Default options:

Are the life cycle funds the solution?

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December 2010

Abstract

To ensure the sustainability of pension systems, many governments are encouraging their citizens to save for retirement through funded DC schemes - both occupational and individual. The pension fund development across the developed and the emerging countries poses the question of the default options. In this paper we aim to address two key questions. First, do lifecycle funds can be settled as default option? How should individuals invest in asset classes with different risk/return properties over the lifecycle? We conclude that lifecycle plans offer attractive features considering a mean variance framework as compared to other default strategies. Looking at the current market offer, these can be strengthened by greater focus in inflation linked bonds.

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1 Introduction and motivation

Aging populations and low economic performances have eroded the state-run pay-as-you-go (PAYG) pension systems. To ensure their sustainability, the governments are now adjusting the PAYG parameters (retirement age, level of benefits and level of contributions) reducing the generosity of state run schemes. Forced to reduce the generosity of PAYG[1] or partially funded defined benefit pension models, many governments are today encouraging their citizens to save for retirement through funded pension schemes - both occupational and individual - to compensate for the lesser future pension payments from the state system. For instance, the Japanese government introduces in 2004 defined contribution pension structures within a pension system exclusively dominated by defined benefit pensions. Comparable changes have been observed in Europe in particular for countries where PAYG systems are well developed as in Germany or France[2]. For instance, the British government should introduce the National Pension Saving Schemes in 2012 in which employees are forced to save for retirement funding purposes unless they are already enrolled into an occupational pension scheme (Chetouane, 2008). The funded pension plans have become a central building block in the redesign of pension systems.

According to the OECD pension statistics, total pension assets in the OECD area stood at just under 70% of the OECD GDP in 2009. Given a rapidly aging population and growing recourse to funding (pension reserves, etc) - not just in the OECD but at the global level too - this market is set to grow strongly in the coming years. Basically, two types of funded pension schemes are proposed to employees in which to invest their pension saving. On the one hand, the defined benefit pension plans (DB plans hereafter) gather pension plans sponsored by a company and in which the pension benefit is promised to the future retiree, and depends on a set of parameters. On the other hand, the defined contribution pension plans - occupational or individual - (DC plans hereafter) are pension schemes in which the pension benefit paid is a function of the amount contributed and financial performances. The key difference between both pension schemes concerns the risk management. Regarding DC pension schemes, all the risks are borne by the individual investors while they were supported by the sponsoring companies within a DB framework. The so-called "pension storm" of increased longevity, disappointing capital market returns, low long-term interest rates and a tighter regulatory environment has damaged the global pension landscape. In this context, companies have steadily deserted DB plans in favor of DC structures. This shift towards funded DC plans raises important issues in terms of pension saving investments for individuals.

[1] PAYG systems provide a defined benefit stream and can hence be described as an unfunded (or notional) defined benefit plan (NDB). While a discussion hereof lies outside the scope of this paper, it is interesting to note that in addition to the trend to shift to funded DC plans; notional defined contribution (NDC) plans are also winning ground as a building block for pension reform. The resulting shift in risks shares a number of common points with the discussion above.

[2] There are no authentic DC pension plans in France but investment vehicles which act as DC plans.

Four risks underlying the pension payment can be identified: the longevity risk, the human capital risk, the inflation risk and the investment risk. Taking into account market information asymmetries and misaligned interest rates, the risk is that the DC model could result in both expensive and suboptimal choices. At top of the list of risks transferred is the longevity risk. Longevity risk in retirement is a so-called "pure" risk to the extent that it involves a probability of loss (the cost of living longer) but with no chance for a gain (assuming that the individual is no longer earning any labor income). While the "law of large numbers" allows a certain level of predictability of longevity for a total population, at the level of the individual it is almost impossible to predict within any meaningful range. Hence, replacing a DB system by a DC system thus removes the "insurance" philosophy of the former. Turning to the second risk, we define human capital as the discounted value of all future labor income. In a funded DC plan, contributions are defined as a share of labor income. As such, if there is a loss of labor income, for example due to a period of unemployment, contributions to the plan and thus future benefits will, all else being equal, be reduced. Note that to the extent that the benefits of occupational DB plans are based on labor income, the shift from occupational DB to occupational DC plans does not involve any transfer of human capital risk, which was already carried by the plan beneficiary. The key variable for any pension plan beneficiary is not the nominal amount of the pension payment, but the purchasing power hereof, this is the inflation risk. In the first instance, the reference measure for protecting future pension payments is indexation to consumer prices. In the second instance, the reference measure is wage inflation. DB plans often offer some degree of protection to both risks to the extent that pension benefits are often calculated on the basis of a terminal salary, and then indexed to consumer prices during retirement[3]. For DC plans, there is no explicit inflation protection and as such this risk, that can also be considered a "pure" risk, is fully transferred to the individual. Investment risk distinguishes itself from longevity, human capital and inflation risks in that it is a "speculative" risk as opposed to a "pure" risk, and can thus yield either a gain or a loss. For funded DB plans - either in the state or private sector regime - the investment risk is carried fully by the plan sponsor, but is transferred to the individual under DC schemes. The question of how to manage the investment risk lies at the core of the discussion in the following sections.

As seen from the discussion above, the move to funded DC schemes entails an important shift of risks to the individual. Individuals enrolled in a DC plan have to choose, according to their risk aversion and their time investment horizon, an investment strategy within a menu offered by the plan. Focusing on the investment choice process, behavioral finance studies bring interesting insights to understand the investor psychology when he faces a large panel of choices. On the one hand, Shea and Madrian (2001) underline the role played by inertia within the retirement plan investment decisions. This result echoes those found by Michell and Utkus (2004) who show that only 10% of the Vanguard Group participants adjust their contributions allocation every year. Besides, Lavigne and Legros (2005) underline that individuals facing a set of pension investment solution usually choose the default option. Beshears et al. (2007) point out that a significant proportion of pension investors choose the default option among the other fund solutions

[3] Practices of cost-of-living adjustment for pension benefits vary greatly from country to country and in some instance from plan to plan.

offered. In the United States, 80% of individuals enrolled in a DC pension plans invest their pension savings in this way. Similar proportion has been found in Australia (Basu & Drew, 2006). In the same vein Garnier and Thesmar (2009) report the Swedish experience where government introduced a funding pension system constraining the employees to save a little fraction of their compensation. The pension plan members can choose between a government retirement funds and private pension funds. A large majority of new participants opt for the default option (83% in 2001 and 96% in 2006). In addition, they reveal that the default option fund performances were much higher than those proposed by asset managers.

In this context, the initial investment decision plays a substantial role as it determines, in most cases, the remaining asset allocation over the entire saving horizon. In light of these results, it appears that future retirees are unable to make a reasoned investment choice. In addition, the OCDE/IORP institutions point out that the lack of financial literacy of individual investors leads them to make inadequate investment choices. This reinforces the necessity for fund managers, or to a greater extent for the regulator, to supervise the pension plan participants in their portfolio choice. In this context the default option choice is crucial. The investment menu proposed by DC plans always comprises a default option which in most cases corresponds to lifecycle funds. The lifecycle fund allocates pension savings according to the age of the investor. In other words, the share of stocks decreases as the investor becomes older. The pension wealth is then auto-piloted until the retirement date. Taking into account the success of the lifecycle solution and the amount of assets invested for retirement pensions, we ask how robust this pension solution is, as compared to other long term investment strategies? In other words, does a lifecycle asset allocation strategy offer a superior risk-return outcome compared to other default options? Besides, the question of the riskless asset is a burning issue in building long term investment solutions. Can the traditional long term government bond yield fulfill this role or should we switch to inflation linked bonds? In addition, can other investment methods, such as portfolio insurance, be considered in this framework?

Based on a Monte Carlo simulation and backtests and calibrated on the American financial markets, our empirical analysis puts forward the role of constant mix and lifecycle funds. Considering the risk tolerance of the investor and in light of several risk indicators, aggressive lifecycle and constant mix strategies can be seen as potential candidates for the default option. While pure equity funds always outperform all other default options, the risk associated with this investment solution is too high to be considered as a default option. The results also confirm the risk inherent to pure fixed income solutions in terms of replacement rate. We find also that autopilot strategies provide better outcome as compared to naive strategies (equally weighted portfolio). These results are enhanced by the introduction of inflation-linked bonds. The inclusion of inflation bond characteristics in the aggressive lifecycle and constant mix strategies lead to replacement rates comparable to those of the pure equity strategies. But the risk inherent to these strategies is much lower than the pure equity one. Finally, the first attempts made to include a portfolio insurance approach do not lead to conclusive results. Further researches have to be performed in this field.

The structure of this paper is as follow. In section 2, we review the theoretical and empirical studies dealing with the relevance of a default option. In section 3, we present the background required for the Monte Carlo and the backtests exercises. We also present the performance indicator which will be used to compare the investment strategies. In section 4, we report and discuss the results. In section 5, we will appreciate how relevant is the use of inflation-linked bonds as the riskless asset. Likewise, the use of a portfolio insurance framework will also be considered. The last section concludes.

2 The literature

The recent figures of the Survey of Consumer and Finance (released in 2007), show that retirement is the first saving motivation of American households. Meanwhile, Thaler and Sunstein (2008) notice that a large fraction of American households consider that their saving effort is insufficient to face retirement issues. In this context, the pension market is expected to gain ground over the coming years. This would encourage or reinforce the development of different types of default pension solutions whose aim is to answer to the different needs of the future retirees.

2.1 The widespread default option solutions

Benartzi and Thaler (2001), observe that DC plan members portfolio choices do not follow the message delivered by the financial theory. They notice that pension plan participants affect their pension wealth in a naive way. Windcliff and Boyle (2005) put forward a heuristic diversification rule called the "1/n pension investment puzzle". This simple rule describes a naive asset allocation where the investor breaks down its portfolio in as many equal proportions as there are available investment asset classes getting thus an "equally weighted portfolio". Under a set of several conditions, the authors show that the equally weighted portfolio can protect the investor against very bad outcomes.

