Cohort mortality risk or adverse selection in the UK annuity market?

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Abstract

The "money's worth" measure has been used to assess whether annuities are fairly valued and also as evidence for adverse selection in the annuity market. However, a deterministic money's worth calculation is problematic from the point of view of a risk-averse life assurer or a regulator worried about the solvency of the life assurer. In this paper we document the considerable uncertainty that annuity providers and regulators face in predicting long-run mortality and show this affects money's worth estimates. We provide a simple model of the effect of cohort mortality risk on the money's worth and show that we cannot identify the effect of our model from that of an adverse selection model. Using a Lee-Carter model of mortality we quantify the effect of cohort mortality risk and show that it is quantitatively important. We conclude that the empirical finding of differences in the money's worth for differing types of annuities may be due as much to risk as to adverse selection.

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1.   Introduction

Ever since the development of the theoretical model of Rothchild and Stiglitz (1976) identifying the rôle of asymmetric information in insurance markets, the search for empirical evidence on adverse selection has yielded conflicting findings depending on the characteristics of the particular market (Cohen and Siegelman, 2010). The market for life annuities has been studied in a number of papers (Mitchell et al, 1999; Finkelstein and Poterba, 2002, 2004) who have examined the pricing of life annuities using the *money's worth* metric i.e., the ratio of the expected value of annuity payments to the premium paid.[1] Two stylised facts that emerge from this literature are that typically (i) the money's worth is less than one; and (ii) the money's worth of back-loaded annuities (where the expected duration is longer) or "real" annuities is less than that for level or conventional annuities. For example, Table 5 of Finkelstein and Poterba (2002) reports that the money's worth of level annuities for 65-year old males is 0.900, but for real annuities is 0.825. These two observations have been interpreted as evidence of adverse selection, that annuitants have more information about their life expectancy than insurance companies, which is then reflected in the equilibrium annuity prices.

However, in this paper we demonstrate that these facts would also be consistent with a model where there were no adverse selection and where the variation in annuity rates for different types of annuity were due to the different costs of supplying annuities. Either because life assurers are prudent or because of regulatory requirements, riskier liabilities such as real annuities have to be priced to ensure sufficient reserves are available and matched to similar real assets and these effects make them more costly. We identify three additional costs for real annuities: greater cohort risk; greater idiosyncratic risk and greater management

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[1] James and Song (2001) provides an international comparison of money's worth studies. Cannon and Tonks (2008, ch. 6) report further money's worth calculations for the United Kingdom and also review other articles on the money's worth for the UK (Murthi, Orszag and Orszag, 1999); for Chile (Rocha and Thorburn, 2006); for Switzerland (Bütler and Ruesch, 2007); for Australia (Knox, 2000); and for Singapore (Fong, 2002). Since then further analyses have been conducted for Canada (Milevsky and Shao, 2011); for Germany (Kaschützke and Maurer, 2011); for the Netherlands (Cannon, Stevens and Tonks, 2012); for Singapore (Fong, Mitchell and Koh, 2011); and for Switzerland (Bütler and Staubli, 2011). The money's worth has also been used in analysis of decision making by Fong, Lemaire and Tse (2011).

costs. The last two additional costs could, in principle, be measured fairly easily if adequate data were available and in this paper we discuss the extent to which this is possible. The first additional cost will be shown to be more problematic. The route by which cohort risk and adverse selection affect annuity prices is the same, namely the duration of the annuity. This makes identifying the importance of the two explanations for annuity prices difficult or impossible. In this paper we quantify thecosts of the risks and show they are sufficiently large to explain much of the observed variations in the money's worth, leaving little or no rôle for adverse selection.

We approach this problem by modelling explicitly the risky nature of an annuity liability. In all of the papers that we have cited the money's worth calculations are "deterministic", in the sense that it is impicitly assumed that survival probabilities for the cohort of annuitiants are known. Even if a life assurer has a sufficiently large pool of annuitants to diversify away idiosyncratic risk, it still needs to forecast the future survival probabilities and these forecasts are obviously risky. Combined with prudential or regulatory reasons to avoid downside risk this means that, the greater the risk of a liability, the greater the reserves needed by the life assurer to ensure that the liability can be met. In this paper we introduce the concept of the *stochastic money's worth* which takes into account the uncertainty faced by annuity providers predicting long-run mortality. We suggest that estimates of the stochastic money's worth are the appropriate risk metric for life assurers.

The attitude of public policy to the pricing of annuities is multi-faceted. Low annuity rates result in lower income streams for pensioners and the risk that they will be entitled to more means-tested benefits; as voters they may also complain more. For these reasons governments wants annuity rates to be high. On the other hand, high annuity rates correspond to higher future annuity payouts by providers for a given premium, raising questions about the providers' future solvency. In its capacity as financial regulator, the government would not want annuity rates too high if this meant that annuity providers might end up failing, because then pensioners would end up in poverty and the government would have to compensate them either through a direct rescue or by paying higher means-tested benefits.

In the United Kingdom, where the compulsory annuity market is large worth £11 billion per year (HM Treaury, 2010b) and plays an important rôle in pension

provision, the government has faced both problems in the recent past, and continues to do so. Annuity rates have fallen consistently since 1994, which has proved politically sensitive, prompting the Department of Work and Pensions to investigate analyse the cause of these reductions (Cannon and Tonks, 2009). In the popular press, low annuity rates have been cited as a reason for removing the compulsory annuitisation requirement in the UK.[2]

At the same time, the UK government is still having to deal with the failure of Equitable Life to provide sufficient reserves for a set of guaranteed annuities sold in the 1980s. This became apparent in the late 1990s and resulted in a court case in 2000 (Equitable Life Assurance Society v Hyman) followed by the Penrose Report of 2004. The most recent "Abrahams" report (Parliamentary and Health Service Ombudsman, 2008) found that the financial regulators in the United Kingdom (initially the Department of Trade and Industry, DTI, and then the Financial Service Authority) had made errors over a ten-year period in the regulation of Equitable Life, dating from the time of the original problem in the 1990s and continuing even after the court case. Although the products sold were not conventional immediate annuities, the conclusion is clear: that maladministration by financial regulators can result in a government liability. The total cost to the UK government is expected to be £1.5 billion (H.M. Treasury, 2010a) or approximately 0.1 per cent of UK GDP.

Even apart from the Equitable Life débâcle and before the financial crisis of 2007, the regulator was encouraging life assurers to price conservatively. For example, the chairman of the Financial Services Authority wrote to life assurers recognising that companies would usually make assumptions based on their own mortality experiences, but adding

"...if this is not possible we would expect firms to consider the different industry views in this area and to err on the side of caution." (FSA Dear CEO letter, April 2007)

A further motivation for appropriately assessing the risks associated with annuities is the proposed EU-wide changes to insurance regulation enshrined in *Solvency II,* which will take effect from 2013. Solvency II applies to the insurance industry the risk-sensitive regulatory approach adopted in the Basel 2 reforms for the banking

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[2] For example: *The Telegraph*, 12 May 2012, "Annuity rates have plunged to such low levels that they are not now a 'viable option' for millions of savers at retirement."

industry. Under the proposal for Solvency II, life insurance companies are required by the regulatory framework to allow explicitly for uncertainty in their valuations:

> *"the technical provision under the Solvency II requirement is the sum of the best estimate and the risk margin, . . ., the best estimate is defined as the probability-weighted average of future cash flows . . . The probability-weighted approach suggests that an insurer has to consider a wide range of possible future events: for example, a 25% reduction in mortality rates may have a small probability of occurrence but a large impact on the cash flows. However, the assumptions chosen to project the best estimated cash flows should be set in a realistic manner, whereas the prudent allowance for data uncertainty and model error should be taken into account in the risk margin calculation."* (Telford et al, 2011; paras. 7.2.1 - 7.2.2.3).

As is made explicit in text books such as Booth et al (2005), actuaries take risk into account when pricing annuities. What is less clear is exactly how life assurers do this, but some idea of the magnitude of the issue can be gained in the UK from the life assurers' Returns. Each life assurer must delare the actuarial assumptions used to value its liabilities, by comparing the mortalities used in its own calculations with the mortalities in the benchmark tables produced by the Institute of Actuaries' Continuous Mortality Investigation. The CMI collects data from all of the major life assurers, aggregates and anonymises it and then analyses the pooled data. So the CMI tables of mortality approximate to the average mortalities across the whole industry. The figures presented in life assurers' FSA returns are then compared to this average and we summarise the figures for the major annuity providers in Table 1 and illustrate in Figure 1.

[Table 1 about here]

[Figure 1 about here]

The biggest problem for researchers is knowing which benchmark series to use. Finkelstein and Poterba (2002) used the "life office pensioner" mortality table, which reports mortalities of members of occupational defined-benefit pension schemes administered by life assurers: the most recent version of this table is PCMA00. Since annuitants are defined-contribution (DC) pensioners, this is clearly the wrong series. The original DC pensions, retirement annuity contracts for self-

employed workers, have their own mortality tables, of which the most recent are RMC00 and RMV00.[3] But the more recent DC personal pensioners are described by PPMC00, which is currently based on a relatively small sample. In Figure 1 we plot all four possible benchmarks.

Up to age 67, most life assurers assume lower mortality than the most widely used mortality tables PCMA00 or RMC00, with only Canada Life assuming higher mortality. But for ages greater than 68, every life assurer assumes lower mortalty rates than all four possible benchmarks. So, even if there is some doubt about which benchmark mortality table to use, every life assurer is assuming that their annuitants' mortality is lower than the average (so life expectancy is greater than average). Some of the variation in assumptions between companies must be due to genuine variations in mortality of the annuitants purchasing from each company, but it is obviously impossible that every company has lower mortality than the average. This is *prima facie* evidence that firms are building some allowance for mortality risk into their valuations, although they are almost certainly making extra allowance elsewhere in their calculations.

In this paper we propose a novel approach to analyse the effect of risk on annuity pricing. Life assurers are typically able to hedge their annuity liabilities with assets such as government or very high quality corporate bonds.[4] RPI-linked annuities are backed by index-linked bonds, predominantly issued by the government.[5] It is

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[3] RMV is for pensioners in receipt of a pension ("vested") and RMC is for both pensioners in receipt of a pension and for those still making contributions ("combined").

[4] Any company providing long-term life assurance business must provide detailed accounts to the regulator referred to as the FSA Returns. Where investing in corporate bonds results in a higher yield (a risk premium), life assurers are not allowed to use this to value their liabilities. For example, see the note in *Norwich Union Annuity Lim*ited, Annual FSA Insurance Returns for the year ended 31st December 2005 (page 53): "In accordance with PRU 4.2.41R, a prudent adjustment, excluding that part of the yield estimated to represent compensation for the risk that the income from the asset might not be maintained or that capital repayments might not be received as they fall due, was made to the yield on assets." The return goes on to say that AAA-rated corporate bonds had yields reduced by 0.09 per cent, A-rate by 0.32 per cent and commercial mortgages by 0.41 per cent.

