPPI-type floor aims at ensuring capital preservation at a given horizon, while a TIPP-type goal aims at insuring against a maximum loss. They are usually of the form: $k * F(t)$
where k is a constant and $F(t)$ a function of time that relates to the investor's objective.
Different floors can be developed to mimic the desired behavior. In a structure where several DARMs are embedded, the floors can help shape the desired return distribution.

The floor is also the special ingredient that allows the extension towards relative performance or risk management amongst two risky assets. It can be constructed in such a way that the portfolio insurance mechanism dynamically switches the portfolio towards the asset with the smallest on-going drawdown. The relative floor is usually designed in such a way that the risk budget has an in-built regenerating capability. Once the possibility of combining two risky assets is established, there is no limit anymore in unbedding several DARMs, each having his own core, satellite, multiplier, floor and allocation constraints.

Finally, the use of stochastic scenarios for the assets (i.e. using the Filtered Historical Simulation approach[2]) allows for the determination of a suitable multiplier for each DARM, that can be adjusted to move in an inverse direction to the risk in the market.

Example of a generalized DARM

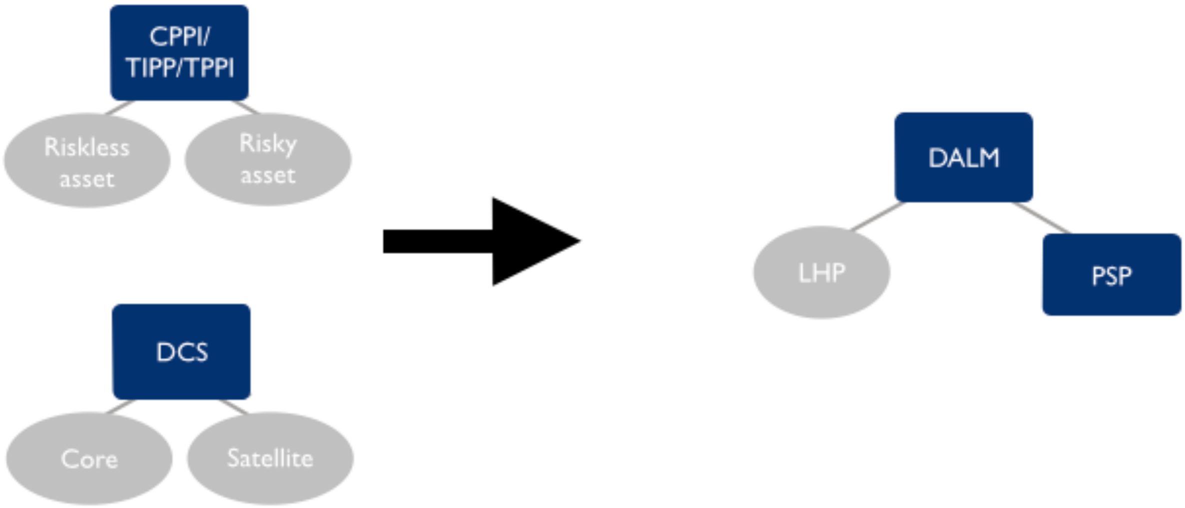


The calculation of the weights w to be given to each asset derives from the initial CPPI/TIPP/TPPI/DCS formula, w=m*C where
w is the weight given to the risky asset or the satellite
m is the multiplier
C is the cushion
In the generalized formulation,
$w_{A5} = w_f * w_4 * w_2$ where
w_{A5} is the weight of Asset 5 in the final portfolio
w_f is the weight given to DARM 4 in DARM final
w_4 is the weight given t o DARM 2 in DARM 4
w_2 is the weight given to Asset 5 in DARM 2

2.3 Presentation of the DALM technique

The DARM extension to the presence of liabilities requires the construction of a liability-hedging portfolio (LHP) that can be used as a "safe asset" and that of a performance-seeking portfolio (PSP) that is used for diversification and performance purposes. It also requires the definition of the right goal. For a pension fund, such a goal could be to make sure that the funding level does not fall below a given level, in line with regulation and/or with the sponsor's company ability to pay extra contributions. The goal translates easily into a TIPP-type floor on the funding ratio.

[2] Grigoriu, Johny, Merelle and Paris (2015) http://active-asset-allocation.com/wp-content/uploads/2015/02/Risk-letter-3-FHS.pdf



The LHP can take several forms, going from a perfect hedge to a liability hedge. The PSP is a final DARM, composed by all the assets the investor wants to include. In order to minimize the monetization risk, the PSP should have a certain behavior, in particular to become more and more correlated to the liabilities when there is stress on the other assets.

To properly calibrate the model in a stochastic environment, we take into account the joint Asset-Liability dynamic. This can be achieved by the use of the same bootstrap matrix in the FHS simulations of the yield curve and the assets. Furthermore, the fixed income asset prices should not be computed from history, but from the simulated yield curve.

3- Case study of DALM construction for a DC pension fund

This example illustrates how the DALM approach can be used by a pension fund (PF) and how the results obtained compare with other available options, meaning a strategic allocation (determined by a traditional ALM study), a traditional LDI approach and a de-risking approach.

Lets suppose a US DC pension fund with a funding ratio of 80% and liabilities with duration of 12.6 years. In the US, the Pension Protection Act requires that if the funding ration (FR) falls below 100%, the deficit should be amortized over 7 years and contributions be made if needed to stick to the amortization plan. We suppose that the PF's internal policy requires an end of year exceptional contribution if the FR falls below 80% in order to bring it back to 80%.