When DC plan members are not able to choose an investment vehicle, the pension funds or the asset management industry are obliged to propose default options in their panel of investment solutions. One of the most widespread investment vehicle provided by the financial or the pension industry is the constant mix solution. The constant mix portfolio solution is a particular case of buy and hold strategies in which the proportions invested into risky and safer assets remain constant over the investment horizon. Basically, constant mix strategies involve stocks and bonds, assuming that bonds is the safer asset and equities the risky asset. Let α be the fraction of risky assets in the portfolio, S_t the stock price index, B_t the riskless asset price index and W_t the financial wealth of the pension fund member. Hence the dynamics of W_t given by:

$$W_t = W_{t-1}\left[1+\left(\overline{\alpha}\frac{S_t}{S_{t-1}}+\left(1-\overline{\alpha}\right)\frac{B_t}{B_{t-1}}\right)\right].$$

Buy and hold or constant mix strategies are based on asset class relative performances observed over a long holding period. Gollier (2007) studies the relationship between risky asset returns and the holding horizon. He demonstrates that long term investors should heavily invest on stocks due to the negative relationship between risk and stockholding. Likewise, Bec and Gollier (2007) consider this issue studying the French case. Their investigations indicate "that the risk associated with equities is actually significantly higher than the one of other assets for short investment horizons, but it becomes statistically undistinguishable from the bonds rolled risk for horizons longer than 5 quarters and from the bonds held to maturity after around six years". This result gives additional support to the Samuelson criticism. Based on a fallacious lecture of the Law of Large Numbers, Samuelson indicates that "the repetition of an investment pattern over many periods does not cause risk to wash out in the long run" (Gollier & Zeckhauser, 1997). In this context, investors can maintain a high exposure to equities whatever their position in the lifecycle. This result receives additional support considering the performance of riskless assets compared to equities. Indeed, using a large sample (from 1802 to 2006), Siegel (2007) shows that American stock returns are on the one hand higher than the returns of treasury bonds and cash assets and on the other hand very stable, whatever the sub-period considered. Furthermore, Siegel (2007) signals that real fixed income returns were negative on average between 1946 and 1981. Davis (1995) also notes that, between 1967 and 1990, the standard deviation of fixed income investment returns was close to the one of equity returns. Using the value at risk indicator, Blake et al. (2004) show that conservative approaches increase the probability of losses.

Despite the theoretical and empirical evidences, this issue is still subject to controversy. Bodie (2001) tackles this question by considering the costs of hedging against the risk of lower earnings compared to risk free interest rate. Bodie shows that the "riskiness of equities increases" as the investment horizon increases, whether the stock returns are mean reverting or not. Enhancing the analysis of Bodie (2001), Lankassen and Propper (2007) find similar results. In addition, the authors point out the fact that these studies neglect two fundamental aspects regarding long term investors: the role played by the human capital and the link between investor age and risk aversion (additional references are given in the next section regarding this two issues). The introduction of these parameters in the long term investment solution design has opened the way for the development of lifecycle or target date solutions.

At this stage, Malkiel (1996) proposes a simple rule which reconnects the investor's age to the portfolio choice over the lifecycle. Broadly speaking, an individual's equity exposure should be equal to 100 minus their age. For example, a 35 year old person holds 65% of the "risky" equity portfolio and 35% of the "safe" fixed income portfolio. These products have grown in popularity in recent years. In the US, 48.5% of 401(k) plans offered a lifestyling fund in 2005 - up from a mere 12.1% in 1996. De facto, the lifecycle funds offer, proposed to DC plan members, has been diversified with more or less prudent investment strategies. The following charts give an illustration of lifecycle funds solution offered to 401(k) plan members in the United States.

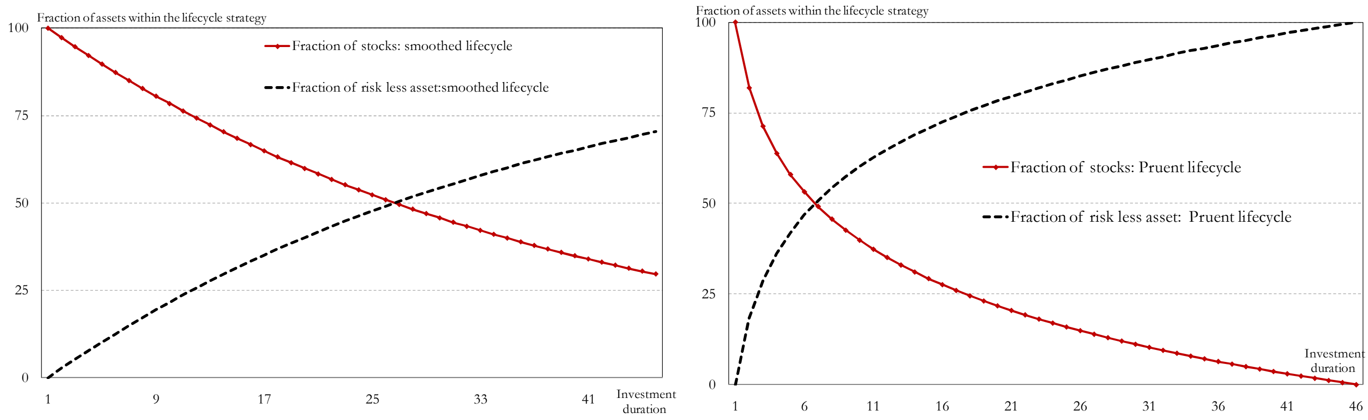

Figure 1: Smoothed and prudent lifecycle investment strategy

2.2 The theoretical foundations of the lifecycle asset allocation

Merton (1969) and Samuelson (1969)[4] showed that under several restrictive assumptions, the optimal portfolio structure is independent of age. However, the introduction of background risks, of less restrictive assumptions on stock return characteristics, of additional constraints such as liquidity constraints, are different ways used by the modern portfolio model to connect investor age and its asset allocation decision. Considering an intertemporal portfolio choice model, the introduction of the purchasing power risk leads the optimal investor to reduce its equity exposure in favor of indexed investment products. Likewise, the introduction of mean reverting stock returns implies that investors may benefit from mean reverting stock returns. In this context, younger investors have to be aggressively invested on stocks and should reduce progressively their equity exposure as they are aging. In addition, Bodie, Merton and Samuelson (1992) introduce human capital as a state variable within a stochastic intertemporal framework. They find that the fraction of an individual's wealth optimally invested in equity should decline with age for two reasons: young investors can adjust their labor supply with greater flexibility and their human capital is greater than that of older persons and is generally less risky than many financial assets. While earnings and stock returns are orthogonal in the short run, Colin-Dufresne and Goldstein (2007) show however that both variables are cointegrated. In the long run, additional income can be generated by substituting risky assets to human capital asset .

For the same reasons, Bakshi and Chen (1994) suppose the existence of a lifecycle risk aversion translating the fact that the investor's risk aversion changes throughout his lifetime. In particular, they suggest that risk aversion increases as the investor is aging. Numerous studies have addressed the relationship between risk aversion and investor age. The investigations performed by Morin and Suarez (1983), Halek and Eisemhauer (2001) and Bellante and Green (2003) put forward such a relationship. Thereby, the propensity to be exposed to risky investments is lower for older investors than for younger ones, giving thus support to lifecycle investment solutions. At this stage, one may question the older investor investment choices. Here again, various authors addressed this issue using different datasete and

[4] Merton (1969) explored the LPS issue using a continuous time model whereas Samuelson (1969) did it in a discrete time framework.

methodologies over various countries. Using numerous waves of the Survey of Consumer and Finance, Yoo (1994), Poterba and Samwick (1997), Heaton and Lucas (2000), Bertaut and McCuller (2000) find that the fraction of riskless assets held by households increases with the age of the household head. In other words, they highlight the existence of a hump shaped pattern between the fraction of risky assets and the investor's age profile. Comparable results have been found using the TIAA-CREFF survey (Bodie & Crane, 1997) and the dataset extracted from the individual retirement accounts asset allocation (401k) dataset (Agnew, Bladuzzi & Sunden, 2003; Holden & Vanderhei, 2009).

2.3 Testing default options

Considering a mean-variance framework, Butler and Domian (1991) use simulated data and show that equity investments provide higher outcomes than lifecycle strategies. To discriminate between a set of financial vehicles, Kim and Wong (1997)use the stochastic dominance measurement. They conclude that the optimal strategy must be aggressively invested in stocks. In the same vein, Hickman et al. (2001) compare the performance of the "100 minus age" rule to basic equity funds. The simple rule only outperforms the equity index in 15% of cases. Likewise, Poterba et al. (2006) analyze the outcome distribution of lifecycle and constant mix investment vehicles. They do not observe significant differences between both distributions. In light of these results, one could conclude that lifecycle strategies are not relevant as the default option for long term investors. However, Hibert and Mowbray (2002) demonstrate that lifecycle investment strategies lead to a reduction of the outcome standard deviation. In addition, Bodie and Trussard (2007) recommend the implementation of a "safe target date fund" which matches the prudent investor. The Bodie and Trussard (2007) results confirm that the "one size fits all" principle cannot be applied without generating suboptimal situations[5]. Antolin et al. (2009) signal that DC pension schemes have experienced huge financial losses due to the financial downturn. In particular, they observe that the financial crisis hits aggressive investment strategies. From an historical database, the author test different default options and reveal that lifecycle strategies, with relatively high equity exposure, offer higher replacement rates than constant mix strategies. Beyond this first result, they insist on the fact that lifecycle strategies show higher resistance during financial downturn episodes. Accounting for the size of the DC pension schemes, and the increasing number of participants, Antolin et al. (2009) recommend redesigning the pension fund investment regulation to avoid older workers and retirees being exposed to such risks.

3 Default options behavior: a simulation exercise

3.1 The framework

We assume that the future retiree begins to save for retirement at 20 years old and that the legal pension retirement age is set at 65. Thus, the investment horizon is 45 years and we assume no early retirement system. We consider that the individual devotes a constant proportion of his earnings to face its

[5] Vigna and Harberman (2002) conclude in the same way using a stochastic control approach.

retirement. While the contribution rate remains constant over the working period the wage growth rate is expected to vary throughout. Actually, the earnings growth rate varies with the age of the individual. Basically, we observe a rapid increase in earnings at early ages (between 20 and 34 years) and a progressive slowdown until the retirement age. The following chart illustrates income earning changes in function of age in the United States.