[5] In the U.K., where annuities are sold that are adjusted to inflation, it is possible to hedge indexed annuities by purchasing government bonds that are indexed to the same price index (i.e., the Retail Price Index, or RPI). The FSA Returns make

impossible for life assurers to match the duration of their assets precisely with the duration of their liabilities, so there are small additional costs to managing cash flow: these may be bigger for RPI-linked annuities where the market for bonds is smaller. An implicit assumption of the remaining large risk faced by a life assurer is the group mortality risk of the annuitants (the number of annuities sold is sufficiently large that idiosyncratic annuitant mortality risk can be ignored). We introduce a stochastic money's worth metric, and compute the distribution of the present value of annuity payments that allows for uncertainty in the cohort mortalities.

It is possible to quantify mortality risk through relatively recently developed stochastic mortality models such as that of Lee and Carter (1992), which allow us to estimate the probability distribution of future mortality. The traditional risk-neutral approach to pricing annuities is to set the price equal to the expected present value of the promised annuity payments, yielding a money's worth equal to unity (or slightly less than unity after allowing for the loadings associated with the annuity provision). The Solvency II / Basel 2 approach often uses the Value-at-Risk (VaR) as a guide to suitable reserving, so a Regulator could use our estimates of the distribution of annuity values to calculate VaRs and examine the effect that VaR pricing would have on conventional measures of the money's worth. By VaR pricing we mean that insurance providers price off the tail of probability distribution of future mortality such that there is a 95 per cent chance of having sufficient assets to meet the actual risky liabilities. This approach allows us to answer two questions:

First, given our estimates of the risks to life assurers, can we say anything about how these will affect the money's worth and how the money's worth will change when yields change?

Second, what are the consequences of the pricing of different annuity products (e.g. real versus nominal) on the money's worth?

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explicit that the different types of annuities are backed by different assets. For example, the note in *Norwich Union Annuity Limited*, Annual FSA Insurance Returns for the year ended 31st December 2005 (page 50): "Non-linked and index-linked liabilities are backed by different assets and hence have different valuation interest rates."

We start by briefly reviewing the theory of adverse selection in the annuity market and discuss whether it is appropriate to characterise the market as being a separating equilibrium. We then describe the conventional money's worth measure and how it is calculated in practice. In section 4 we then review the evidence for the money's worth in the UK. In section 5 we show how a probability distribution of the value of an annuity can be constructed using a stochastic mortality model. We use this to measure the risk for annuities and the consequences when a researcher calculates the money's worth based on a deterministic projection of mortality while annuity providers are pricing to take into account the financial risk associated with mortality risk and a given set of interest rates.

2.   Money's Worth Calculations

2.1   The (Conventional) Money's Worth

The conventional measure of the value of an annuity is the money's worth (Warshawsky, 1988; Mitchell et al, 1999). To fix notation and frame the discussion we consider the simplest annuity product: a *level annuity* which pays constant nominal payments at monthly intervals,[6] purchased by a man currently just age 65, with the first payment being made at the point of purchase. In a world of perfect certainty the present value of an annuity with monthly payments would be evaluated at time t as

(1) $$\text{annuity value} \equiv \sum_{x=65}^{\infty} R_{t,(x-65)} I_x \quad x = \left\{ 65, 65\tfrac{1}{12}, 65\tfrac{2}{12}, \ldots \right\}$$

where $R_{t,j}$ is the discount factor evaluated at time t which determines the present value of £1 received j periods hence and $I_x \in \{0,1\}$ is an indicator variable showing whether or not the annuitant is alive at age x. Notice that we measure age and time in years, but payments are monthly. The expression in (1) is the value of an annuity (usually denoted a in actuarial notation). Define the *annuity rate* as the ratio of annual payments to the actual purchase price of an annuity as A. For example, in the UK in July 2009, the Prudential would sell an annuity for £10,000 to a 65-year old man which would pay a monthly income of £61, or £732 annually for

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[6] Other frequencies of payment (eg quarterly) are possible, but rare in the UK.

life: the annuity rate would be $A_{2009,65} = 732/10,000 = 0.0732 = 7.32\%$. Then the money's worth in a world of perfect certainty would be

$$(2) \qquad \text{Money's Worth}_{\text{Perfect Certainty}} \equiv \frac{A_{t,65}}{12} \sum_{x=65}^{\infty} R_{t,(x-65)} I_x \quad x = \left\{ 65, 65\tfrac{1}{12}, 65\tfrac{2}{12}, \ldots \right\}$$

A life assurer with zero transactions costs could set the annuity rate so that the money's worth would equal one and just break even, sometimes called "actuarially fair" pricing. In reality both $R_{t,j}$ and I_x are unknown. Not only must the life assurer make assumptions about how to price the annuity, but the economist analysing the data with the benefit of hindsight must try to decide what the life assurer should have used to calculate the money's worth at the time. If the life assurer and the economist were to make the same assumptions, then absent transaction costs, the money's worth would still equal unity so long as the life assurer were risk neutral, and hence expected profits were zero.

A real annuity makes payments which rise over time in line with a price index. In the UK this is the Retail Price Index (RPI).[7] The government issues inflation-indexed bonds, also linked to the RPI. Denoting $R^*_{t,j}$ to be the discount factor at time t for a payment of £1 in time t's real terms j periods hence and the annuity rate $A^*_{t,65}$ as the annuity rate based on the initial annual payment divided by the premium the money's worth of a real annuity is

$$(3) \qquad \begin{aligned} &\text{annuity value} \equiv \sum_{x=65}^{\infty} R_{t,(x-65)} s_x \qquad s_x \equiv \prod_i p_{t+i,x} \\[2mm] &\text{Money's Worth} \equiv \frac{A_{t,65}}{12} \sum_{x=65}^{\infty} R_{t,(x-65)} s_x \end{aligned}$$

where $p_{t+i,x}$ is the one-period survival probability for the annuitant who is age x in period $t+i$ (that is the probability of living one more period conditional on being alive at the beginning of the period) and s_x is thus the probability of the current 65-year-old living to age x or longer.

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[7] The official measure of inflation is calculated from the Consumer Price Index (which is based on a different basked of goods and uses a different methodology). Although recent UK government policy has been to use the CPI for pensions, it still issues bonds indexed to the RPI.

The discount factor $R_{t,j}$ is usually inferred from the yield curve on government bonds and it is assumed that this rate of return is risk free. There are two possible justifications for using these interest rates. The first is that annuity payments are meant to be secure (i.e. the chances of default are minimal) and the interest rate on government bonds is typically the best possible guess at the "safe" rate of interest. For some countries the interest rate on government debt would not be risk free, but this seems a reasonable approximation for countries such as the U.K. for which debt is typically AAA rated. Where life assurers use commercial bonds (or commercial mortgages), they must adjust the higher rates of return for the greater risk and, to a good approximation, the risk-adjusted rates of return on commercial bonds or mortgages are likely to be the same as the rates of return on government bonds[8] Secondly, life assurers approximately match their annuity liabilities with government bonds and regulation may even compel them to do so.[9]

2.2   Estimating the Survival Probability

Equation (3) requires us to know the probability of living $p_{t+i,x}$, or equivalently the probability of dying, referred to as the mortality.[10] The estimation of these variables is a staple of actuarial textbooks (Bowers *et al*, 1997; Pitacco et al, 2009), but forecasting these variables is more problematic and usually relies on extrapolating the past trend, since models based on the causes of death are insufficiently precise to be used for prediction purposes. The resulting estimate is subject to uncertainty from a variety of sources.

First, the estimates will be based on data available up to time t (or possibly earlier if there are lags in data collection): other than measurement errors, there is also the

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[8] Details of the notional yields, credit ratings and corresponding adjustments are reported in the FSA returns. Price risk is relatively unimportant since bonds are typically held to maturity.

[9] CGFS (2011) provides a review of international insurance regulation and notes that this matching can be *duration matching* which only partially matches liability and asset cash flows and *cash-flow matching* which perfectly matches the flows. The footnotes of various FSA returns note that perfect matching is impossible and that there is a small residual risk. This matching is likely to be more difficult for RPI-linked annuities and we return to that issue in section 3.4.

[10] More formally, mortality μ is the continuous-time analogue of the one-year death probability $q \equiv 1 - p = \int d\mu$.

problem that death rates are not quite the same as death probabilities and there may be considerable sampling error if the death rates are based on relatively small samples (which is often the case for the highest ages).

Second, there is model uncertainty. Most models' starting point is Gompertz's Law, i.e. the empirical regularity is that the logarithm of death rates tends to increase approximately linearly. The caveats to this are that: the decline is only approximately linear; the speed of decline depends upon age; and there are occasional structural breaks, which may apply either to the whole population or to just some cohorts. There is also some doubt as to whether one should look at the logarithm of the death rate or a logistic function and whether the decline is a stochastic or deterministic trend (Cairns *et al*, 2009).

Third, when looking at pensioners there may be additional changes in the data generating process either because the health of annuitants changes relative to that of the population as a whole or because the health of pensioners is different from others and pension coverage changes - these are two different forms of a selection effect. In many countries sufficiently detailed data for p are simply unavailable and the U.K. is unusual in having reliable data over a long time period, with the further advantage that data are available for different groups of pensioners, so that different life expectancies can be carefully aligned with the relevant financial assets and the third problem of selection effects can be eliminated (or at least attenuated). The professional actuarial bodies publish official tables which can be used by regulators in determining the valuation of annuity providers' liabilities. These official tables can then be used to calculate the money's worth.

In the U.K., since 1924, life offices have provided their firm-level data to a central committee of actuaries who anonymised and pooled this information to create a large enough data set to enable reliable statistical analysis and long-term projections. Until 1999 (ie the "92" series) the projections were only updated infrequently. The "interim adjustments" to this series in 2002 allowed for three different scenarios and the "00" series of 2006 did not even attempt to project mortality into the future but simply described the evolution of the data up to 2002. Furthermore, there are three different sets of mortality data. First, a high proportion of annuities are sold to individuals with a "personal pension", ie who have their own pension fund which has accumulated free of tax and who are

required by law to buy an annuity. However, the number of male pensioners with personal pensions in payment (rather than in accrual) was as low as two thousand as recently as the period 1987-90, so effectively there is too small a data set for meaningful projections or forecasts. The pre-cursor to the "personal pension" in the UK was the "retirement annuity contract" for whom there are more annuities in payment (more than 600,000 from 1987-90 onwards). But this second set of contracts were primarily purchased by the self-employed so the selection into this group is likely to be different to that of personal pensioners and the run of data is still fairly short. The only large data set is for the pensioners who have occupational pensions which are administered by life assurance companies (the "life office pensioners"), and this constitutes the third set of data on mortality, but again this group may be selected differently from that of the personal pensioners.

2.3   Money's Worth Calculations

Figure 1 illustrates our updated annuity rate data for 65-year old men in the UK compulsory purchase market[11] and the money's worths for annuities for three ages are illustrated in Figure 2. For the whole period we calculate the money's worth using four different mortality tables from the *Institute of Actuaries* (the annuity rate data and interest rate data are the same for all five series). These series are based on the same data as Cannon and Tonks (2010), but are calculated monthly rather than on annual averages.[12] Each new actuarial table results in an increase in the money's worth due to longer projected life expectancy, and we only plot the series for those time periods when the relevant actuarial tables were most likely to be used. The evidence in Figure 2 suggests that there was a fairly small decline in money's worths for a male aged 65, but little change for males aged 70 or 75: in fact the range of money's worths fell considerably.

