Valuing Life at Gunpoint, or as a Statistic, or both*

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Abstract

We calculate, and estimate closed-form expressions for the value of life *at gunpoint*, i.e. the maximal amount an individual would be willing to pay to avert certain death, as well as for alternative life valuation (Value of Statistical Life, VSL). Our estimates show that marginal valuation of life diminishes rapidly in incremental death risk. Consequently, linear extrapolation under VSL overestimates how much to pay to avert one's own death, and confirm that the VSL is best interpreted as a collective willingness to pay to avert an infinitesimal risk increase, and not as a value attributed to one's own life.

Keywords: Values of Human Life, Value of Statistical Life, Willingness to pay, Equivalent Variation, Endogenous Morbidity, Endogenous Mortality, Structural Estimation.

JEL Classification: J17, D91, G11.

1 Introduction

1.1 Motivation and outline

This paper asks a question most people would rather avoid having to answer: How much would you value your own life or, equivalently, what is the maximum you would be willing to pay in order to survive in a credible "your money or your life" threat? We refer to the corresponding amount as the *Gunpoint Value of Life* (GPV).

One of course is rarely caught up in a genuine money-or-life situation, and those who were are often unable or unwilling to share their experience. We therefore employ a revealed- rather than stated-preference perspective. As the maximal willingness to pay (WTP) to ward off an unfavorable economic change, the Hicksian Equivalent Variation (EV, Hicks, 1946) provides a natural theoretical framework to elicit our life vs death valuation.

Towards that aim, we rely on a dynamic model of health- and financial-related decisions that we developed elsewhere (Hugonnier et al., 2013). This life cycle framework focuses on endogenously-determined health status affecting labor income as well as exposure to sickness and death risks, in a setting where agents have strict preference for life over death. Importantly, it yields closed-form expressions for the choice variables (health spending, and insurance, consumption, and portfolio), and therefore for the indirect utility from which the Hicksian EV can be computed. We offer three theoretical contributions. We first calculate the willingness to pay to avert any exogenous increase in the probability of dying. The second contribution is the closed-form Gunpoint Value, i.e. the EV that leaves the agent indifferent between remaining alive and *certain* death. Third, we recover a theoretical expression for the Value of a Statistical Life (VSL), a widely-used life valuation alternative. Finally, we conduct a structural estimation of the model by resorting to PSID data counterparts to the optimal choice variables. Relying on these deep parameter estimates, our fourth contribution is thus empirical, and consists of closed-form estimates of the WTP, the GPV, and the VSL.

Our main theoretical findings are as follows. First, both welfare, as well as the optimal choice variables are functions of net total wealth, i.e. the sum of financial wealth plus the economic value of the health capital accumulated by the agent. As for Human Capital (HK) models, the latter is the net present value of the health-dependent labor income.

Unlike HK models however, that value is adjusted downwards for morbidity risk exposure, and for minimal subsistence requirements. Second, exposure to exogenous death risk affects welfare through the marginal propensity to consume (MPC) only. Its effect on the latter crucially depends on the elasticity of inter-temporal substitution (EIS). At high (low) EIS, the agent responds to a shorter horizon by increasing (decreasing) consumption, and the MPC is unaffected at unit EIS.

Third the willingness to pay to avoid an increase in the exogenous death risk exposure is a weighted average of net total wealth, and a mortality risk adjustment term capturing endogenous risk exposure and mortality risk aversion, and whose sign depends on the EIS. As exogenous death risk exposure increases, the mortality adjustment term loses relevance, and the WTP converges to the morbidity-adjusted net total wealth.

Fourth the Gunpoint Value is also equal to the latter, and can thus be interpreted as the limiting WTP as death becomes certain. Assuming perfect financial markets, and barring any bequest motive, the agent pledges total financial wealth, plus the capitalized value of any future income streams, net of minimal survival consumption levels in order to survive. In particular, if we abstract from organ donation, health is non-transferable, and fully depreciated by death. Agents would therefore compute and be willing to forego the shadow value of the human capital that they possess in order to survive. Importantly, this valuation encompasses the value of the fundamental services procured by health: the capacity to work, to produce future health, its durability, as well as the ability to ward off sickness risks. Because death is certain in a Gunpoint valuation, elements such as the endogenous ability to alter exposure, or death risk aversion, or even *how* death obtains are irrelevant in computing the maximal willingness to pay to save oneself.

Fifth, the Value of a Statistical Life can be computed in closed-form as the (negative) of the marginal rate of substitution between exogenous death risk exposure and wealth or, equivalently, as the slope of a tangent of the WTP calculated at base exposure. We show it is an increasing function of net total wealth, and of a morbidity adjustment term whose sign also depends on the EIS. Similarly, the effect of death risk exposure on the VSL crucially depends on the elasticity of inter-temporal substitution and cannot be established ex-ante.

Our main empirical results are the following. First our point estimates are realistic, and indicate that the EIS is larger than one, i.e. the marginal propensity to consume is increased in response to higher death risk. Second, the estimated WTP in an increasing, and strongly concave function of the increment in the exogenous risk of dying. The limiting value for the WTP – i.e. when death becomes certain – corresponds to the GPV. Given low observed financial wealth, our calculations reveal that the Gunpoint Value of Life mainly encompasses human capital, and is 360K $ for an individual in Good health, and 3^{rd} wealth quintile, and ranges between 88 K$ (Poor health, and 1^{st} quintile) to more than 729 K$ (Excellent health, and 5^{th} quintile). Our human capital estimates are somewhat lower than alternative HK estimates due to the morbidity, and subsistence adjustments. Third, as for the GPV, the Value of a Statistical Life is an increasing function of health and wealth. The estimated VSL is much higher than the GPV, corresponding to 7.88 MM$ (Good health, 3^{rd} quintile), and ranging between 2.06 MM$ (Poor health, 1^{st} quintile) and 15.82 MM$ (Excellent health, 5^{th} quintile), values that are well in line with reduced-form estimates from the empirical VSL literature.