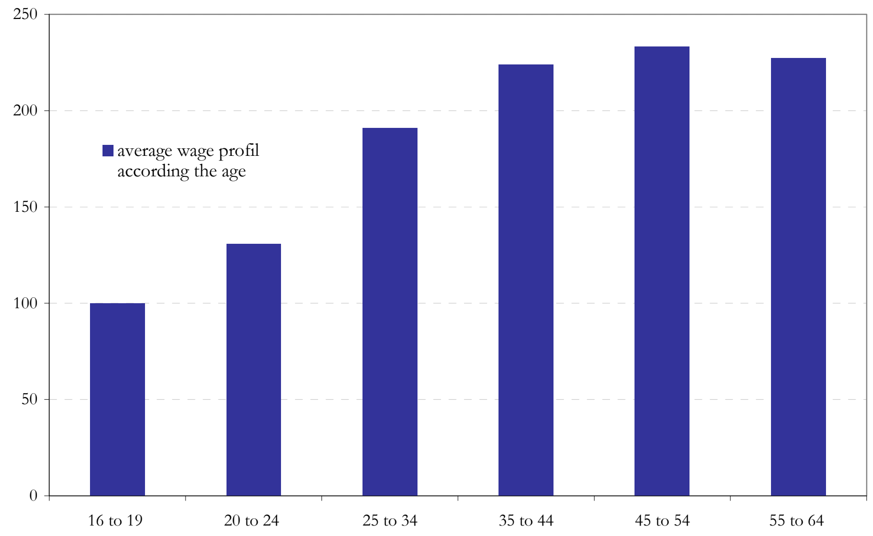

Figure 2: United States - Rebased average earning changes per age (sources: BLS)

The employee contributes each year to the pension scheme at a fixed and predetermined rate. We assume that the contribution rate is set at 10% of the investor's earnings. In line with the DC pension plan behavior, the accumulated contributions are invested in the financial markets throughout financial vehicles selected by the pensioners. We assume that the retirees do not move from an investment vehicle to another during the investment period despite the absence of fees or transaction costs. This assumption is not limitative as inertia affects the household asset allocation decision. Thus, the selected investment vehicle at the beginning of the process remains exactly the same during 45 years.

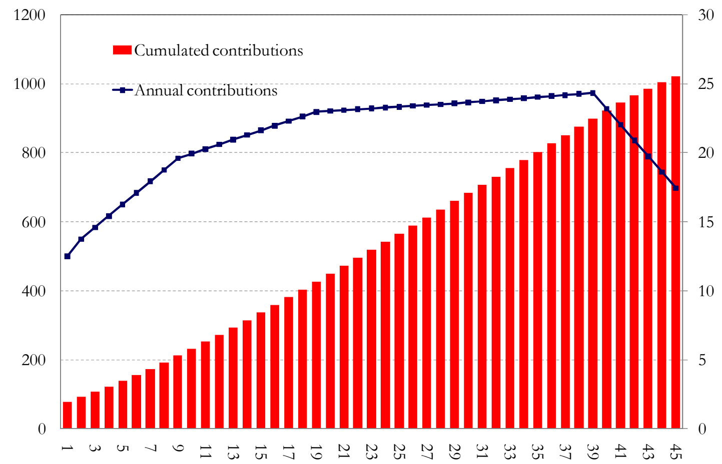

Figure 3: Saving and cumulated contributions to the DC plans (Sources: BLS and author's calculations)

In this context, the pension plan member financial wealth dynamics can be written as follows:

$$W_{i,t+1} = \left[W_{i,t}\left(1 + r_t^i\right)\right] + \overline{\gamma}A_t \ ,$$

$$W_{i,T} = W_0 \prod_{i=1}^{T}\left[1 + r_{t+i}\right] + \sum_{j=1}^{T-1} \overline{\gamma}A_j\left(1 + r_j\right) + \overline{\gamma}A_T \, ,$$

where W_t is the financial wealth, $r_{i,t}$ the investment return, $\bar{\gamma}$ the contribution rate and A_t the investor's earning. Having established the contribution profile of a DC plan member and hence his pension wealth, we briefly present the solution categories proposed by DC schemes. We analyze the pure investment solutions, the constant mix solutions, the autopilot investment strategy (or the "100 minus age" solution), the lifecycle funds and finally an investment solution based on a naive approach. The following table gives further details regarding the default options tested in this study:

	Initial equity exposure	Average equity exposure	Average long term bond exposure	Average cash (or short term asset) exposure
Naive divesification	**Equidiversified portfolio (1/n)**			
Aggesive constant mix with nominal bonds	70%	70%	30%	-
Prudent constant mix with nominal bonds	40%	40%	60%	-
Vulgate "100 minus age" w/nominal bonds	75%	52%	48%	-
Aggressive life styling w/nominal bonds	100%	73%	27%	-
Prudent life styling w/nominal bonds	100%	22%	78%	-
Smoothed life styling w/nominal bonds*	100%	56%	44%	-
Pure equities	100%	100%	-	-
Pure nominal bonds	0%	-	100%	-
Pure Money market	0%	-	-	100%

Table 1: Investment strategies sample

To perform our study, we need to simulate a stock price index, a cash index and finally a long term bond index. The following section is dedicated to the presentation of the instruments used to simulate the pension default options.

3.2 Simulation framework

We assume that asset returns are randomly distributed which implies no serial correlation. Moreover, as we consider low frequency data, namely, annual data, we can thus assume that stock returns are not characterized by mean reversion (Basu, 2009). We assert in addition that financial asset real prices and yields can be represented by stochastic processes. In this context, we choose to use a Monte Carlo tool to simulate a large number of market data. The short term real interest rate is modeled as an Ornstein-Uhlenbeck process which corresponds to a mean reverting process. This representation provides a satisfactory framework to describe real short term interest rates. In our framework, we consider the real short term interest as a driver for bond, stock and inflation linked bond indices. In this context, the real short term interest rate dynamics is thus written as follows:

$$dr_t = a\left(\beta - r_t\right)dt + \sigma_r dW\left(t\right),$$

where parameters α, β and σ_r are non-negative and constant and r_t is the current level of the real interest rate. The parameter a represents the mean reversion degree with which the interest rate dynamics reverts to an average value β. A high value of a translates a rapid return of r_t to the long term nominal interest rate. The coefficient σ_r is the diffusion of the stochastic process and $W(t)$ is a Weiner process. The solution of the previous dynamics is given by:

$$r_t = r_0\, e^{(-at)} + \beta a \int_0^t e^{-a(t-s)}\, ds + \sigma \int_0^t e^{-a(t-s)}\, dW_s\left(\beta - r_t\right)dt,$$

$$r_t = r_0\, e^{(-at)} + \beta\left(1 - e^{(-at)}\right) + \sigma\, e^{(-at)} \int_0^t e^{-at}\, dW_s.$$

The discretization of the previous relationship leads to the following expression of the short term interest rate dynamics which will be used to simulate our cash data:

$$r_t = r_0\, e^{(-at)} + \beta\left(1 - e^{(-at)}\right) + \sigma\sqrt{\frac{\left(1 - e^{(-2at)}\right)}{2a}}\varepsilon_t.$$

Stock market price paths are usually modeled with Geometrical Brownian Motion processes. Let S_t be the stock index price at time t. The GBM process and its exact discretization are written as follows:

$$\frac{dS_t}{S_t} = \mu_S dt + \sigma_S dW_t,$$

$$S_t = S_0\, exp\left\{\left(r - \frac{\sigma_t^2}{2}\right)dt + \sigma_t\sqrt{dt}\varepsilon_t\right\}.$$

where μ is a constant which represents the drift of the stochastic differential equation and σ is the diffusion associated to the Brownian motion $W(t)$. In addition, we choose to correlate the equity market increments to the one of the real short term interest rate, using the Cholevski decomposition. This assumption implies the following changes in the expression of the GBM:

$$\frac{dS_t}{S_t} = \mu_S dt + \rho dW_t^r + \sqrt{\left(1 - \rho^2\right)}\sigma_r dW_t^S\ ,$$

$$S_{t+dt} = S_t\, exp\left\{\rho\left(r - \frac{\sigma_t^2}{2}\right)\sqrt{dt}\varepsilon_t^r + \left(1 - \rho^2\right)\sigma_t\sqrt{dt}\varepsilon_t\right\}.$$

In the same vein, we link the long term bond increments to the real short term interest rate one. Hence, the dynamic of the bond index is given by:

$$\frac{dB_t}{B_t} = \mu_B dt + \rho dW_t^r + \sqrt{\left(1-\rho^2\right)}\sigma_B dW_t^B \ ,$$

$$B_t = B_0 \exp\left\{\rho\left(r - \frac{\sigma_t^2}{2}\right)\sqrt{dt}\varepsilon_t^r + \left(1-\rho^2\right)\sigma_t^B\sqrt{dt}\varepsilon_t\right\}.$$

The simulations parameters are calibrated on the United States financial data. The main difficulty lies in the choice of the most relevant window for the calibration of the processes. Keeping the purpose of this study in mind, we finally choose to calibrate our equations using a rolling sample equal to the holding period, that is, 45 years. We evaluate the average real rate of return and the average standard deviation over 45 years rolling periods going from 1850 to 2005 (or 110 overlapping periods), for a stock index, a cash index and a long term bond index. The data come from the "Global Finance Data" and they were provided by the Société Générale Asset Management. The figures used to calibrate the stochastic process are given in the following table:

	Real Stock index yield[1]	***Real Cash index yield***	***Real "nominal bond" index yield***
Mean	6,2%	1,6%	2,0%
Standard Deviation	18,5%	4,9%	7,3%

[1] Real stock return account for real dividends
[2] 10 years inflation expectations are considered to build ILB index yield

Table 2: Monte Carlo calibration

3.3 The risk indicators

To compare the various investment vehicles, we use several measures of investment risk and return. The performance of an investment vehicle can be presented in different ways. In our framework, we measure the investment yield by calculating the number of retirement years funded, taking the average earning over the working period as the amount of pension benefit received. Let $\Gamma_{i,T}$ be the number of years paid by investment strategy *i*. Then, we have

$$\Gamma_{i,T} = \frac{\overline{E}}{W_{i,T}}.$$

This measure of the investment vehicle performance is debatable as pension fund members compare their pension benefits to their last earning. For instance, Poterba et al. (2006), Basu (2009), Antolin et al. (2010) use the retirement wealth ratio, which compares the terminal pension plan member earning to the terminal financial wealth, as a benchmark to evaluate investment strategy performances. Considering this performance indicator, we calculate the standard deviation:

$$\sigma_i^2 = \frac{1}{N}\sum_{i=1}^{N}\left(r_i - \mu_i\right).$$

Moving on to the measures of risk, it must be kept in mind that the standard deviation gives the same weight to negative and positive outcomes. To avoid this limitation, we calculate the lower partial moment and focus in particular on the semivariance indicator, which was introduced by Markovitz in (1959). The semivariance indicator takes only into account outcomes that are below the average[6]. Knowing the semivariance, we can evaluate the downside risk for each default option. The lower partial moment, the semivariance and the downside risk expressions are given by the two following relationships:

$$Lpm_i = \frac{1}{N}\sum_{i=1}^{N}\left(Max\left(0, r_i - \mu_i\right)\right)^{\gamma} \rightarrow SVar_i = \frac{1}{N}\sum_{i=1}^{N}\left(Max\left(0, r_i - \mu_i\right)\right)^{2},$$

$$dws_i = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(Max\left(0, r_i - \mu_i\right)\right)^{2}}.$$