[Figures 2 and 3 about here]

Comparing Figures 2 and 3, it is apparent that the decline in annuity rates of about 2.5 per cent between 1994 and 2000 does not correspond to as large a change in the

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[11] The data are discussed in more detail in the appendix.

[12] The money's worths are slightly higher, partly because of the effect of the averaging and partly because a small error was removed from the original calculations: the first payment is made on the day of purchase and not one month in later.

money's worth: this politically sensitive fall is predominantly explained by falls in interest rates and increases in life expectancy. The much more notable change is the apparent reversal of money's worths by age in the period from 2001 to 2004: from 2004 onwards the ordering of money's worths by age returns to the pattern found in Finkelstein and Poterba (2002), but the range is very much smaller. Cannon, Stevens and Tonks (2012) analyse the Dutch annuity market and find this inverse pattern of money's worths by age there for the period 2001-2010.

[Figures 4 and 5 about here]

Figure 4 shows the money's worth for annuities with guarantee periods: these appear to make little difference. Figure 5 shows the money's worths for level, real and escalating annuities: unfortunately we do not have a full set of data before 1998. We are able to confirm the findings of Finkelstein and Poterba (2002) that backloaded annuities (real and escalating) have lower money's worths than level annuities. Other than the fact that the money's worth tends to be lower than in Finkelstein and Poterba (2002), the qualitative behaviour of level annuities verus escalating or real is largely the same. Comparing the beginning of the period to the end (the two periods when we are relatively confident about the appropriate mortliaty table to use), there is some slight evidence that the money's worth has fallen and that the gap between the nominal and real money's worth has risen. The results for the relative money's worths of real and escalating are more mixed: the gap between them is often small and sometimes the money's worths of real annuities is slightly higher than for escalating, rather than lower.

Overall, our analysis for the money's worth over the whole period largely confirms that of Finkelstein and Poterba (2002). The caveats are that the differences in money's worth by age or guarantee period have disappeared by the end of the period.

3.   Adverse selection in Annuity Markets

3.1   Annuity rates and the money's worth

In this section we outline a simple framework within which we analyse the annuity market. Consider a simple model where agents live for up to two periods: all agents live in period 1 for certain, but the probability of living into period two is p^{type} where the super-script *type* refers to the fact that the survival probability may depend on

an annuitant's characteristics. Notice that a high value of p^{type} corresponds to a high-life-expectancy individual, which is high risk from the perspective of the life assurer. Most of this section will be discussing the precise nature of p^{type}.

Suppose the annuitant can choose an annuity which, per £1 premium in period 0, makes a payment of a_1^{type} in period 1 and a payment (conditional on being alive) of $a_2^{type} = \phi^{type} a_1^{type}$ in period 2. Life assurers sell annuities to a large pool of annuitants and this means that there is minimum idiosyncratic risk and in a competitive market they set the annuity rate to satisfy the break-even condition

$$(4) \qquad 1 = \frac{a_1^{type}}{1+r} + \tilde{p}^{type} \frac{a_2^{type}}{(1+r)^2} \Rightarrow a_1^{type} = \frac{(1+r)^2}{1+r+\tilde{p}^{type}\phi^{type}}$$

where r is the interest rate and $\tilde{p}^{type}$ is the survival probability used by the life assurer to value the liability: for a life assurer selling a annuities to a large number of personal pensioners it is the proportion of pensioners who live to period 2.

An economist analysing annuity data typically has information only on annuity rates for different annuity products, i.e. information on a_1 and ϕ. Using this information the money's worth for nominal annuities – whether level or escalating – is calculated as

$$(5) \qquad MW = a_1 \left\{ \frac{1}{(1+r)} + \bar{p}\frac{\phi}{(1+r)^2} \right\}$$

where $\bar{p}$ is the survival probability of used by the economist. As we have seen in section 2, there may be some problem in deciding exactly which survival probability to use.

The evaluation or real (inflation-indexed) annuities is almost identical. If the real interest rate is r^*, then the life assurer's pricing equation will be the real analogue of equation (5), namely

$$(6) \qquad 1 = \frac{a_1^{type}}{1+r^*} + \tilde{p}^{type} \frac{a_2^{type}}{\left(1+r^*\right)^2}$$

where $\phi = 1$ as real annuities are not escalating in real terms. The researcher's estimate of the money's worth will be the real analogue of (5), namely

(7)
$$MW^* = a_1 \left\{ \frac{1}{1+r^*} + \frac{\bar{p}}{\left(1+r^*\right)^2} \right\}$$

Equations (6) and (7) are the same as equations (4) and (5) but for the replacement of r^* for r: since $r^* < r$, we can compare the money's worths of real and nominal annuities just by looking at $\partial MW / \partial r$.

3.2 A simple model of adverse selection

Discussion of adverse selection in insurance markets normally centres on whether agents purchase full or partial insurance. In the UK it is virtually obligatory for a personal pensioner to purchase an annuity with at least 75 per cent of the value of the pension fund (up to 25 per cent can always be taken as a tax-free lump sum). In principle it is possible to avoid annuitisation until age 75 and even then one can avoid annuitising all of one's pension wealth. In practice deferring or avoiding annuitisation is unattractive for most individuals, except perhaps the very wealthy who are doing so for complicated tax reasons. If one defers annuitisation until 75 one must enter "drawdown", which is expensive and limits the amount of the fund that can be accessed. In particular this would severely limit the possibility for individuals who believed they had short life expectancy to bring forward consumption. To avoid annuitisation one must first demonstrate that one has a secure pension income (perhaps purchased from a portion of the pension fund) and then pay 55 per cent tax on that part of the pension fund which is not annuitised: the 55 per cent tax effectively reclaims all of the tax priviliges that a higher-rate tax payer would have received from saving in a pension fund.[13]

For these reasons we can treat annuitisation as effectively compulsory for nearly all personal pensioners and the only choice available is the timing of annuitisation and the type of annuity purchased. Evidence from the Associationof British Insurers shows that the vast majority of annuities are purchased at the conventional retirement ages of 55, 60 or 65, suggesting that timing is driven almost entirely by retirement (very few annuities are purchased at age 75, further evidence that the option to defer annuitising until then is not very important).

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[13] The details of compulsory annuitisation rules have changed several times in the last decade. The most recent rules are described in HM Treasury (2010b). Earlier variants are discussed in Cannon and Tonks (2009).

Finkelstein and Poterba (2002, 2004) suggest that an adverse selection separating equilibrium could be achieved through agents with different life expectancies buying different products, since product type is the only remaining choice open to a personal pensioner. In the model of section 3.1, suppose that there are two types, distinguished only by their survival probabilities, which are p^h for high-life-expectancy individuals and p^l for low-life-expectancy individuals, where $p^h > p^l$. Agents maximise respectively:

$$U^h = u\left(c^h\right) + p^h \delta u\left(c^h\right)$$

(8)

$$U^l = u\left(c^l\right) + p^l \delta u\left(c^l\right)$$

For simplicity of exposition we also assume that agents have the same felicity function $u(\bullet)$, that $r = \delta$ and that there is no saving product other than the annuity and that there is no borrowing.

[Figure 6 about here]

The budget constraints and indifference curves are plotted in Figure 6, which looks like the standard two-period-model diagram for the Rothschild-Stiglitz model of insurance. However, there are a subtle differences: first, whereas in the R-S model the two axes show consumption in the two different states of the world, in this diagram the vertical axis shows the certain consumption in period one and the horizontal axis shows how much the individual consumes conditional on surviving into period two; second, at any point on the budget constraint an annuitant is fully insured in the sense that he will not outlive his resources (the only way to be under-insured against longevity risk is to invest some wealth in non-annuity form, which cannot be shown in this diagram). So different points along a budget constraint do not show different levels of insurance but different consumption paths through time: for example point A has the same level of consumption in both periods whereas point B has a higher level of consumption in period one at the expense of lower consumption in period two.

In a separating equilibrium, a life assurer would offer two contracts, at A and B. High-life-expectancy annuitants would choose contract A, whereas low-life-expectancy agents would choose contract B.

The diagram illustrates the requirements necessary for a separating equilibrium to exist: the two types of annuitant must have indifference curves with different slopes and it must be possible to offer a contract such as B, where consumption in period one is higher than consumption in period two. How much consumption must be higher in period one than period two depends upon the convexity of the high-life-expectancy individual's indifference curve. The separating equilibrium is achieved through agents' substitution of consumption between periods rather than variations in the amount of insurance purchased.

We can now determine the consequences for the money's worth. If information were available on survival probabilities of both high- and low-risk annuitants then the money's worths for both contracts should equal one (from the fact that the contracts are actuarially fair). In reality, the only survival probability available to the economist is that for the whole pool of annuitants. If this were measured correctly, then the observed survival probability would be

(9) $$p = \theta p^l + (1-\theta) p^h$$

where θ is the proportion of the annuitant population which is low-life expectancy.

In fact we can say more than this. Table 2 shows the number of annuities sold by leading life assurers in the UK taken from the FSA returns, which require life assurers to distinguish sales of nominal and real annuities: clearly the proportion of real annuities is tiny. If it were really the case that low-life-expectancy annuitants purchase level annuities, then it would follow that $\theta \approx 0.99$.

[Table 2 about here]

Regardless of the value of θ, the calculated money's worth for each type is

(10) $$MW^{type} = \frac{a_1^{type}}{1+r} + \left(\theta p^l + (1-\theta) p^h\right) \frac{a_2^{type}}{(1+r)^2}$$

The high-life-expectancy annuitant chooses contract A where the annuity payments are the same in both periods, so

(11) $$MW^h = \frac{1 + r + \theta p^l + (1-\theta) p^h}{1 + r + p^h} < 1$$

The money's worth for the low-life expectancy annuitant is more complicated because the payments are different in the two periods and the precise pattern of payments depends upon the indifference curve of the high-life-expectancy type. But if we write simply that $a_2^l = \phi a_1^l$ for some $\phi < 1$, then

(13) $$MW^l = \frac{1+r+\phi\theta p^l + \phi(1-\theta)p^h}{1+r+\phi p^l} = 1 + \frac{\phi(1-\theta)(p^h - p^l)}{1+r+\phi p^l} > 1 > MW^h$$

which confirms that the money's worth should appear to be greater for low-life-expectancy individuals than for high-life expectancy individuals. Although equation (13) suggests that $MW^l > 1$, the fact that $\theta \approx 0.99$ means that in this model it will actually be very close to one: the fact that it is virtually never that high suggests a much larger rôle for transactions costs in explaining the money's worth.