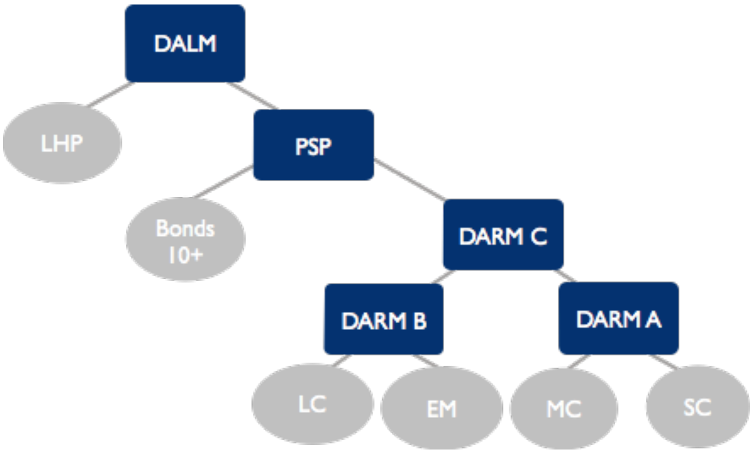
The study is conducted in a stochastic environment. We generate 1,000 scenarios for the various asset classes that we use (bonds and equities) as well as the yield curve and the liabilities of the pension fund, with a FHS approach.

The assets that we take into account for the study are the following: US corporate bonds 10+ (same duration as the PF liabilities), US Large Caps, US Mid Caps, US Small Caps and

Emerging Market equities. They are the same in all strategies in order to make the strategies comparable.

We analyze each strategy (DALM, strategic allocation, LDI and de-risking) in the light of the 1,000 different scenarios, and the results are presented in quartiles form for each one of them. All strategies are rebalanced on a monthly basis. The details of the four strategies is as follows:

DALM: we construct a DALM portfolio in which the LHP is invested in US corporate bonds 10+. The liability hedging portfolio is a duration hedge only, thus the portfolio is not protected against non-parallels shies in the yield curve. We also construct a PSP that is a generalized DARM. The DALM floor is a 86% maximum drawdown floor on the funding ratio (allowing for a maximum funding loss of 14%). We use a fixed multiplier of 4.



Strategic allocation: it is a 50% allocation to US corporate bonds 10+ and 50% allocation to equities.

Traditional LDI strategy: 100% invested in US corporate bonds 10+.

De-risking: initial allocation identical to the strategic allocation. With every 2% increase in the absolute funding ratio, we switch 5% of the equities to bonds until the portfolio becomes 100% invested in US corporate bonds 10+.

In order to analyze the results of the study, we look at the **funding ratio** at the end of 10 years for each strategy and the **level of contributions** that were necessary.

Funding ratio - end of 10 years					
Scenarios	Q 97.5%	Q 75%	Q 50%	Q 25%	Q 2.5%
(1) Traditional diversification 50%/50%	1,95	1,41	1,19	1,01	0,84
(2) Traditional LDI	1,38	1,06	1,01	0,97	0,85
(3) Strategy 50/50 -> 100% Bonds	1,43	1,08	1,02	0,98	0,86
(4) DALM	2,21	1,44	1,22	1,04	0,88

The table shows the funding ratios obtained by the pension fund at the end of 10 years. Thei calculation takes into account the contributions made by the sponsor company. We observe a statistical dominance of the DALM Strategy.

Cumulated contributions (PV) - end of 10 years					
Scenarios	Q 2.5%	Q 25%	Q 50%	Q 75%	Q 97.5%
(1) Traditional diversification 50%/50%	0	37,9	100	176,2	372,8
(2) Traditional LDI	51,6	94,5	104,7	123,4	203,2
(3) Strategy 50/50 -> 100% Bonds	0,9	48,3	95,4	145,1	281,8
(4) DALM	0	24,7	64,6	114,4	211,8

Table showing the present value of the contributions that have been necessary to make over the 10 years in order to obtain the funding ratios showed in the previous table. The DALM strategy that allows to have at the end of the 10 year period the best funding ratios is also the least expensive in 97,5% of the scenarios; in 2,5% of the most extreme scenarios, the cost of the strategy is only slightly greater than the LDI strategy.

4- Conclusion

The aim of the paper is to present a generalized form of portfolio insurance, called DARM. It can be applied from a risk perspective to all kind of portfolios (single or multi asset, sectors, geographies…) using all kind of underlyings (direct investments, smart beta funds, ETF, actively managed funds). It blends together different researches done over the past 30 years or so, to improve the initial CPPI approach. The DARM approach allows for the use of portfolio insurance techniques in a world with n asset classes, deriving from the diversification opportunities a much lower monetization probability and a much higher expected return than any two-assets portfolio insurance technique.

The DARM extension to the presence of liabilities, called DALM, proves to be an efficient alternative to the alternative approaches used by pension funds. The US DC pension fund case study shows to which extent the use of a DALM strategy with the clearly defined objective of protecting a certain level of funding ratio can be beneficial for pension funds, as it leads to improved funding throughout the distribution when compared to other investment strategies. It is a useful alternative from the sponsor point of view.

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