To evaluate the probability of losses, asset managers refer to the Value at Risk indicator. For a predetermined level, the Value at Risk measures the potential losses given an investment horizon. Assuming a Gaussian distribution, the Value at Risk, for the risk threshold α is given by:

$$VaR_i\left(h, p\right) = \alpha_p \sigma_{h,i},$$

where h is the holding period. We admit however that the use of the Value at Risk over such horizons is subject to criticisms (see Blake et al., 2004, for additional details regarding this issue). In a second stage, we use relative measures of risk and performance in order to build a ranking of the studied investment vehicles. We start by calculating the Sharpe ratio defined (Sharpe, 1966) by the following quantity:

$$Sr_i = \frac{\left(\kappa_{i,t} - R_f\right)}{\sigma_{\Gamma_{i,t}}},$$

where R_f is the risk free rate of return and $\kappa_{i,t}$ the investment return. In the same vein, we use the Roy ratio (1952) in which the investment strategy return is compared to a benchmark. When we consider the number of years of retirement funded by the investment strategies as our performance indicator, the target used as the minimum acceptable return (MAR) is the life expectancy at 65 years old. The last figures of the National Vital Statistics System (from the Centers for Disease Control and Prevention) show that the life expectancy at 65, in 2005, reached 18.2 years. We use the performance of the pure long term bond strategies as well, as a benchmark to calculate the Roy ratio. Broadly speaking, the Roy ratio is given by:

$$Roy_{i,t} = \frac{\left(\Gamma_{i,t} - Mar_t\right)}{\sigma_{\Gamma_{i,t}}}.$$

[6] Note however that the semivariance indicator weighs identically extreme losses and slight losses. In other words, this measure does not discriminate between financial crashes and downwards financial markets for instance.

Investment strategies can also be selected using their empirical cumulated distribution function. Using the stochastic dominance properties, we can determine graphically the investment strategy which stochastically dominates the others. Let S_i and S_j be two investment strategies. X_t stochastically dominates Y_t at the first order if:

$$E\left[u\left(S_i\right)\right]=\int_a^b u_t\left(w\right)dF_{S_i}\left(w\right) \succ_1 E\left[u\left(S_j\right)\right]=\int_a^b u_t\left(w\right)dF_{S_j}\left(w\right).$$

The first order stochastic dominance may appear restrictive in most cases. Hence, we consider the second order stochastic dominance definition. S_i stochastically dominates S_j at the second order if:

$$\int_a^w F_{S_i}\left(s\right)ds \leq \int_a^w F_{S_j}\left(s\right)ds \; for \; w\in\left[a,b\right].$$

In the coming sections, we will present and discuss the results from the Monte Carlo simulations using both traditional measures and the indicators mentioned above. In a second stage, we present the results derived from the backtests using the same indicators.

4 The results

4.1 Monte Carlo simulation exercises

As we consider rolling periods, we generate 300 paths for each market variables getting thus 54000 time series, leading to 2 160 000 observations for each investment strategies. The Monte Carlo simulations show, without any surprise, that the portfolio entirely invested in equities finances the highest number of retirement years (Table 3). Secondly, it can be observed that investment strategies based on a heavy exposure to stock markets lead to higher outcomes and outperform the other default options. For instance, an investor who invests his contribution in equity investment vehicles over the entire working period generates a terminal financial wealth which corresponds to 15 years of earnings. The simulations indicate that aggressive lifecycle investment vehicles provide better outcomes compared to the aggressive constant mix strategy. Besides, the performance of the autopilot or linear lifecycle strategy is very close the aggressive constant mix one which is unexpected as the proportion of bonds in much lower than in the previous one.

	Mean	Min	Max	Quartile 1	Quartile 3
Naive divesification	8,18	1,89	24,55	5,99	10,00
Pure nominal bonds	6,93	0,92	42,41	3,58	8,51
Pure Money market	6,97	1,28	23,75	4,96	8,49
Pure equities	14,57	1,64	144,90	8,17	18,61
Aggressive constant mix with nominal bonds	10,97	1,62	55,26	7,05	13,67
Prudent constant mix with nominal bonds	8,74	1,29	36,04	5,51	10,95
Aggressive life styling w/nominal bonds	12,83	1,64	144,90	7,83	16,21
Smoothed life styling w/nominal bonds	9,86	1,64	55,57	6,59	12,12
Prudent life styling w/nominal bonds	7,83	1,62	36,32	4,86	9,56
Vulgate "100 minus age" w/nominal bonds	10,25	1,64	50,00	6,81	12,66

Table 3: Monte Carlo Simulation results for the investment vehicles in terms of years of retirement funded

This result suggests that there is a benefit in time diversification. Excluding equities, pure investment strategies provide the lowest performance in terms of funded years of retirement. Naïve diversification of the portfolio, frequently used as the last resort investment vehicle by pension plan members, presents returns comparable to prudent approaches. Furthermore, the pure long term bond solution provides the worst outcome. Looking closer at the outcome distribution, we can notice that the dispersion of the first quartile is small while it is much more pronounced for the third quartile

	Roy ratio (MAR = life expectancy at 65)	Roy ratio (MAR = Bond portfolio asset)	Sharpe Ratio
Naive divesification	-3,301	0,411	0,241
Aggessive constant mix with nominal bonds	-1,327	0,741	0,248
Prudent constant mix with nominal bonds	-2,151	0,407	0,175
Vulgate "100 minus age" w/nominal bonds	-2,038	0,254	0,247
Aggressive life styling w/nominal bonds	-1,484	0,822	0,236
Prudent life styling w/nominal bonds	-4,543	0,383	0,206
Smoothed life styling w/nominal bonds	-1,764	0,620	0,238
Pure equities	-0,391	0,823	0,207
Pure nominal bonds	-2,270	-	-
Pure Money market	-3,975	0,013	0,109

Table 4: Other performance measures – Monte Carlo simulations

Table 4 gathers relative performance indicators, i.e. the Roy ratio in which we consider alternatively the life expectancy at 65 and the long term bond yield as the minimum acceptable return, and the Sharpe ratio[7]. While the previous results point out the weakness of pure long term bond investment strategies, the results derived from the Roy ratio calculations put this observation into perspective. Actually, the contributions invested in prudent lifecycle products or in a 100% cash options give the weakest performances while aggressive equity options ensure the highest return. This normalized indicator shows

[7] The Sharpe ratio is calculated considering the investment return, in a literal sense, and the riskless interest rate at each date.

that these two strategies do not succeed to provide a sufficient financial wealth to meet DC plan member needs over their remaining lifetime. Considering the bond portfolio asset as a benchmark, the Roy ratio (2nd column) signals the low performance of the autopilot option. The Sharpe ratio, in which the difference between the default option return and the risk free rate (here the cash asset) is normalized by the standard deviation, challenges the previous pictures. Actually, the Sharpe ratio associated to the pure equity solution is substantially lower than the aggressive constant mix and lifecycle fund. But, more surprisingly, the autopilot and the smoothed lifecycle strategies display a Sharpe ratio comparable to the one of the aggressive solutions. In light of these findings, these investment strategies appear most relevant for long term investors.

	Standard deviation	*Semi variance*	*Risk shortfall*
Naive divesification	3,036	0,079	0,280
Aggessive constant mix with nominal bonds	5,447	0,130	0,360
Prudent constant mix with nominal bonds	4,405	0,065	0,256
Vulgate "100 minus age" w/nominal bonds	4,916	0,079	0,280
Aggressive life styling w/nominal bonds	7,175	0,158	0,398
Prudent life styling w/nominal bonds	2,343	0,062	0,250
Smoothed life styling w/nominal bonds	4,726	0,083	0,287
Pure equities	9,282	0,231	0,480
Pure nominal bonds	4,964	0,016	0,127
Pure Money market	2,825	0,076	0,276

Table 5: Risk indicator based on the Monte Carlo simulations

To refine our judgment, these results have to be considered in light of risk parameters. Moving on to risk indicators (reported in Table 5), pure equity and aggressive lifecycle investment strategies present much higher standard deviation than the other investment strategies. Most surprisingly, holding pure nominal bond portfolio does not protect the investor against bad outcomes, as in Blake (2004). The lowest standard deviation is observed for the prudent lifestyling investment strategy. The assessment is quite different when the risk indicator only comprises negative outcomes. In this context, the pure long term bond investment vehicle appears to be the safest default option while the pure equity portfolio is still the riskiest. Comparing now the lifestyling options to the constant mix approaches in terms of risk, the aggressive constant mix options show better figures while risk adverse profiles should prefer naïve diversification or smoothed lifestyling portfolios indifferently.

	Value at risk			
	10%	5,0%	2,5%	1,0%
Naive divesification	3,89	5,03	5,86	7,07
Aggessive constant mix with nominal bonds	6,97	9,02	10,51	12,69
Prudent constant mix with nominal bonds	5,64	7,29	8,50	10,26
Vulgate "100 minus age" w/nominal bonds	6,29	8,14	9,49	11,45
Aggressive life styling w/nominal bonds	9,18	11,88	13,85	16,72
Prudent life styling w/nominal bonds	5,63	7,29	8,49	10,25
Smoothed life styling w/nominal bonds	6,05	7,82	9,12	11,01
Pure equities	11,88	15,37	17,91	21,63
Pure nominal bonds	6,35	8,22	9,58	11,57
Pure Money market	3,62	4,68	5,45	6,58

Table 6: Risk indicator - Value At risk

We finally end the analysis by paying attention to the Value at Risk measure. The Value at Risk indicates the probability to see the investment option return below a certain level, considering various confidence levels and rolling periods of 45 years. In other words, there is a 5% chance of seeing the pure equity returns goes below the "15 years" level. Likewise, there is a 5% chance of seeing the naïve diversification providing a replacement level equivalent to 5 years of earning. Reported in Table 6, the evaluations confirm the results given by the previous indicators. At a 5% level of risk, we note that the lowest Value at Risk is reached for cash investment vehicles whereas the highest is reached for pure equity. The Value at Risk points out the superiority of lifecycle funds over the constant proportion ones. In light of this indicator, the autopilot strategy provides better outcome than the naïve diversification and the smoothed lifecycle fund solutions.

To conclude, the Monte Carlo simulations bring interesting and unexpected insights. The simulation exercises put in perspective the superiority of the pure equity investment solution. Corrected from the risk indicator, the aggressive lifecycle strategies provide comparable outcomes to the most aggressive solutions and higher replacement levels as compared to the aggressive constant mix. In the same vein, the autopilot lifecycle fund outperforms pure fixed income strategies, which provide the lowest outcomes, and the equally weighted portfolio investment. Finally, risk tolerant and risk adverse investors can find in the lifecycle funds a relevant answer to the retirement funding challenges.