3.3 Forecast rather than known survival probabilities

In this section we consider the simplest possible model of cohort mortality risk where there is no annuitant heterogeneity (and hence no adverse selection), but the life assurer does have to reserve against cohort mortality risk. A simple way to characterise this would be to assume that the life assurer to use a Value-at-Risk method based on an appropriate percentile of the perceived distribution of cohort survival probabilities so that the annuiy price equation were

(14) $$1 = \frac{a_1}{1+r} + \tilde{p}_{C\%}\frac{a_2}{(1+r)^2}.$$

Combined with equation (5) this suggests that the resulting money's worth would be

(15) $$MW = \frac{1+r+\overline{p}\phi}{1+r+\tilde{p}_{C\%}\phi} < 1 \quad \text{since } \overline{p} < \tilde{p}_{C\%}$$

We can draw three inferences from this equation. First, all annuity types woudl have a money's worth less than one. Second, since $\partial MW/\partial\phi < 0$ escalating annuities would have a lower money's worth than level annuities. Third, since $\partial MW/\partial r > 0$ real annuities would have a lower money's worth than level annuities (the relationship between escalating and real annuities is ambiguous).

A further problem with real annuities is that the number of annuitants is much smaller, as we have seen in Table 2. A consequence of this is that sales of real

annuitants suffer not only from cohort mortality risk but also idiosyncratic mortality risk.

This simple model suggests that observed pattern of money's worths may be due to pricing annuities to account for cohort mortality risk rather than due to adverse selection. Unfortunately it is impossible to identify the two effects, but in section 4 we shall consider whether the magnitude of reserving is significant. Before those calculations we make a few additional points about the money's worth.

3.4 Final comments on the plausibility of the adverse selection model and the rôle of costs

In this section we make some qualitative remarks about the adverse-selection separating-equilibrium characterisation of the annuity market and the issue of costs.

Our discussion of adverse selection refers back to Figure 6. First, contract B may not be allowed by regulators. Annuities in the compulsory purchase market have to be recognised by the tax authorities (HMRC). In practice the types of annuity allowed are level (constant in nominal terms), RPI-linked (constant in real terms) and escalating (nominal payments rising at (typically) 3 or 5 per cent per year). In a low-inflation environment, a level annuity does not allow for much front-loading: translated to Figure 6 it might be that the most heavily front-loaded contract available is at point D. Unfortunately, offering contracts A and D will not result in a separating equilibrium. A corollary of this is line of reasoning is that it will be easier to achieve a separating equilibrium in a high-inflation environment, but the variation in inflation is too small during our period of observation to attempt to use this result.

A second problem is that life assurers are unable to observe the consumption choices of the annuitants and the annuity payments may be very poor proxies for consumption. Most annuitants will have both non-annuitised wealth and additional annuity wealth which will not be observed by the life assurer. At a minimum, annuitants are likely to have the UK's Basic State Pension, plus additional means-tested benefits derived from the Minimum Income Guarantee. Some annuitants will own a house (non-annuity wealth): those without housing wealth will receive Housing Benefit (since this will either be received until death or superceded by long-term care assistance, it is close to annuity wealth). On top of

this, some annuitants will also have occupational pensions and other personal pensions: it is possible to have more than one personal pension fund and not necessary to combine them at the point of annuitisation.[14] This means that a life assurer observes a_1/a_2 on what may be a relatively small part of an annuitant's total wealth but not c_1/c_2 which is what is needed to effect a separating equilibrium.

We make a final comment about the effect of costs. Life assurers are involved in many sorts of insurance and long-term fund management and there are presumably large economies of scope. This means that it would be very difficult to allocate precisely all of the relevant costs to a firm's annuity business alone, let alone the costs of particular types or cohorts of annuitants.

As noted by Finkelstein and Poterba (2002), it is probable that the costs of managing real annuities are higher than for level annuities. HM Treasury and Bank of England (1995) describe several reasons why the market for RPI-indexed bonds is thinner and less liquid than for conventional bonds and the differences are considered sufficiently important that the bonds are issued in different types of auction (Debt Management Office, 2013). Because fewer RPI-indexed bonds are issued this will amost certainly make complete cash-flow matching more difficult. However, we are unable to quantify the cost of this.

4.   The Stochastic Money's Worth

In the previous section we showed how the pattern of observed money's worths might be due to life assurer's reserving against cohort mortality risk. In this section we quantify this effect. This requires us to quantify the uncertainty in forecasting mortalities or, in the notation of section 3, the uncertainty in forecasting p. There are two components to this: first, one has to know that one has the correct data, the correct model and to be sure that the forecasting method will not be compromised by structural breaks; and second, given the previous considerations one has to have a stochastic model. We shall be ignoring all of the first set of considerations: we are using life office pensioner data (rather than personal pensioner data); from the array of potential models (eg described in Cairns et al, 2009), we simply choose a model which is widely used and understood; the period after our data end was

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[14] Data are not yet available for researchers to be sure of the distibution of different pension funds across pensioners and life assurers would not have this information.

characterised by significant changes in models and forecasts due to the perception of structural breaks or cohort effects. By concentrating on the uncertainty within a particular model we are under-estimating the effect of uncertainty on the money's worth.

The model we use for this exercise is that of Lee and Carter (1992) model, which has been widely accepted as a starting point for mortality analysis.[15] The LC model has a flexible (non-parametric) relationship between age and log-mortality which is assumed to be constant: projection of mortality consists of simple shifts in the log-mortality-age curve. More specifically, the one-year death probabilities are modelled as

$$(16) \qquad \ln\left(1 - p_{xt}\right) = \ln q_{xt} = \alpha_x + \beta_x \kappa_t + \varepsilon_{xt} \qquad \varepsilon_{xt} \sim \mathcal{N}\left(0, \sigma^2\right)$$

which can be estimated by least-squares from a singular-value decomposition method (see Pitacco et al, 2008; Girosi and King, 2009, for an exposition).[16] There are a variety of identification estimation issues which we discuss in the Appendix. Regardless of the estimation procedure, forecasting is based upon

$$(17) \qquad \Delta \kappa_t = \lambda + \psi_t \qquad \psi_t \sim iid\left(0, \sigma_\psi^2\right)$$

where the parameters λ and σ_ψ^2 are estimated in a second-stage regression (and where a more complicated dynamic process than a random walk is also possible).

A simpler procedure is to ignore the fact that mortality is following a stochastic trend: Girosi and King (2009) suggest that it is common in practice to project the κ_t terms using

$$(18) \qquad \Delta \kappa_{t+s} = \lambda s$$

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[15] As a robustness check to our analysis we consider an alternative to the Lee-Carter approach: the Cairns-Blake-Dowd (2006) model, which builds on the empirical observation that the relationship between log-mortality and age is approximately linear, and uses this as a restriction in the estimation strategy.

[16] It would be possible to estimate the LC model using maximum likelihood rather than least squares, but throughout most of our period LC models were estimated by least squares.

which ignores the fact that the κ_t terms will be evolving randomly. We shall refer to this as the *deterministic projection* of the Lee-Carter model. Alternatively, additional forms of uncertainty can be included in the model: for example the parameters λ and σ_ψ^2 must themselves be estimated and we shall incorporate estimates of the parameter uncertainty in our estimates of $\hat{\lambda}$ and $\hat{\sigma}_\psi^2$, referred to as a model with *uncertain parameters*.

Our estimate of the Lee-Carter model uses the UK's life office pensioner mortality data, which is the largest and most commonly used data for UK private pensions. The data we use are for 1983-2000:[17] the typical exposed to risk for a given age in a given year is in the range 5,000-10,000, although there are fewer for very high ages. The total exposed-to-risk in 1983 is 356,552 and in 2000 it is 289,019. This period, and the years immediately following it, demonstrated significant falls in mortality, requiring substantial revision to life tables as documented in Cannon and Tonks (2008, section 6.2).[18] Consistent with Gompertz's law the alphas and betas are approximately linear in age, and the kappa is a stochastic trend. The fact that beta depends upon age shows that the trend in log-mortality is age dependent.

Using the estimated alphas and betas and with projected kappas, we can project survival probabilities into the future using numerical methods (we conduct a Monte Carlo with 100,000 replications). Figure 7 shows the survival fan chart for a male aged 65 at the end of the period of our data (2001). Such fan charts have been discussed in Blake, Dowd and Cairns (2008): there is relatively little uncertainty about the survival probability for the first few years: the probability of dying is small and there is little scope for uncertainty. However, by age 75 there is considerable uncertainty. Note that an annuity which was more backdated (had longer duration) would have a higher proportion of its present value paid in the period of greater uncertainty and thus would be a risker liability for a life assurer.

[Figure 7 about here]

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[17] Although detailed data on pensioner mortality were collected in the United Kingdom from 1948 the data prior to 1983 have been lost (CMI, 2002).

[18] In this data set no 60-year old male died in 1998, so the log mortality was not defined: we replaced the zero value by 0.5 (which corresponded to the lowest mortality rate observed elsewhere in the data set). A variety of alternative assumptions resulted in almost identical conclusions.

Using the distribution of survival probabilities from Figure 7 we then estimate the distribution of the value of an annuity paying £1 per year and illustrate this in Figure 8 for different interest rates, assuming that the yield curve is horizontal (the same interest rate at all terms). As expected, Figure 8 shows that, as the interest rises and the duration of the annuity falls, both the expected value of an annuity and the standard deviation fall.

Table 3 shows the consequences for the money's worth if a life assurer prices annuities from the relevant centile of the distribution of annuity values and the researcher uses the expected annuity value. When priced from the median, the money's worth is approximately one, since the median and expectation are virtually the same. When the life assurer prices from the 99th centile, the money's worth is less than one and the discrepancy is larger the lower the interest rate (since the duration of the annuity rises and is where there is greater uncertainty).

The left-hand panel of the table shows the effect when there is no idiosyncratic mortality uncertainty: the figures are based purely on the uncertainty in the distribution of projected cohort mortality uncertainty. The right-hand panel quantifies the effect of idiosyncratic uncertainty that would be faced by a life assurer who sells only 400 annuities: in each Monte Carlo replication the future one-year death probabilities are generated for all ages and then the actual number of deaths are drawn from a Bernoulli distribution with the projected probability).

[Table 3 about here]

We can now use the table to quantify the possible effect on the money's worth. Suppose the nominal interest rate were 9 per cent and the real interest rate were 4 per cent, figures roughly consistent with the ten-year government bond yields of 1994 in Figure 2 and a life assurer were pricing off the 95th centile (i.e. a VaR of 95 per cent). Then the money's worth for nominal annuities would be 0.969 (from the left hand panel of the table) and the money's worth for real annuities would be 0.934 (from the right hand panel of the table), a difference of 3.5 per cent. Slightly more than half of this arises from the real interest rate being lower than the nominal rate and the rest arises from the imperfect diversification of having only 400 real annuitants. If the life assurer were pricing from the 99th centile, then the difference would be just over 5 per cent. Despite being an under-estimate of the effect of cohort mortality risk due to ignoring model uncertainty and ignoring

additional costs of real annuities, this is about half of the observed difference in the money's worth in 1994 from Figure 5.

In Figure 9 we illustrate our final calculations making use of the actual interest rates that were used in the money's worth calculations in Figures 2-4. Notice, however, that we are using a constant set of mortality projections for the whole period, so our results are not directly comparable with the earlier graphs. Instead, Figure 9 isolates the effect that actual interest rate changes would have had on money's worth calculations had annuities been priced on the 95th centile (all calculations are for a male aged 65 and we make no allowance for idiosyncratic mortality risk). Figure 9 reinforces our back-of-envelope calculations in the previous paragraph: a significant part of the difference between nominal and real money's worths could be due to cohort risk.

[Figure 9 about here]

In our discussion of Figure 5 we noted that there appeared to be a slight fall in the nominal money's worth between 1994 and 2012 and that the gap between the real money's worth and the nominal money's worth had risen. Both of those features are also evident in Figure 9: this arises from the fall in interest rates which not only reduces the expected value of annuity payments (an effect capture in money's worth calculations) but also increases the uncertainty (which is not captured in money's worth calculations).

The striking difference between Figures 5 and 9 is the relative behaviour of real and escalating annuities, since escalating annuities have a consistently lower money's worth than real annuities in Figure 9, but the reverse is true in Figure 5. The empirical finding is equally problematic for the adverse selection model of annuities, which would make the same qualitative predictions as our model (as noted by Finkelstein and Poterba, 2002). Although not reported in the FSA Returns, we believe from discussions with practitioners that the number of escalating annuities is similar to the number of real annuities and therefore the difference cannot be due to idiosyncratic risk. This underlines the fact that administrative costs of real annuities have to be significantly higher than for nominal annuities for any model to fit the observed money's worths.

5.   Summary and Conclusions

In this paper we have updated our money's worth calculations for the UK compulsory purchase market – the biggest annuity market in the world – to 2012. This provides a starting point for re-visiting the idea that there is adverse selection i the annuity market, following the original analysis of Finkelstein and Poterba (2002). Some of their corroboratory evidence, such as the money's worth varying by age or guarantee period are no longer valid. However, their most important result, that backloaded annuities have a lower money's worth than frontloaded annuities is still true in the UK annuity market.

Finkelstein and Poterba's explanation for this was that there is adverse selection and that a separating equilibrium is achieved via longer-lived individuals purchasing backloaded annuities. When calculating the money's worth using the mortalities of all annuitants pooled together (i.e. the only mortality data that are available), this would result in backloaded annuities having a lower money's worth.

In this paper we shows that an alternative model yields exactly the same qualitative conclusions. Our model relies upon the fact that life assurers need to reserve against the uncertain evolution of cohort mortality, both for prudential reasons and because they are required to do so by government regulation. Because back-loaded annuities have a higher proportion of payouts in the more distant future, they are inherently riskier products and require greater reserves.

Because our model yields the same conclusions as the Finkelstein-Poterba model it is impossible to identify the magnitude of the two effects from the data alone. To address this problem we have quantified the importance of cohort mortality risk using the Lee-Carter model, although arguably this might under-state the effect as we are effectively ignoring the issue of model uncertainty. Our results suggest that a substantial proportion of observed differences in money's worths for different annuity products may be due to the relative risk. Combined with other costs of annuity supply, which are conventionally ignored in money's worth calculations, this suggests a much smaller effect for adverse selection.

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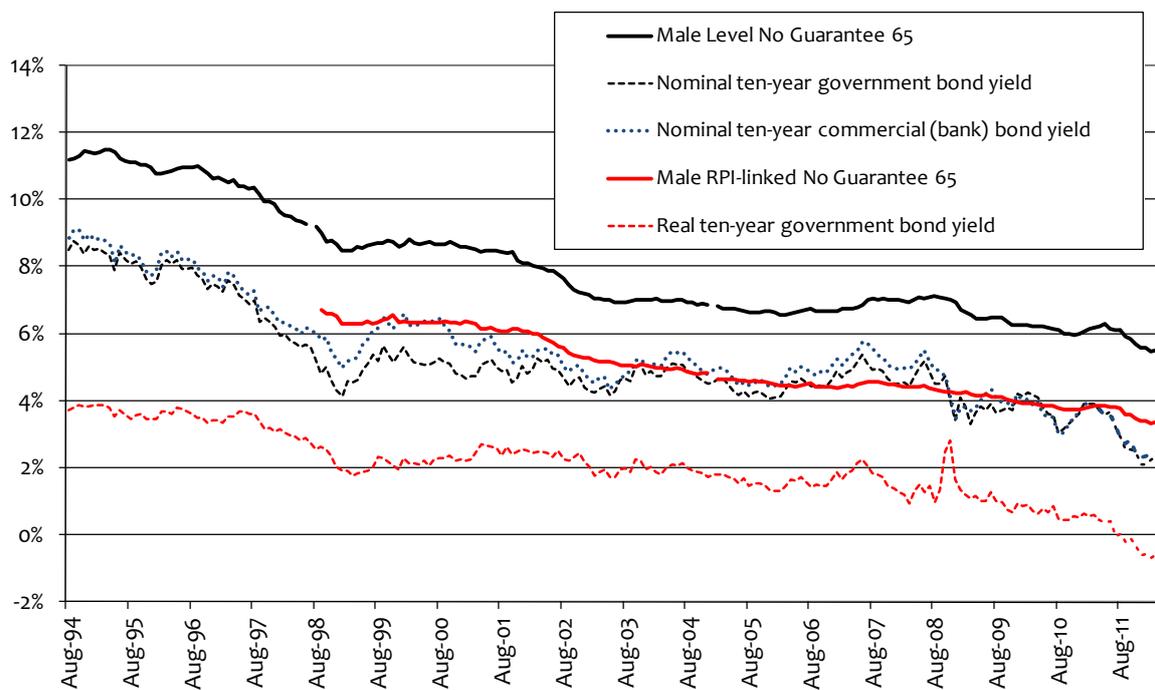
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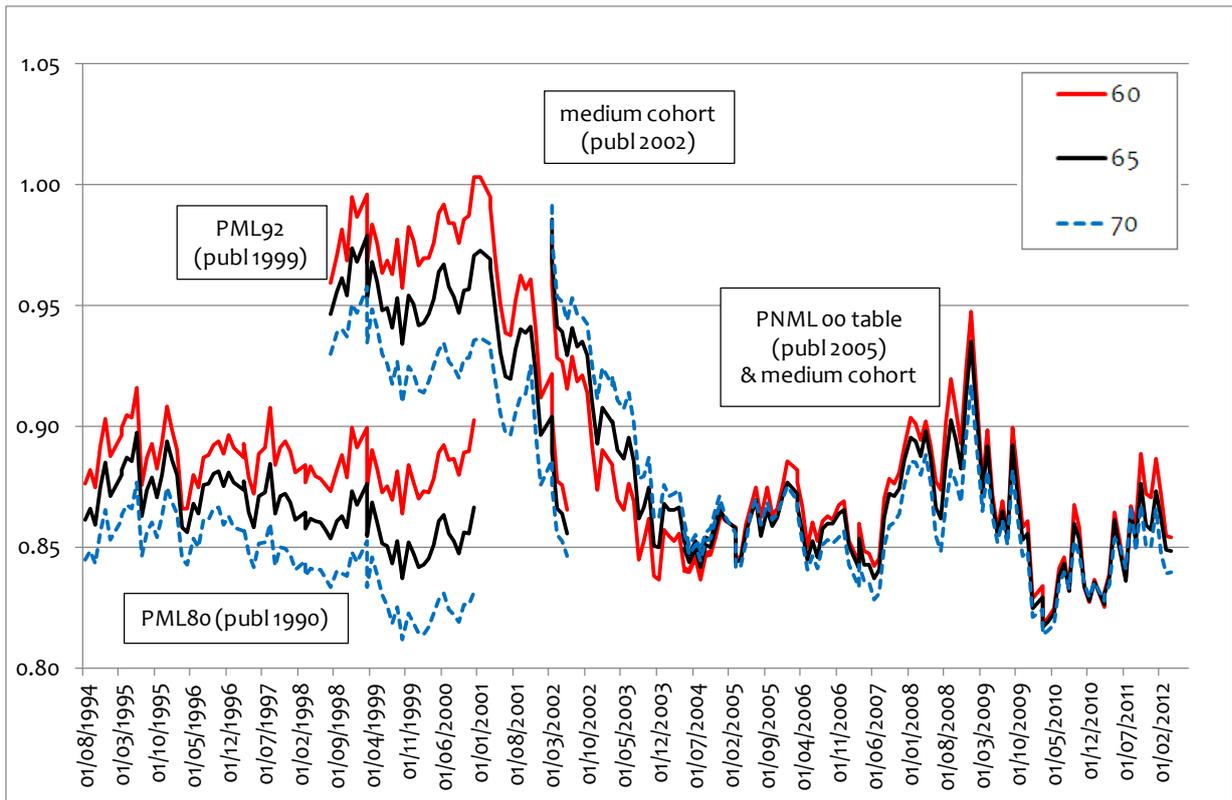
Figures and graphs