4.2 Backtesting the strategies

4.2.1 The background

Considering the previous investment vehicles, we propose to test the relevance of the results with a historical database which starts in 1840. We thus apply each successive 45 year periods of market history to the individual, given the lifetime earning profiles and applying the same constant contribution rate of 10%. As previously, the labor supply is assumed to be fixed with no options to alter pension savings in response to financial market performances. Furthermore, we do not take into account the probability to be in employment as it is assumed that the contributions are paid every year. The data used in the model are historical data for the US equity market, long-term nominal government bonds and short-term money market bills. The backtests are performed on real rates of return, calculated using the American consumer price index. The overall idea is thus to test how the described investment strategies did over all the 45 year periods of market data from 1850 to 2005. Given that the final 45 year period starts in 1960, the simulation covers a total of 110 overlapping periods. In line with the previous simulation exercise, we use a similar indicator to compare and evaluate the different investment option outcomes.

4.2.2 The results of the backtests

Unsurprisingly, the fixed income strategies offer the safest option when long term bonds are used (Table 7). Nevertheless, the relative security offered by this strategy has a cost in terms of returns as the nominal bond portfolio can only cover seven years of retirement. However, the money market option appears decidedly less safe than intuition may have suggested. This is because money market instruments provide neither interest rate hedge due to their short maturity nor any inflation hedge. The more surprising outcome of these simulations is that the aggressive life styling strategy appears superior to the aggressive constant mix in a significant way measured in returns. Conversely, the prudent lifecycle investment is less profitable than the constant mix one. In light of these outcomes, lifecycle approaches could play the role of default options for DC plans members. These backtests confirm the superiority of autopilot strategies over naive strategies in terms of replacement rate as well. This results has however to be taken with caution as we do not introduce the risk in our discussion.

	Mean	*Min*	*Max*	*Quartile 1*	*Quartile 3*	*Standard deviation*
Naive diversification	10,04	3,16	19,24	7,04	12,18	4,11
Pure nominal bonds	7,27	1,74	13,91	5,46	9,40	3,12
Pure money market	7,73	2,21	15,92	4,59	11,18	4,21
Pure equities	16,57	3,06	37,49	8,18	12,85	9,22
Aggressive constant mix with nominal bonds	13,59	3,71	26,96	7,98	18,38	6,17
Prudent constant mix with nominal bonds	10,63	3,03	19,67	7,48	13,29	4,25
Aggressive life styling w/nominal bonds	14,98	3,06	33,32	8,35	20,91	7,46
Smoothed life styling w/nominal bonds	11,82	3,06	24,39	7,78	14,36	4,91
Prudent life styling w/nominal bonds	8,86	3,06	18,01	6,44	11,43	3,62
Autopilot life styling w/nominal bonds	12,50	3,44	24,82	7,82	16,30	5,33

Table 7: Investment strategy backtests: Returns in terms of years of retirement funded.

Looking at the standard deviation indicator, the autopilot strategy presents, as expected, a higher risk profile compared to the naive strategy and the corrected indicators of risk tend to confirm this result. In addition, we notice that the autopilot and smoothed lifestyling strategies are as risky as the aggressive constant mix, even though their exposure is significantly lower. This observation reduces the interest of such options for retirement. Note moreover that the results highlighted by the Monte Carlo simulation regarding the pure fixed income are confirmed by the backtests exercises.

	Standard deviation	Semi variance	Downside risk
Naive diversification	4,11	2,90	1,70
Aggressive constant mix with nominal bonds	6,17	3,59	1,90
Prudent constant mix with nominal bonds	4,25	2,70	1,64
Autopilot life styling w/nominal bonds	5,33	3,51	1,87
Aggressive life styling w/nominal bonds	7,46	4,58	2,14
Prudent life styling w/nominal bonds	3,62	2,42	1,56
Smoothed life styling w/nominal bonds	4,91	3,52	1,88
Pure equities	9,22	5,72	2,39
Pure nominal bonds	3,12	1,95	1,40
Pure money market	4,21	3,16	1,78

Table 8 : Backtests - Alternative risk measures

Moving on to the relative performance indicators, reported in Table 9, we observe that whatever the investment option considered, the Roy ratio (in the first column) is negative. This translates the fact that there is no investment option which succeeds in generating a sufficient amount of financial wealth to ensure an income over the remaining lifetime of DC members. The two other indicators reveal that both aggressive constant mix and lifecycle funds hardly outperform the pure equity strategies.

	Roy ratio (MAR = life expectancy at 65)	Roy ratio (MAR = Bond portfolio asset)	Sharpe Ratio
Naive divesification	-1,98	0,68	0,75
Aggressive constant mix with nominal bonds	-0,75	1,02	1,20
Prudent constant mix with nominal bonds	-1,78	0,79	0,85
Autopilot life styling w/nominal bonds	-1,07	0,98	1,19
Aggressive life styling w/nominal bonds	-0,43	1,03	1,21
Prudent constant mix with nominal bonds	-2,58	0,44	0,55
Smoothed life styling w/nominal bonds	-1,30	0,93	1,14
Pure equities	-0,18	1,01	1,20
Pure nominal bonds	-3,51	-	0,05
Pure Money market	-2,49	0,11	-

Table 9: Backtests - alternative measures of performance

The backtest exercises confirm the main results derived from the pure simulation exercise. While pure equity strategies remain the most profitable for DC plan members, the backtests put in perspective their supremacy as aggressive options provide comparable returns, in terms of retirement years, for a slightly lower level of risk. We notice yet that historical data give additional support to the fact that fixed income solutions are not the safest solutions and do not succeed in offering a sufficient income to cover the longevity risks borne by DC plan participants. Keeping in mind the conclusion of Bodie and Trussard (2007) who insist on the "one size does not fit all" principle, the backward simulations suggest that the best default option solution for risk adverse investors is the prudent constant mix approach. On the other hand, the lifecycle solution could be an advisable default option in particular for pension participant whith a high risk tolerance.

At this stage, we only consider the traditional asset classes in the construction of the default option. In the next section, we extend this analysis by discussing the choice of the riskless asset. We substitute the long term bond asset by an inflation-linked bond asset. We also discuss the relevance of portfolio insurance tools and provide preliminary results.

5 The riskless asset: a key issue for pension investors

The determination of the riskless asset for the construction of investment options for DC plan members remains a key issue[8]. Long term nominal government bonds are usually considered as the riskless asset. However, this asset class does not hedge against the inflation risk which is one of the main concerns for long term investors. Some economists sustain the idea of a new inflation regime characterized by a higher and/or more volatile inflation rate. Besides, the recent development of the financial markets gives an additional support to this insight. Indeed, some argues that the current monetary policy (Quantitative Easing and low interest rates) may lead to higher inflation pressures. Otherwise, the relative scarcity of natural resources and to a lesser extent of the labor force should drive price indices up. Accounting for the fact that long term investors are in essence inflation risk averse, this context should lead them to increase the demand for protection against the inflation risk. One possible answer is to consider the inflation-linked bonds as the riskless asset in the structuring their portfolio.

5.1 A potential candidate: the inflation-linked bonds

Broadly speaking, an inflation-linked bond is a fixed income product which pays a fixed real interest rate over the entire bond lifetime and in which the principal is completely hedged against the price increase. The inflation-linked bond provides a perfect guarantee against the risk of purchasing power losses as compared to traditional long term fixed income products. While the economy is facing deflation, the investor is ensured to get back his initial capital. These inflation hedging products appeared for the first time at the beginning of the eighties in the United Kingdom (1981). Since then, all the developed countries have issued inflation-linked bonds. Several reasons led governments to issue inflation-linked bonds (Beletski, 2006; La Bruslerie, 2002):

- By issuing inflation-linked bonds, governments can reduce the cost of financing their debt and deficit;
- in addition, inflation-linked bonds (ILB) issuances facilitate the matching between government assets and liabilities as both tax incomes and government spending are highly correlated to the inflation dynamics;
- in a high inflationary environment, the issuance of ILB signals to the market the government willingness to incurve and stabilize inflation pressures.

[8] The financial downturn, started in 2007, has progressively affected the sovereign debt sector. Consequently, numerous European countries have been downgraded (Greece, Ireland, Portugal and Spain) from January 2009. Despite the deterioration of the situation, in particular, the probability of see European countries being in a default situation remains extremely weak or nil. These troubles cause erratic changes of long term interest rates. This implies management of the interest rate risks.

Turning to the investor side, several authors argue in favor of a greater use of the ILB in particular for long term ones such as the insurance companies or the pension funds. Considering the United Kingdom case study, Mamum and Visultanachoti (2005) observe that the use of inflation-linked bonds within a diversified portfolio enhance its performance. Garcia and Rixtel (2007) notice however that the inflation-linked bond market suffers from liquidity matters which can affect its valuation. They also notice that inflation-linked bond markets have gain ground rapidly over the last years even though the inflation dynamics in the developed economies have remained moderate. This trend can be explained by long term investor's, in particular the pension funds, appetite for this asset category.

Inflation-linked bonds were introduced in 1997 in the United States. Consequently, the inflation-linked bond index used in our study is only a proxy. To evaluate the ILB returns, we derive an ILB index by calculating a real interest rate and inflation expectations. To do so, we base our analysis on the survey of professional and forecasters provided by the Federal Reserve Bank of Philadelphia. This quarterly macroeconomic survey is carried out among economists who provide their forecasts for the main economic variables, for different horizons. In particular, the survey provides both one year and 10 year consumer price indices and GDP deflator forecasts (Figure 4). Available since 1990, the 10 year inflation expectations display a downward trend until the beginning of 2000 and remain stable around 2.5% until 2010. Considering the United States CPI index, we observe that long term inflation expectations can be proxied by a simple five year moving average (Figure 4). In other words, we can assert that inflation expectations are built considering an adaptive expectations framework. In light of this result, we are able to build our inflation-linked bond index and deduce a proxy of the ILB investment yield (this point requires additional research). This index and the ILB returns will be used to calibrate the Monte Carlo simulation and backtest the simulated default options. The results are presented in the coming section.