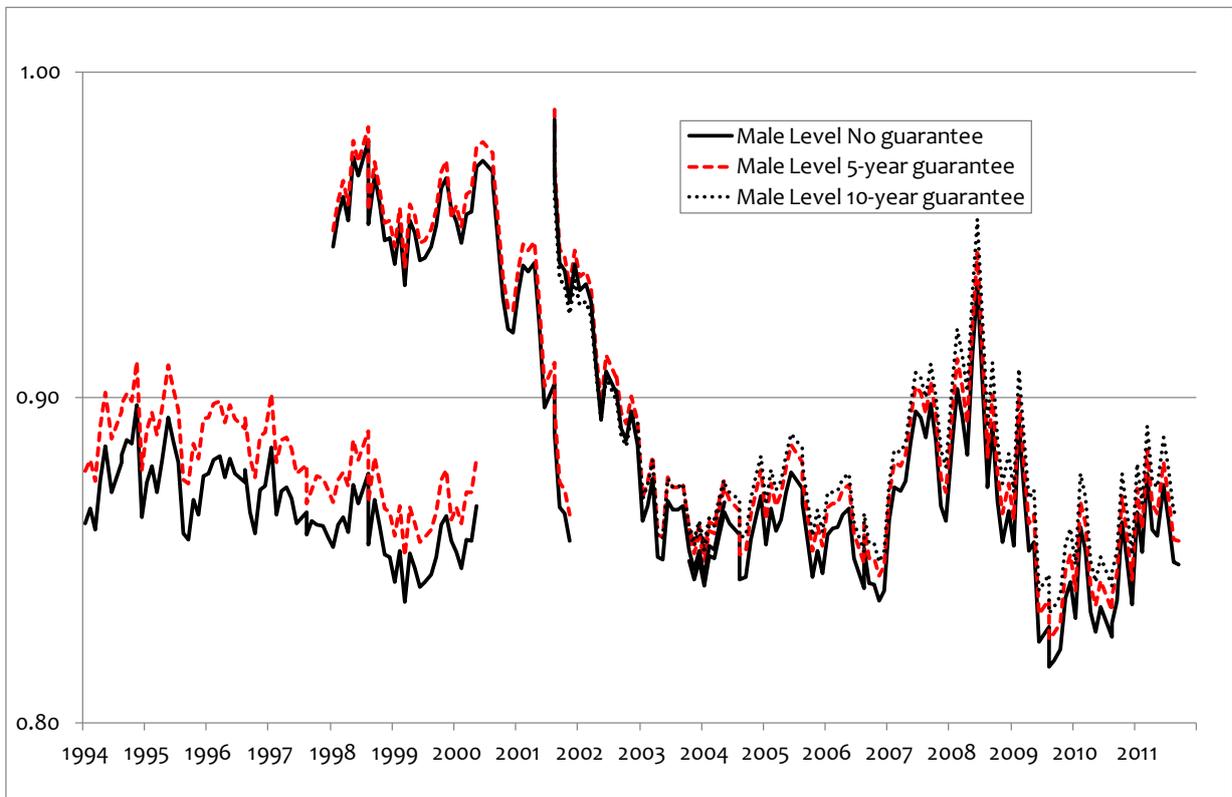**Figure 1   Mortality assumptions of life assurers**
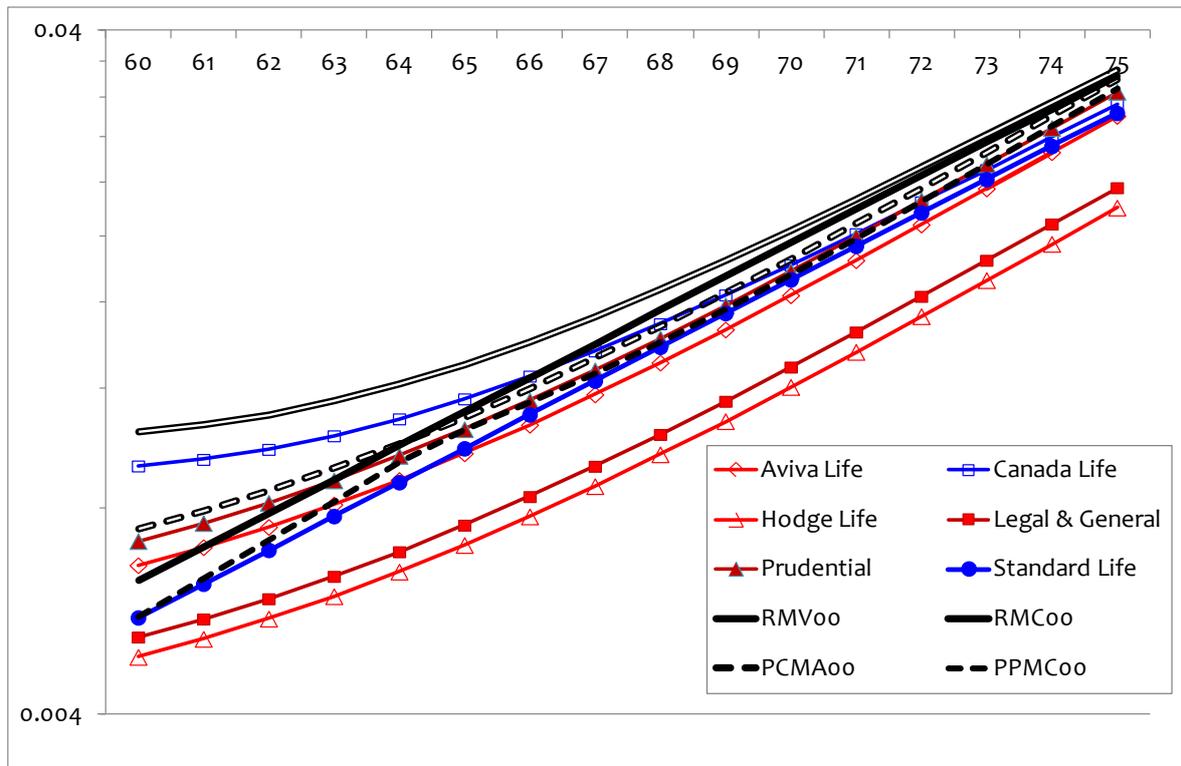
Figure 2   UK Annuity Rates (Male, Compulsory Purchase) and Bond Yields
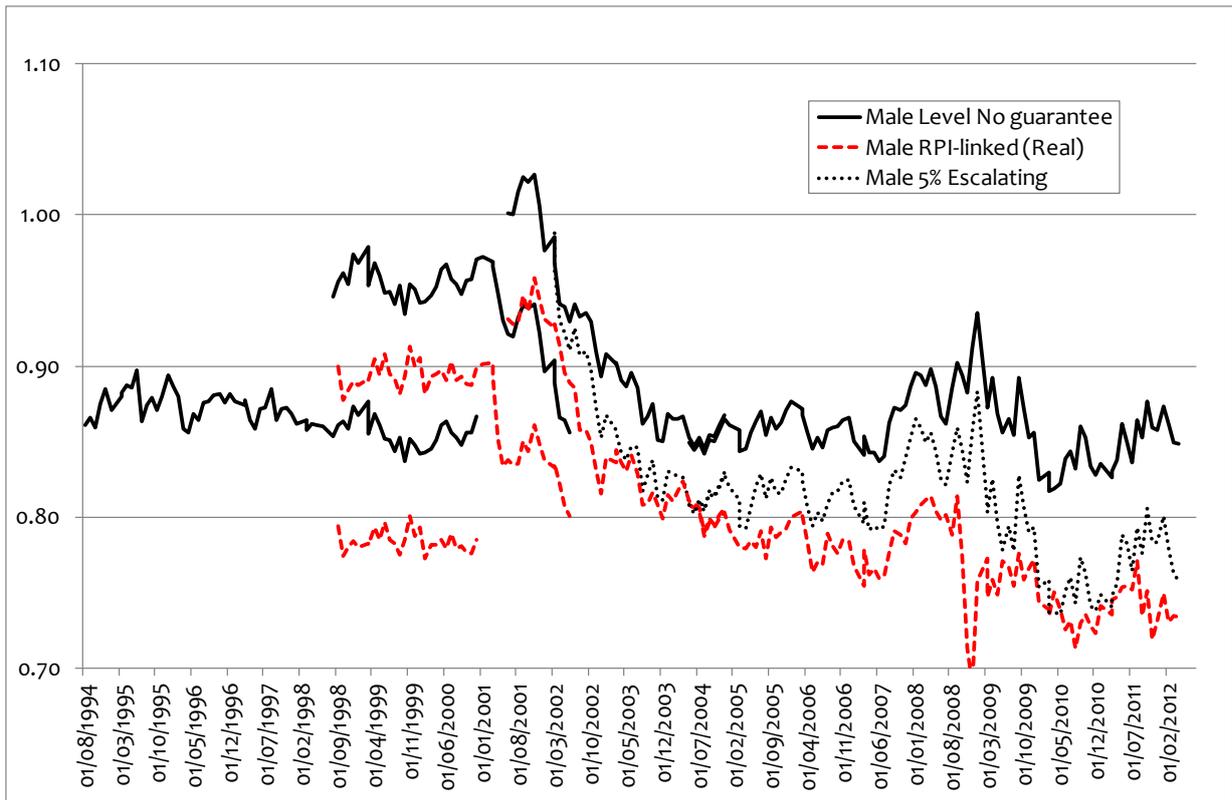
Figure 3: Money's worth calculations, level annuities for different ages
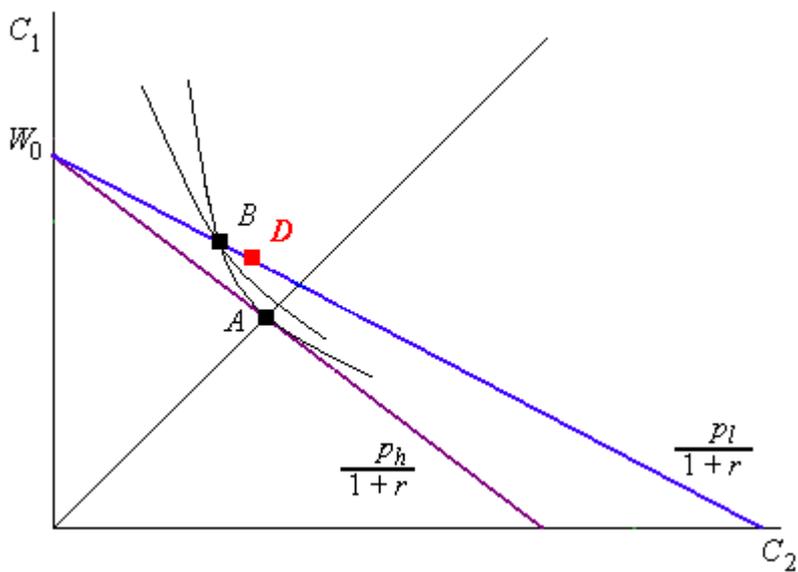

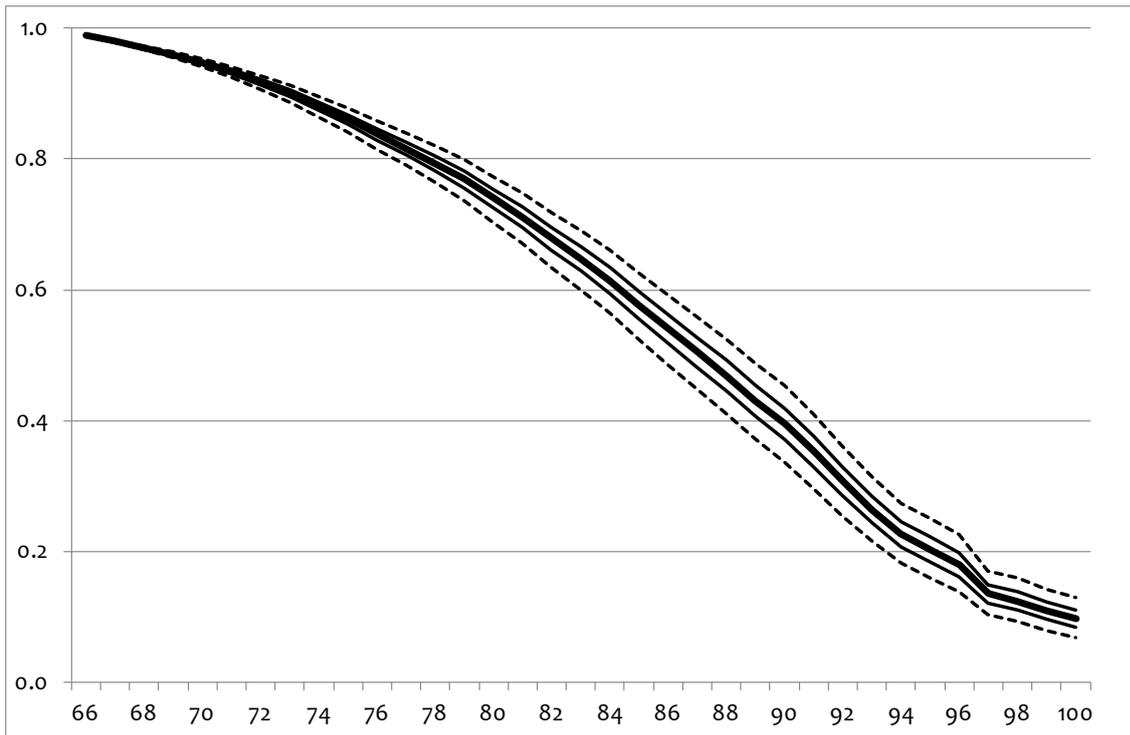**Figure 4: Money's worth calculations, different guarantee periods, male, 65**