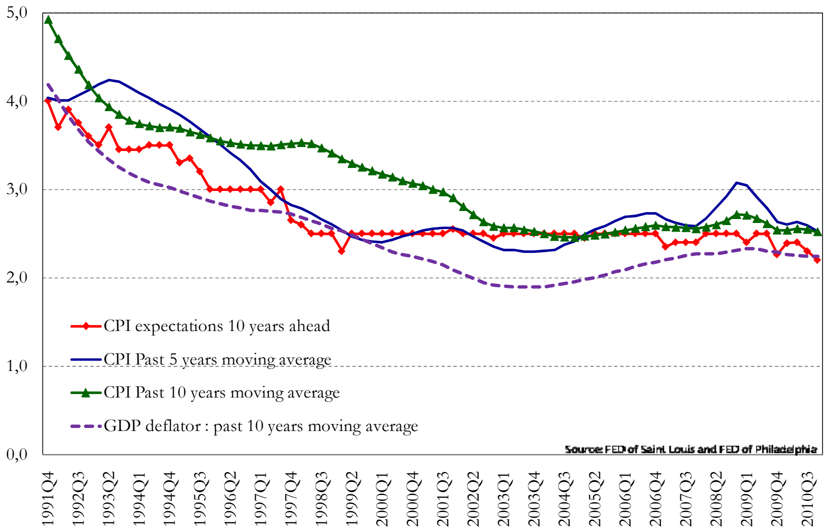

Figure 4: Inflation expectations at different forecasting horizons

Our aim is to perform the same simulation exercises replacing the nominal bond with an inflation-linked bond. First of all, we use Monte Carlo[9] simulations and the various indicators presented previously. Then, we test the results of the Monte Carlo simulations with a historical dataset.

5.2 Using an ILB index as the riskless asset

The following tables gather the Monte Carlo simulations results. In absolute terms, the performance of the default options structured with inflation-linked bonds is on average higher than those using nominal bonds. Although the real rate of return of the inflation-linked bonds provides a lower coupon than the traditional fixed income one, the nominal is always hedged against price increases[10]. In addition, the lower volatility of inflation-linked bonds tends to diminish the bad outcome occurrences, improving thus the performance of the solutions using this asset (Table 11). The combination of these two properties drives up the average long term performance of the default solutions structured with these inflation hedging products. In comparison to the previous simulation exercises, it can observed that prudent investment strategies derive a higher benefit from the inflation-linked assets due to the weight of safer assets within the portfolio.

	Mean	*Min*	*Max*	*Quartile 1*	*Quartile 3*	*Standard deviation*
Naive divesification with inflation link bonds	9,34	2,64	22,48	7,22	11,29	2,91
Pure Inflation linked bonds	8,84	3,43	15,89	7,32	10,46	2,25
Aggressive constant mix with inflation link bonds	12,48	2,28	55,53	8,33	15,59	5,69
Prudent constant mix with inflation link bonds	10,73	3,05	28,72	8,21	12,99	3,47
Aggressive life styling w/Inflation linked bonds	13,63	1,64	144,90	8,41	17,28	7,45
Smoothed life styling w/Inflation linked bonds	11,66	1,64	70,99	8,84	14,13	4,29
Prudent life styling w/Inflation linked bonds	9,84	1,64	29,08	8,58	11,35	2,34
Autopilot life styling w/Inflation linked bonds	11,97	1,94	65,58	8,64	14,74	4,74

Table 10: Simulated default options with inflation-linked bonds

[9] The ILB index return is simulated using the following relationship

$$\frac{dB_t^{ILB}}{B_t^{ILB}} = \mu_{ILB}dt + \rho dW_t^r + \sqrt{\left(1-\rho^2\right)}\sigma_B dW_t^{ILB} \ .$$

In line with the previous simulation exercise, we calibrate the rate of return of an ILB index using rolling periods between 1850 and 2005. The average rate of return and the average standard deviation reach respectively 1,9% and 4,8%. These figures are used to simulate ILB returns.

[10] While durable deflation exists (Japan for instance), it remains exceptional in particular over several decades. However, we have to keep in mind that our calibration takes into account high inflation periods where inflation-linked products would have outperformed the basic fixed income products. For instance, the recent surge of the inflation rate in 2008, caused by higher energy prices, made the indexed bond far more profitable than the nominal bond.

	Standard deviation	*Semi variance*	*Risk shortfall*
Aggressive constant mix with inflation link bonds	5,692	0,128	0,358
Prudent constant mix with inflation link bonds	3,468	0,062	0,250
Autopilot life styling w/Inflation linked bonds	4,739	0,071	0,266
Aggressive life styling w/Inflation linked bonds	7,449	0,156	0,395
Prudent life styling w/Inflation linked bonds	2,343	0,062	0,250
Smoothed life styling w/Inflation linked bonds	4,288	0,080	0,283
Pure Inflation linked bonds	2,250	0,026	0,160
Naive divesification with inflation link bonds	2,908	0,071	0,266

Table 11: Risk indicators for simulated default options with inflation-linked bonds

Turning to the stochastic dominance criterion, we plot the cumulative distributions of the investment strategies outcomes (see **Erreur ! Source du renvoi introuvable.** in Appendix B). According to this criterion, there is no strategy which stochastically dominates, at the first order, the other investment vehicles. This result challenges the superiority of the 100% equity investment. Consequently, we use the second order stochastic dominance to conclude. As expected, it appears that investment strategies based on a heavy exposure to equities stochastically dominate the others, at the second order. Nevertheless, this criterion ranks second the investment strategies based on inflation-linked bonds. The pure fixed income investment solution brings up the rear.

We perform the same exercise testing the relevance of this kind of asset class with a historical dataset. The results, reported in Table 12 and Table 13, confirm the findings derived from the Monte Carlo simulations. Comparing the outcome from the two backward simulations, the aggressive lifecycle solution generates a outcome comparable to the pure equities investment strategy. Meanwhile, the standard deviation and the different risk indicator are lower than those recorded for the 100% equity investment. In line with the inflation-linked bonds properties, the backtest also reveals that the safest default option consists in investing the contribution stream in a prudent lifecycle investment strategy.

	Mean	*Min*	*Max*	*Quartile 1*	*Quartile 3*	*Standard deviation*
Naive diversification with inflation link bonds	10,87	4,26	17,85	7,94	13,18	3,61
Pure Inflation linked bonds	9,14	4,49	14,90	6,88	10,87	2,72
Aggressive constant mix with inflation link bonds	15,00	4,41	28,48	9,52	19,95	6,40
Prudent constant mix with inflation link bonds	12,63	5,32	19,14	8,72	16,52	4,07
Aggressive life styling w/Inflation linked bonds	15,97	3,06	33,23	9,46	22,32	7,85
Smoothed life styling w/Inflation linked bonds	13,73	3,06	22,00	10,42	17,53	4,68
Prudent life styling w/Inflation linked bonds	10,89	3,06	15,03	9,11	12,79	2,58
Autopilot life styling w/Inflation linked bonds	14,30	3,76	23,74	10,24	18,35	5,30

Table 12: Backtested investment vehicles with inflation-linked bonds

Investment vehicles	*Standard deviation*	*Semi variance*	*Downside risk*
Aggressive constant mix with inflation link bonds	6,40	3,55	1,88
Prudent constant mix with inflation link bonds	4,07	1,82	1,35
Autopilot life styling w/Inflation linked bonds	5,30	2,50	1,58
Aggressive life styling w/Inflation linked bonds	7,85	4,52	2,13
Prudent life styling w/Inflation linked bonds	2,58	1,27	1,13
Smoothed life styling w/Inflation linked bonds	4,68	2,07	1,44
Pure Inflation linked bonds	2,72	1,67	1,29
Naive diversification with inflation link bonds	3,61	2,08	1,44

Table 13: Backtested investment vehicles – alternative risks measures

5.3 Enhancing the discussion: the portfolio insurance strategies as a buffer

The default options offered to participants are frequently heavily exposed to the stock market. For instance, the default solutions offered within the Australian superannuation funds are considered conservative when the fraction of stock is lower than 70%, moderately aggressive when this proportion is between 70% and 80%, and aggressive when the weight of equities is greater than 80% (Basu, 2009). While investment solutions based on an important weight of stocks within the portfolio provide the highest outcome, the risk of substantial wealth losses are not nil. As explained by Antolin et al. (2009), DC pension members may record irreversible losses due to financial crashes that may occur at the end of the accumulation period. To limit the bankruptcy risk, capital guarantee solutions based on the portfolio insurance framework can be proposed to non risk adverse investors. The portfolio insurance product purpose is twofold: protect the accumulated wealth and benefit from a bull market environment. Three different portfolio issuance strategies are widely used: the stop loss, the Constant Proportion Portfolio Insurance (CPPI) and the Option Based Portfolio Insurance (OBPI). Due to severe constraints in terms of time horizon, we will only deal with the two first cases[11]. The stop loss strategy considers both risky and cash assets in which the wealth is invested. We define C_t as the portfolio value invested in cash assets and A_t a portfolio value invested into risky assets. Stop loss strategies consists in the following binary rule: if $C_t > A_t$, then the portfolio is entirely invested in the riskless assets. Otherwise, the whole portfolio is invested in risky assets (see **Erreur ! Source du renvoi introuvable.** and **Erreur ! Source du renvoi introuvable.** in Appendix C for an example). The investment return is then given by:

$$W_t = Max\left(C_t, A_t\right).$$

We evaluate the stop loss strategies using our long term historical dataset. The results, given in the following table, are in line with our expectations as this investment solution succeeds in reducing drastically the risk of holding equities. The risk indicators relative to the stop loss option appear very low and are comparable to the most conservative strategies. Consequently, the outcome generated by this

[11] The OBPI implementation would require long maturity options raising thus issues relative to the liquidity constraints. Otherwise, this OBPI requires rolling strategies which pose in turn the question of the cost of the strategy.

strategy is inferior to constant mix and lifecycle solutions. The introduction of lifecycle bonds leads to a similar diagnosis.

	Mean	*Min*	*Max*	*Quart 1*	*Quart 3*	*Standard deviation*	*Semi variance*	*Downside risk*
Stop loss	9,40	3,57	17,57	6,29	12,28	3,79	1,89	1,38
Stop loss inflation link bond	11,37	6,71	16,12	10,03	13,04	2,25	1,38	1,16

Table 14: Stop loss solution performance

Despite a significant reduction of the risk, this option suffers from important drawbacks. The cost of the investment strategy is substantial in particular when the financial markets become volatile. The asset manager is thus constraint to rebalance more frequently the investment portfolio reducing even more the performance, due to transaction costs. Besides, the stop loss strategies are "trend dependent" which translates the passive character of this solution. In other words, it does not allow the asset manager to benefit from investment opportunities. This last issue has been addressed by the CPPI.

Introduced by Perold and Sharpe (1988), the CPPI strategy allows the investor to allocate his wealth dynamically over the investment horizon taking risk aversion and the remaining time horizon into account. Basically, the construction of a CPPI strategy requires the following steps (see Bertrand, 2004 for additional details):

1. The investor defines a "floor equivalent to the lowest acceptable value of the portfolio".
2. The investors defines the buffer which is equal to the excess of the portfolio value over the floor.
3. The risky asset allocation is then equal to the buffer times "the multiplier" which describes the risk aversion of the investor. The lower the multiplier, the more risk adverse the investor is.
4. The remaining wealth is invested in cash or other liquid short term assets.