Figure 5: Money's worth calculations, different types of annuity, male 65
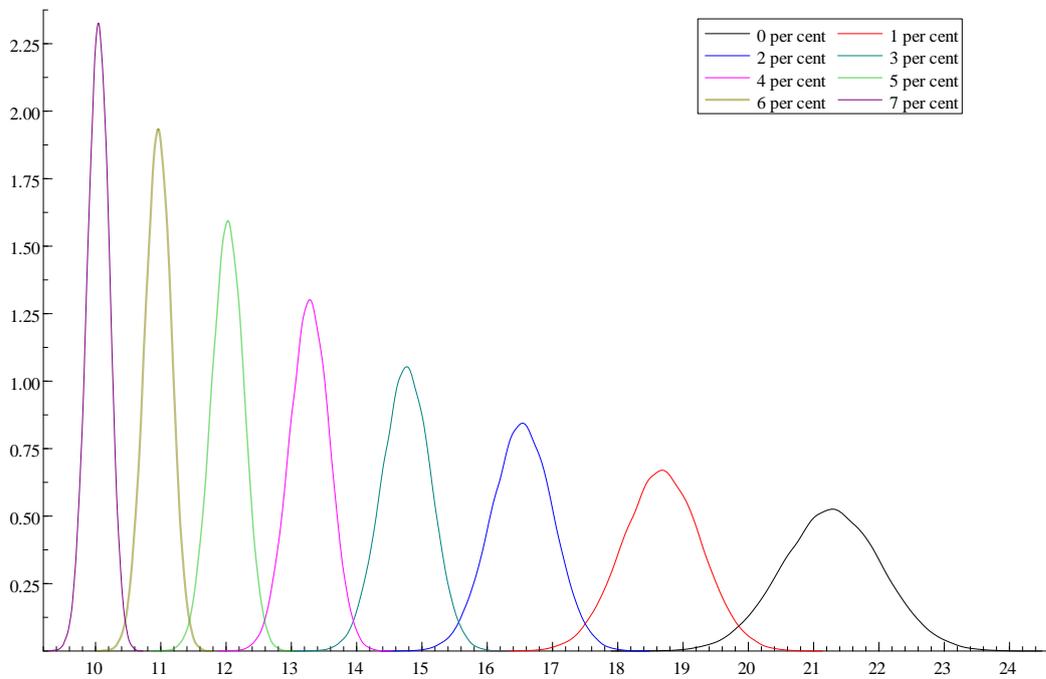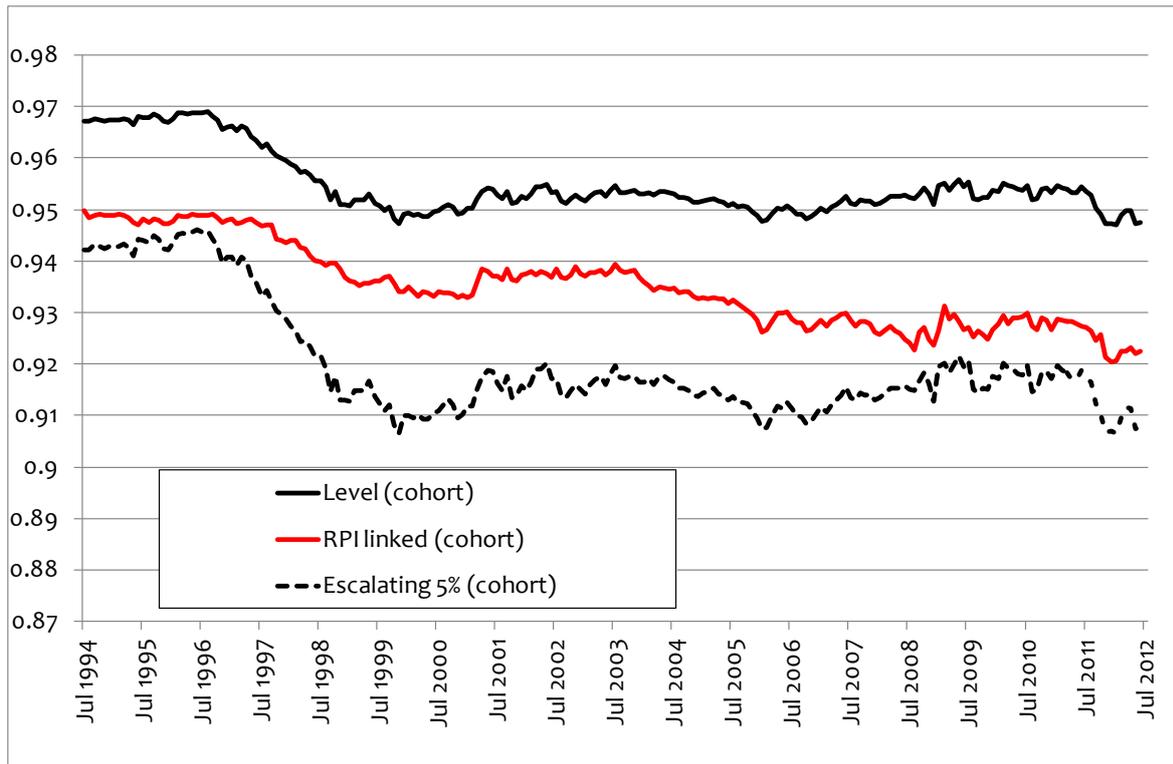

Figure 6: Diagram of separating equilibrium

Figure 7: Fan chart of survival probabilities, male 65

Figure 8: Annuity Value Distributions, male 65

Figure 9:   Money's worths using actual yields

Tables

Table 1: Summary of mortality assumptions in the FSA returns

Company	Mortality assumption
Aviva Life	88.5% of PCMA00
Canada Life	89% of RMV00 (plus further adjustments)
Hodge Life	65% of PCMA00
Legal and General	69.5% of PCMA00 (plus further adjustments)
Prudential	95% of PCMA00
Standard Life	88.4% of RMC00

Table 2: Purchases of different annuity types, 2011

Source: various FSA Returns, Appendix 9.3, Form 47, rows 400 and 905

Company	Nominal		Real	
	No of purchases	Average purchase	No. of purchases	Average purchase
Aviva Annuities	58,692	£32,696	1521	£33,707
Canada Life	13,440	£32,696	909	£72,939
Hodge Life	859	£31,029	none	
Legal & General	25,928	£25,102	845	£18,073
Prudential	37,006	£25,653	307	£33,707
Standard Life	20,361	£14,844	245	£20,804

Table 3: Stochastic Money's Worth Calculations

	Diversification (many policies)				Imperfect diversification (400 policies)			
Quantile:	0.50	0.90	0.95	0.99	0.5	0.9	0.95	0.99
Interest rate								
-1%	1.000	0.933	0.915	0.884	1.001	0.933	0.914	0.879
0%	1.000	0.940	0.924	0.896	1.001	0.936	0.919	0.886
1%	1.000	0.946	0.932	0.906	1.001	0.939	0.923	0.891
2%	1.000	0.952	0.939	0.916	1.001	0.942	0.927	0.897
3%	0.999	0.956	0.945	0.924	1.001	0.945	0.930	0.902
4%	0.999	0.961	0.950	0.932	1.000	0.948	0.934	0.907
5%	0.999	0.964	0.955	0.938	1.000	0.950	0.937	0.912
6%	0.999	0.968	0.959	0.944	1.000	0.953	0.940	0.916
7%	0.999	0.971	0.963	0.949	1.000	0.955	0.943	0.920
8%	0.999	0.973	0.966	0.954	1.000	0.957	0.945	0.924
9%	0.999	0.976	0.969	0.958	1.000	0.959	0.948	0.927
10%	0.999	0.978	0.972	0.961	1.000	0.961	0.950	0.930

Table shows the ratio of the relevant quantile of the annuity distribution to the mean, from equation (9). The projection is made from the Lee-Carter model, assuming parameter uncertainty. The left-hand panel assumes that there are sufficiently many policies that the only risk is from projecting the cohort mortality: the right-hand panel combines cohort mortality risk with additional risk from having only 400 policy holders.

Appendices

Description of the data

Data on UK annuity rates for males and females at various ages are taken from MoneyFacts over the period August 1994 to April 2012 and update the annuity series that we have published previously: the construction of these series is described in more detail in Cannon and Tonks (2008). These are compulsory-purchase annuities which are bought as part of a pension scheme (where tax relief has been given on the accumulation of the pension fund via an EET scheme).

From about 2011 some life assurers started to price annuities based on the postcode of the annuitant (life expectancy varies by region and postcode has considerable predictive power): where life assurers did this the annuity rate data we have is for a "typical" postcode – but the definition of "typical" is decided by the life assurer and we do not know what definition is used. Notice further that from December 2012 it became impossible to price annuities for men and women differently under the ECJ directive, so there would be little point in extending our analysis to later time periods as there will be further changes to the market due to unisex pricing.

In Figure 1 we illustrate the annuity rate series for a 65-year old male over time compared with government bond data, and summary statistics of this data for nominal and real variables is presented in Tables A1 and A2. All bond data is taken from the Bank of England web-site. It can be seen that nominal annuities approximately track the nominal bond yield and analogously for real annuities: annuity rates are highly correlated with long-term bond yields, and the average difference in these two series over the sample period was 2.86%. We also compare the two sub-periods up to the financial crisis (Northern Rock bank run in August 2007) and since the onset of the crisis. Since the crisis, both short-term (base rate) and long-term government bond yields have fallen, and this has been reflected in a fall in annuity rates. Real annuities have payments that rise in line with the UK's Retail Price Index.

Table A1: Monthly Time Series Properties of Nominal Pension Annuity for 65-year old males and various alternative bond yields

	Annuity Rate for 65-year old males	Long-term: 10 year Government Bond Yield	Short-term: Bank of England Base rate	Interest rates on retail term deposits	Difference in Annuity Rate and Government Bond Yield
Panel A: Aug 1994 – April 2012					
Mean	7.96%	5.10%	4.43%	3.21%	2.86%
St.Dev.	1.70%	1.49%	2.09%	1.54%	
Correlation	0.93				
Panel B: Aug 1994 – July 2007					
Mean	8.54%	5.59%	5.34%	3.81%	2.95%
St. Dev.	1.63%	1.38%	1.07%	1.18%	
Correlation	0.92				
Panel C: Aug 2007 – Apr 2012					
Mean	6.40%	3.77%	1.88%	1.62%	2.63%
St. Dev.	0.49%	0.79%	2.12%	1.23%	
Correlation	0.88				

Table 1a presents descriptive statistics on the monthly time series of average annuity rates in the CPA market, long-term and short-term government bond yields and rates on retail term deposits, over the period 1994 to 2012 and for the two sub-periods.

Table A2: Monthly Time Series Properties of Real Pension Annuity for 65-year old males and various alternative bond yields

	RPI-linked Annuity Rate for 65-year old males	Long-term: 10 year Real Government Bond Yield	Difference in Real Annuity Rate and Real Government Bond Yield
Panel A: Sept 1998 – April 2012			
Mean	4.93%	1.60%	3.34%
St.Dev.	0.95%	0.77%	
Correlation	0.81		
Panel B: Sept 1998 – July 2007			
Mean	5.43%	2.02%	3.41%
St. Dev.	0.78%	0.35%	
Correlation	0.71		
Panel C: Aug 2007 – Apr 2012			
Mean	4.01%	0.80%	3.20%
St. Dev.	0.34%	0.73%	
Correlation	0.88		

Table 1b presents descriptive statistics on the monthly time series of average real annuity rates in the CPA market and real long-term government bond yields over the period 1994 to 2012 and for the two sub-periods.

The remaining data that we need to estimate the money's worth are the mortality projections. In previous money's worth calculations (Cannon and Tonks, 2004, 2008, 2009) we attempted to update the mortality projections at the time that the life assurers did so. We did not try to infer the mortality tables used by the life assurers from the footnotes of the FSA Returns because of a variety of problems: each FSA Return contains a variety of assumptions; there are a large number of companies; and until recently the footnotes of the FSA Returns were not easily available. Instead we started using each new table from the Institute of Actuaries from a year before the publication date, on the argument that the broad outline of these data may have been known to life assurers before actual publication (and life assurers would also have been able to analyse the mortality experience of their own annuitants).

The PML80 ("Purchased Male Life") table was published in 1992 ("80" refers to the base year). Although it projected gradual increases in life expectancy, by the late 1990s it had become clear that the downward trend in mortality of pensioners was much stronger and the PML92 tables (published 1999) revised life expectancy up by almost two years. Further analysis of the reduction in mortality both for pensioners and people of below pension age (for which pension data were unavailable: life assurance data was used instead), suggested a "cohort" effect, ie a discrete downward jump in mortality for people born after about 1930. This led to a set or "interim adjustments" published in 2002: the most widely used "medium cohort" adjustment is illustrated here. In 2005 information on the most recent annuitant mortality was published (the "00" table), which did not have an accompanying projection for changes into the future. Accordingly at that time many life assurers used the "00" table as a base and then used the "medium cohort" projection from 2000 (or some other year) onwards.

Implementation of the Lee-Carter method

Lee and Carter (1992) introduced this model and good expositions are Girosi & King (2008, pp.34.ff) and Pitacco et al (2008, pp.169-173 & 186.ff.). This specification does not completely identify the parameters, so identifying restrictions (which have no effect on the analysis) are used:

$$\sum_t \kappa_t = 0 \quad \text{and either}$$

$$\sum_x \beta_x = 1, \quad \text{in the original Lee-Carter (1992) paper}$$

$$\text{or} \quad \sum_x \beta_x^2 = 1, \quad \text{in Girosi-King (2008)}$$

Assuming that the errors are Normally distributed, then estimation is as follows (taken from GK, but with slightly different notation):

$$x = \{1, \ldots, X\} \quad \text{nb slightly different from typical actuarial notation:}$$
$$\text{typically start age age 60, so } x \equiv age - 59$$
$$t = \{1, \ldots T\}$$
$$\mathbf{M} = (m_{x,t}) \in \mathbb{R}^X \times \mathbb{R}^T \quad \text{nb: ages in rows, time in columns}$$
$$\tilde{\mathbf{M}} \equiv \mathbf{M} - \bar{\mathbf{m}}_x \mathbf{1}' \qquad \bar{\mathbf{m}}_x = \{\bar{m}_x\} \in \mathbb{R}^X$$

The estimation of the intercept term is straight-forward and intuitive: given the constraint that $\sum_t \kappa_t = 0$, just take the row means to get

$$\hat{\alpha}_x = \bar{m}_x = T^{-1} \sum_{t=0}^{T-1} m_{xt}$$

Consider a model with l principal components:

$$m_{xt} = \alpha_x + \kappa_{1t}\beta_1 + \cdots + \kappa_{lt}\beta_l + \varepsilon_t$$
$$\beta_i \in \mathbb{R}^X$$
$$\underset{X \times T}{\tilde{M}} = \underset{X \times l}{\beta} \underset{l \times T}{\kappa} + \varepsilon_t$$

By the singular-value decomposition theorem

$$\underset{X \times T}{\tilde{\mathbf{M}}} = \underset{X \times T}{\mathbf{B}} \underset{T \times T}{\mathbf{L}} \underset{T \times T}{\mathbf{U}'}$$

where $\mathbf{L}$ is a diagonal matrix with the singular values put in descending order. The first l principal components are the first l columns of $\mathbf{B}$. The Lee-Carter model is just this model with $l = 1$. So

$$\text{Lee-Carter} \begin{cases} \hat{\beta} = B_1/1'B_1 \\ \hat{\kappa} = (1'B_1)L_{11}U_1' \end{cases} \qquad \text{Girosi-King} \begin{cases} \hat{\beta} = B_1/\|B_1\| \\ \hat{\kappa} = \hat{\beta}'\tilde{M} \end{cases}$$
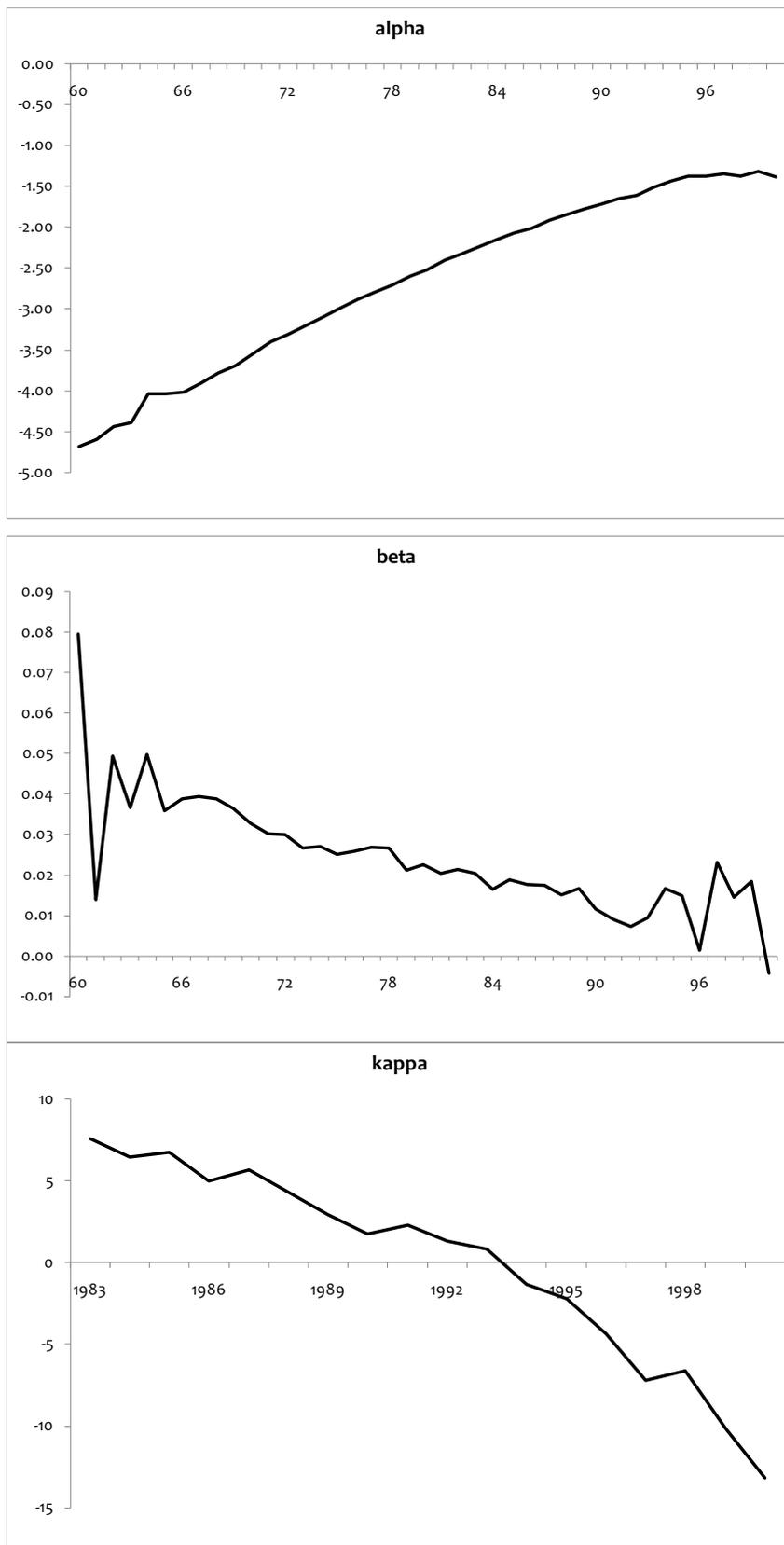
Lee and Carter suggest an adjustment to the time-effects to ensure that the expected deaths match the actual deaths, called second-stage estimation, so that

$$\forall t : \kappa_t \Longleftarrow 0 = \sum_x ETR_{xt} e^{\bar{m}_x + \beta_x \kappa_t} - \sum_x D_{xt}$$

and we have followed that procedure.

Our estimates of the parameters are shown in the figures below.

Figure A1:   Lee Carter Parameters for the UK Pensioner Data

Appendix on Regulation

From Aviva Life FSA Return 2011 we get the following quote

INSPRU 1.1.34R(1): "The assets [held by a firm to cover its technical provisions]...must...be of a sufficient amount, and of an appropriate currency and term, to ensure that the cash inflows from those assets will meet the expected cash outflows from the firm's insurance liabilities as they become due."

. . .

The assets backing the liabilities do not precisely match them by term. The fund has significant holdings in Equity and Property assets that have no set maturity date. These assets are held to provide strong performance for policyholders. Instead, the fund aims to ensure that cash outflows can be met through:

· Incoming premiums

· Income from its assets (dividends, rental income, coupon payments)

· Receipts on redemption from matched fixed interest stocks

· Holding appropriate volumes of cash and liquid assets

· Sales of assets.

There is a risk that the fund may have to sell assets at an unfavourable time, e.g. when prices are temporarily depressed. If the fund has to sell unmarketable assets it may also depress the price through the sale itself. The risk will be very short term; if our liquidity was genuinely an issue then we could sell illiquid assets for cash over the period of a year without distorting the price we would receive (and have used in our asset valuation).

The risk only impacts business where it cannot be passed on to policyholders. For example, the risk is negligible on assets backing Unit Linked business . . .

To mitigate this risk the fund holds liquid assets (cash, certificates of deposit). It also ensures that a significant proportion of its other assets are invested in highly marketable stocks, in particular government bonds (but also shares issued by large companies).

CPI-linked gilts consultation

On 29 June 2011 the DMO launched a consultation on CPI-linked gilts to build an evidence base to inform the Government's consideration of whether to issue such instruments. The consultation closed on 22 September 2011.

The DMO published its response on 29 November 2011 and announced that the Government had decided not to issue CPI linked gilts in 2012-13. It judged that issuance of such gilts in the near term would be unlikely to be cost effective and involve a number of risks. T

Table A1: Impossibility of hedging one type of annuity with another

	0%	2%	4%	6%	8%
0%	1				
2%	0.9992	1			
4%	0.9969	0.9992	1		
6%	0.9932	0.9970	0.9992	1	
8%	0.9880	0.9932	0.9970	0.9993	1

Table shows the pairwise correlations of annuity values with different interest rates but the same mortality. Calculations are based on the same simulations illustrated in Figure ###.