The main characteristics of CPPI strategies are that it offers a capital guarantee while enabling the investor to benefit from the increase of the stock market. In other terms, the portfolio value under a CPPI strategy is given by the following expression:

$$V_{t,t-1} = \overline{m}P_{t-1} \times \frac{S_t}{S_{t-1}} + F_{t-1}e^{r(t).\Delta t},$$

$$V_{t+1,t} = E_{t,t-1} \times \Delta S_t + \left(V_t - \overline{m}C_{t-1}\right)e^{r(t).\Delta t},$$

where V_t is the value of the CPPI investment, $\overline{m}$ is the multiplier, P_t is the cushion, S_t the risky asset price index, F_t is the floor exclusively invested into cash or riskless asset and $r(t)$ is rate of return of the cash investment.

But several drawbacks can be identified. As for stop loss strategy, the CPPI is a trend dependent strategy meaning that investors buy the risky asset when it raises and sell it when it decreases. The CPPI option is not relevant for high volatility markets. The main risks associated with the CPPI strategy are a complete meltdown of the cushion and a burst of the floor which would lead the whole portfolio to be converted into cash assets.

6 Conclusion

The financial market downturn has provoked substantial losses for pension participants highly exposed to stock markets. The financial crash does not challenge the role of pension funds as governments are progressively leaving the retirement funding. Taking into account the recent development of DC plans, this study addresses the default options issue, proposed to DC plan participants. In addition, we raise the question of the riskless asset choice considering the inflation-linked bonds as the safer asset. Using Monte Carlo simulations, we show that lifecycle solutions can act as the default option. The aggressive lifestyling investment is particularly attractive to risk tolerant investors compared to aggressive constant mix or pure equity solutions. For risk adverse investors, autopilot or smoothed lifecycle are better suited. Nevertheless, naive portfolios or fixed income strategies cannot be proposed as a default option. In a second stage, we test this default option using a long term historical dataset and find. Otherwise, we notice that the use of an inflation-linked bond within investment solutions improve significantly the performance and reduce the risk. This outcome is explained by the intrinsic properties of the inflation-linked bonds, in particular their capacity to perfectly hedge against the inflation risk. We introduce portfolio insurance elements to appreciate the opportunity of such strategies. The preliminary tests do not provide satisfying results, but further investigations are needed to conclude.

This study can be extended in different ways. We assumed a constant contribution rate paid by the DC plan members. In this respect, this assumption is quite restrictive as we can assert that the saving contributions are linked to the age of the investor. Consequently, the contribution rate can be modeled as a function (linear or exponential) of the age. Besides, as financial market data are generated through a simulation tool, earnings data can be generated in the same way. Contributions paid to feed the investment vehicle could therefore become randomly distributed. The indicators used to evaluate the investment solutions can be improved. In particular, we need to discriminate between financial crashes and slight decreases of financial market prices. We could then recalculate the Roy and the Sharpe ratios. Besides, by introducing additional assumptions, we can use a utility function within an intertemporal framework to evaluate the surplus associated to each investment solution. To a greater extent, this framework allows us to evaluate ruin probabilities. Concerning backtest exercises, the stability has to be checked performing the backward evaluations over shorter horizons. We are currently testing the stability of each default option.

Regarding the investment opportunities, additional asset categories have to be considered. In particular, real estate portfolios have gained ground in the investment panel offered by asset managers. Except for homeowners, this asset category can be included in the construction of lifecycle solutions. In the same vein, can hedge funds be considered in the retirement funding issues? Turning to the financial tool, an additional attention could be paid to the portfolio insurance strategies. Stop loss and CPPI strategies can be extended by introducing upper and lower thresholds. This would reduce the sensitivity of these solutions to market volatility. Otherwise, CPPI strategies can be enhanced as well, by linking the cushions to market volatility.

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Appendix A – Data calibration

Median usual weekly earnings* full employed workers for both sex - in current dollars

	16 to 19	20 to 24	25 to 34	35 to 44	45 to 54	55 to 64	65 and more
2000	297	383	549	625	669	620	463
2001	305	394	576	657	693	638	488
2002	305	399	591	668	706	674	502
2003	311	402	594	687	723	708	516
2004	309	406	604	713	743	725	560
2005	318	411	610	731	748	742	569
2006	324	423	621	748	773	765	583
2007	337	450	643	769	790	803	605
2008	349	467	666	804	822	825	644

*(second quartile) United States Departement of Labor - Bureau of Labor Statistics

Table 15: Saving and cumulated contributions to the DC plans (sources: BLS and author calculations)

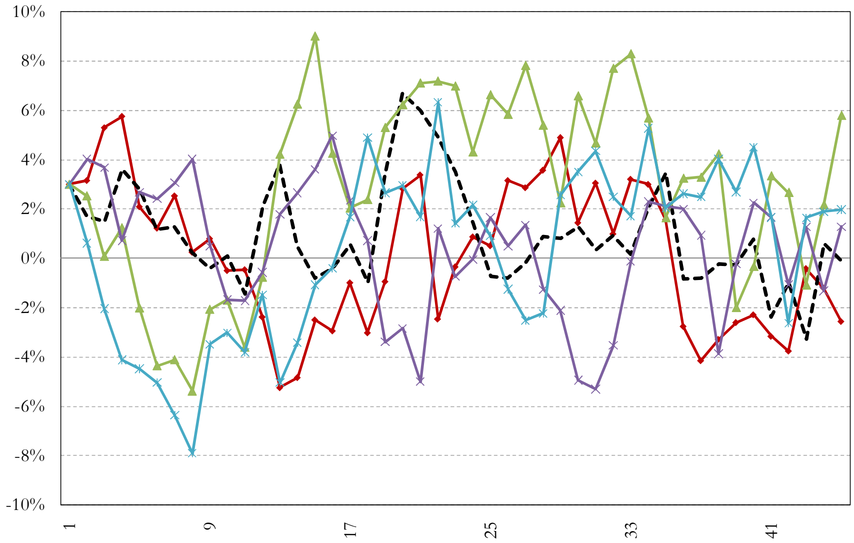

Figure 5: Simulated paces for short term interest rates

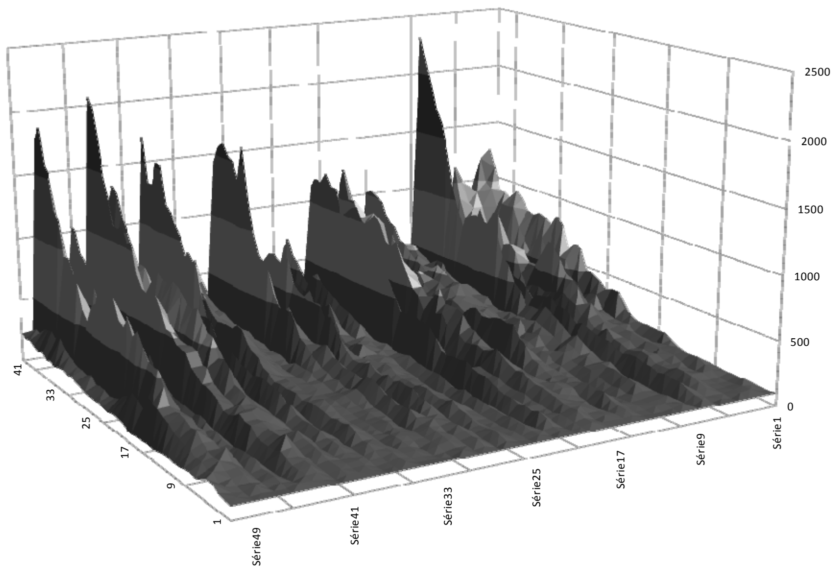

Figure 6: Simulated stock prices paces

Appendix B – Stochastic dominance charts

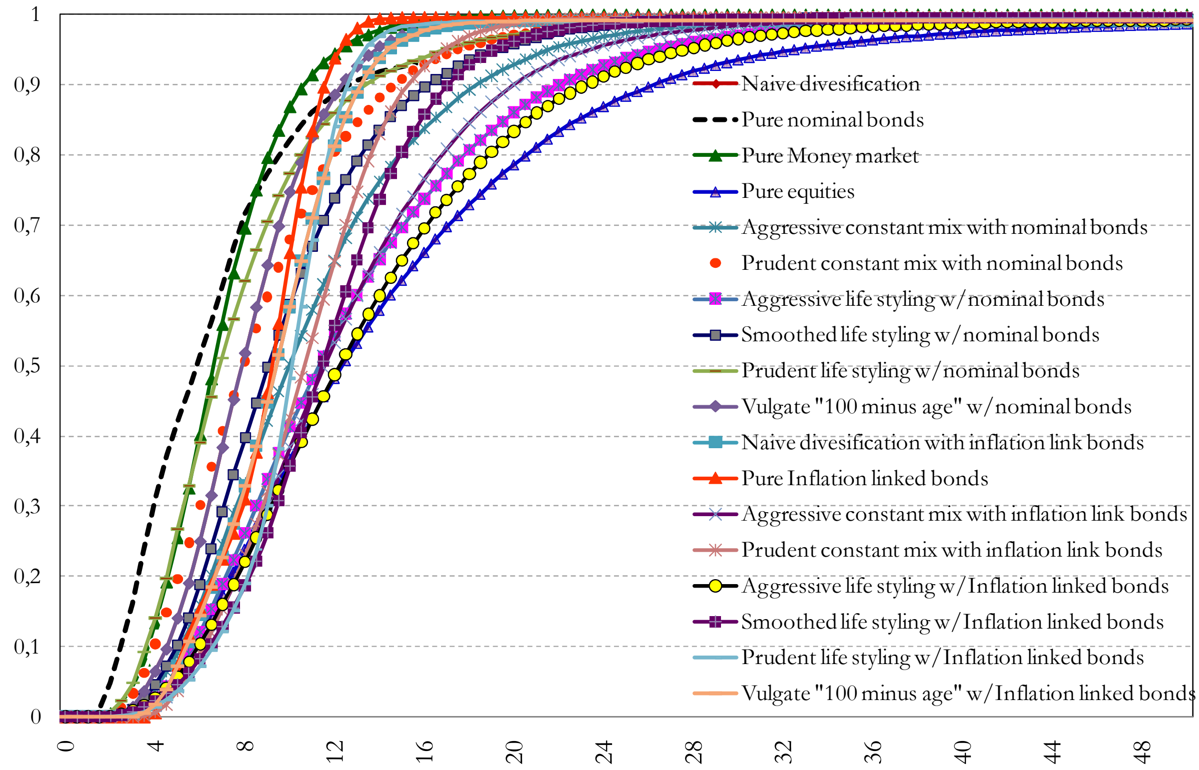

Figure 7: Monte Carlo investment vehicles – Cumulative distribution function

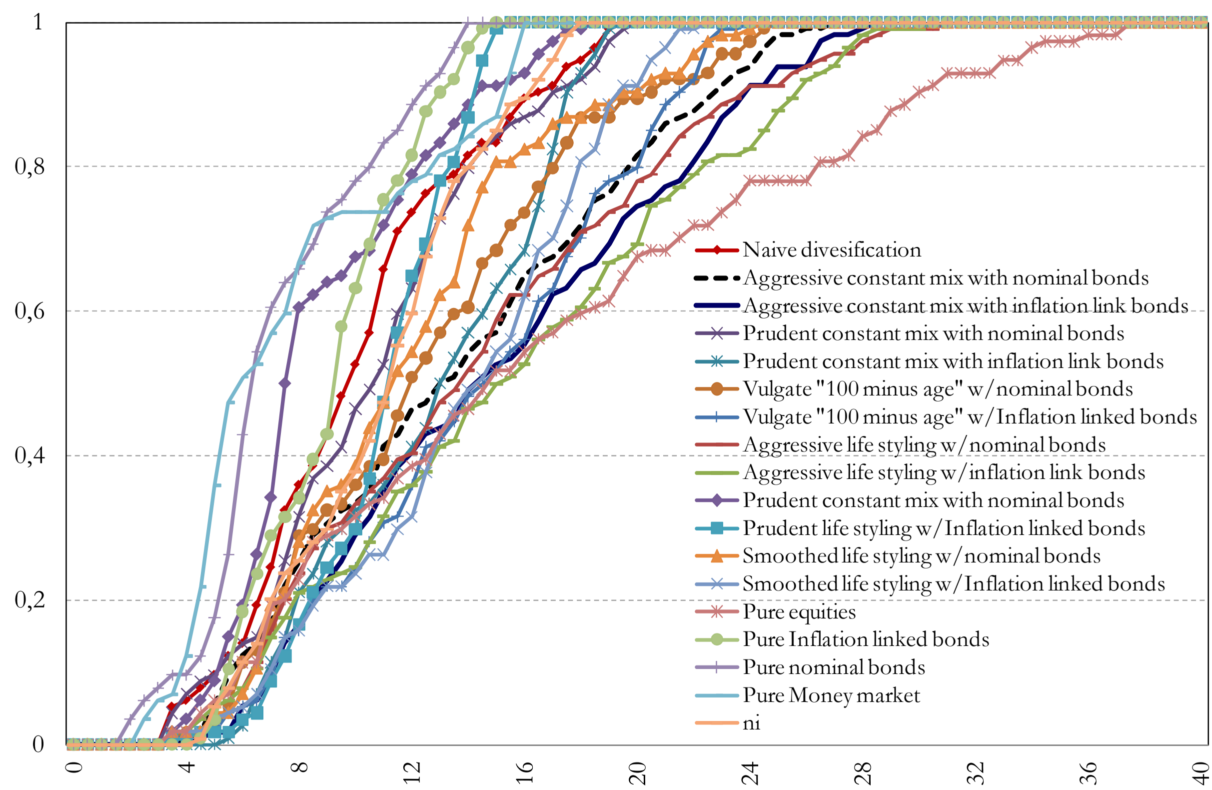

Figure 8: Backtested investment vehicles – cumulative distribution function

Appendix C – Additional results

	Roy ratio (MAR = life expectancy at 65)	Roy ratio (MAR = Bond portfolio asset)	Sharpe Ratio
Aggressive constant mix with inflation link bonds	-1,0056	0,9736	0,3009
Prudent constant mix with inflation link bonds	-2,1543	1,0944	0,4165
Vulgate "100 minus age" w/Inflation linked bonds	-3,0478	0,8264	0,5697
Aggressive life styling w/Inflation linked bonds	-1,4289	0,8995	0,0484
Prudent life styling w/Inflation linked bonds	-4,5474	1,2398	0,1540
Smoothed life styling w/Inflation linked bonds	-1,5252	1,1024	0,3761
Pure Inflation linked bonds	-4,1619	0,8463	0,4470
Naive divesification with inflation link bonds	-1,8702	0,5071	0,2340

Table 16: Alternative performance measures for simulated default options with inflation link bonds

	Value at risk			
	10%	5,0%	2,5%	1,0%
Aggressive constant mix with inflation link bonds	7,29	9,42	10,99	13,26
Prudent constant mix with inflation link bonds	4,44	5,74	6,69	8,08
Vulgate "100 minus age" w/Inflation linked bonds	6,07	7,85	9,15	11,04
Aggressive life styling w/Inflation linked bonds	9,54	12,33	14,38	17,36
Prudent life styling w/Inflation linked bonds	3,00	3,88	4,52	5,46
Smoothed life styling w/Inflation linked bonds	5,49	7,10	8,28	9,99
Pure Inflation linked bonds	2,88	3,72	4,34	5,24
Naive divesification with inflation link bonds	3,72	4,81	5,61	6,78

Table 17: Value at risk measures for simulated default options with inflation link bonds

	Roy ratio (MAR = life expectancy at 65)	Roy ratio (MAR = Bond portfolio asset)	Sharpe Ratio
Aggressive constant mix with inflation link bonds	-0,50	1,21	1,55
Prudent constant mix with inflation link bonds	-1,37	1,32	1,64
Vulgate "100 minus age" w/Inflation linked bonds	-0,74	1,33	1,60
Aggressive life styling w/inflation link bonds	-0,28	1,11	1,29
Prudent life styling w/Inflation linked bonds	-2,83	1,40	1,69
Smoothed life styling w/Inflation linked bonds	-0,96	1,38	1,57
Pure Inflation linked bonds	-3,33	0,69	0,89
Naive divesification with inflation link bonds	-2,03	1,00	1,19

Table 18: Alternative performances measures using inflation link bonds (from the backtests)

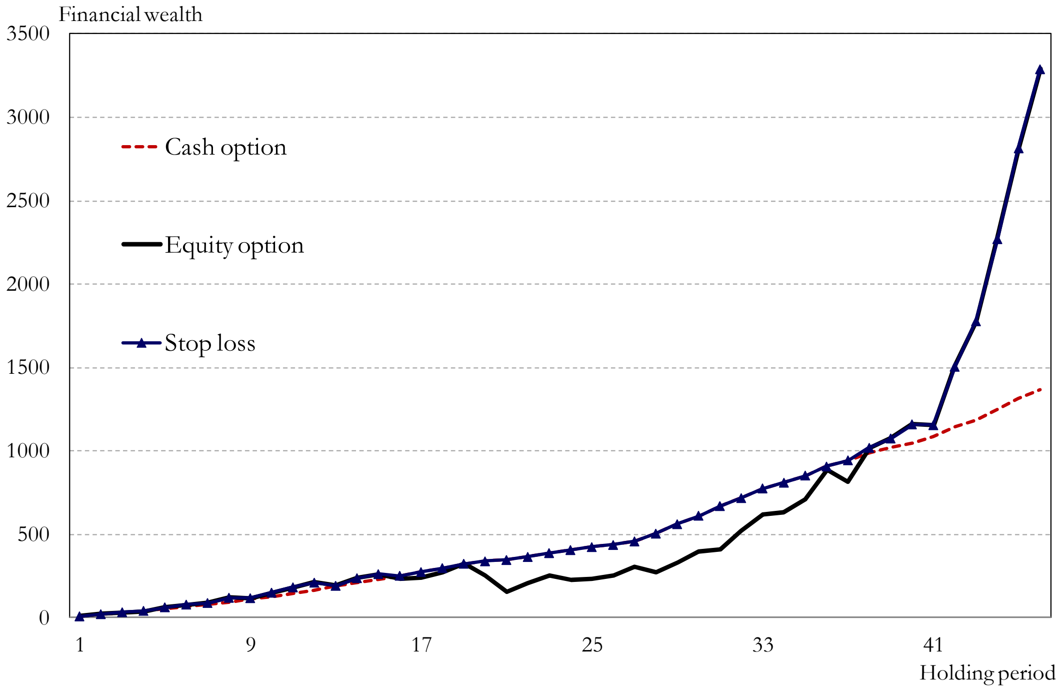

Figure 9: Stop loss strategy example extracted from the historical database

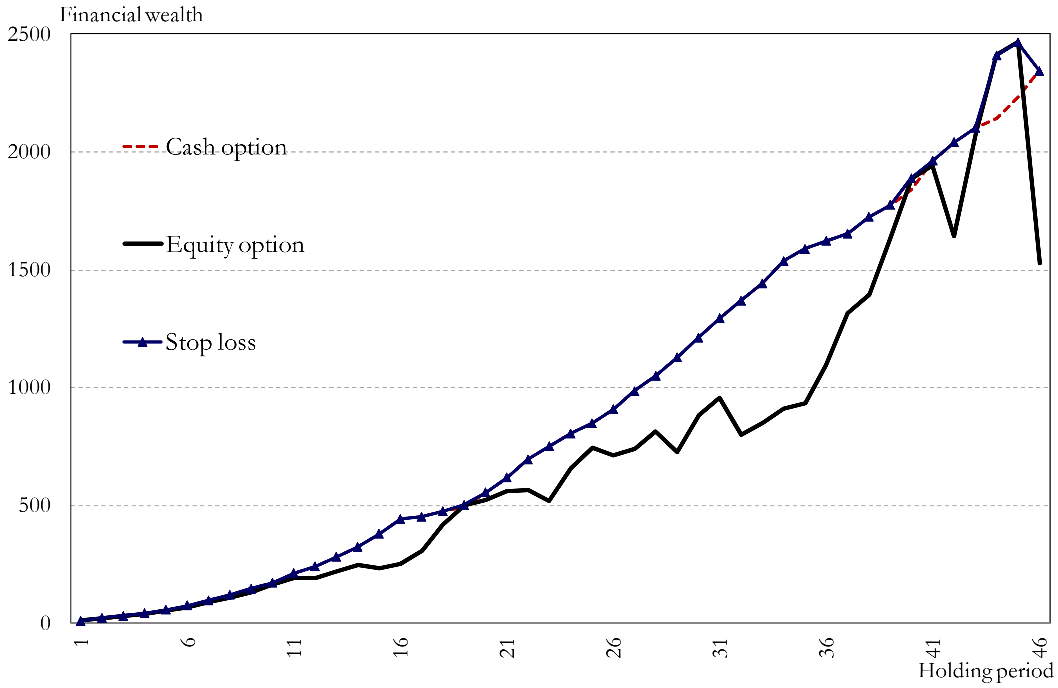

Figure 10: Stop loss strategy example extracted from the historical database

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