Two main reasons for these wide differences between Gunpoint, and Statistical Life values can be invoked. First, the WTP was found to be very concave in exogenous death risk. Consequently, its limiting value is much lower than a slope of a tangent evaluated at the origin. Put differently, the diminishing marginal utility of life implies that linear extrapolation under the VSL sharply overstates the limiting valuation corresponding to the GPV. Second, and more fundamentally, these discrepancies highlight the (well-recognized) caveat that VSL is best interpreted as a collective value that society is willing to pay to save one unidentifed life, rather than what a single person would be willing to pay to save his own. Whereas linear aggregation of small WTP's to avert infinitesimal increases in death risks, and equally affecting large populations will lead to large VSL, the GPV is ultimately limited by the individual capacity to pay, i.e. the value of all assets, be them human or financial. That private capacity is finite, and much lower than its collective counterpart.

Given that they yield two such different answers to the common question of how much is worth a human life, which of the GPV or the VSL should be used? We argue that both are relevant, although in different settings, i.e. the GPV and VSL are complements, rather than substitutes. The VSL is more appropriate than the GPV when gauging a collective WTP for changes in death risks affecting large subsets of the population (e.g. in public safety decisions). The GPV should be relied upon for gauging a value one would ascribe on his own life (e.g. the continuation of life support measures, wrongful death litigation, or life insurance).

The rest of the paper is organized as follows. After presenting the related literature in Section 1.2, we reproduce the key elements of the Hugonnier et al. (2013) model in Section 2 for completeness. The main theoretical contributions regarding the WTP, the GPV, and VSL are outlined in Section 3. The empirical strategy is discussed in Section 4, with deep parameters, and values of life estimates being presented in Section 5. A conclusion in Section 6 reviews the main findings and suggests areas for future extensions.

1.2 Related literature

Evaluating the price of a human life has long been at the forefront of economic research.[1] The main methodologies may be classified as Human Capital, and the Value of a Statistical Life. The HK models evaluate the human capital embodied in the expected discounted net value of the lifetime labor income flows, and that are foregone upon death.[2] Well-known issues related to this approach include the treatment of non-labor activities, the appropriate rate of discounting, and the endogeneity of survival probabilities.[3]

As for HK models, we do calculate the net present value of income streams that are lost upon death. Unlike HK models however, that value is computed in closed-form, i.e. accounting for all potential endogeneities linked to the income stream and/or the rate of discounting. Furthermore, whereas our modeling strategy does allow for labor income flows, this hypothesis is not restrictive for two reasons. First, by assuming that labor income is health-dependent, the reduced capacity to work for unhealthy individuals is implicitly take into account. Since health is an adjustable variable, any endogeneity of labor income is thus implicitly taken into account. Second, Hugonnier et al. (2013) show that the base model with health-dependent labor income and health-independent preferences can be rewritten as an equivalent one with health-independent income, and direct preference for health. Put differently, our model choice is equivalent to one with only exogenous income (which could be zero), and where agents directly value better health in the instantaneous utility function. Finally, the rate of discounting is also

[1]For example, Landefeld and Seskin (1982) reference human-capital based evaluations of the value of life dating back to Petty (1691).

[2]See Jena et al. (2009) for partial- and general-equilibrium HK life values.

[3]Conley (1976) provides additional discussion of HK approaches while Huggett and Kaplan (2016) address the discounting issues.

internally determined, and explicitly incorporates the endogeneity of exposure to death risk. Put differently, agents fully internalize their adjustable longevity in discounting any revenue flows.

The VSL alternative relies instead on explicit and implicit evaluations of the Hicksian WTP for a small reduction in fatality risk which is then linearly extrapolated to obtain the value of life.[4] Explicit VSL uses stated preferences for mortality risk reductions obtained through surveys or lab experiments, whereas implicit VSL employs a revealed preference perspective in using decisions and outcomes involving fatality risks to indirectly elicit the Hicksian compensation.[5] Examples of the latter include responses to prices and fines in the use of life-saving elements such as medical treatment, smoke detectors, speed limitations or seatbelts regulations. Implicit VSL research also exploits the fatality risk and wages nexus in labor markets to identify the death-income tradeoff. In particular, the Hedonic Wage (HW) variant of VSL evaluates the equilibrium willingness to accept (WTA) compensation in wages for given increases in work dangerousness. Controlling for job/workers characteristics, the wage elasticity with respect to job fatality risk can easily be estimated, and again extrapolated linearly to obtain the VSL (e.g. Aldy and Viscusi, 2008).

Ashenfelter (2006) provides a critical assessment of the VSL's theoretical and empirical underpinnings. First, the assumed exogeneity of the change in fatality risk can be problematic. For instance, safer roads will likely result in faster driving, which will in turn increase the number of fatalities. Second, agency problems might arise and lead to overvaluation in cost-benefit analysis when the costs of safety measures are borne by groups other than those who benefit (see also Sunstein, 2013; Hammitt and Treich, 2007, for agency issues). Third, and related, whose preferences are involved in the risk/income tradeoff and how well these arbitrage are understood often remains an open question. For example, high fatality risk employment may attract workers with low risk aversion and/or high time discount rates; generalizing the wages risk gradient to the entire population

[4]A canonical example (e.g. Aldy and Viscusi, 2007) has each agent $i = 1, 2, \ldots, N$ individually willing to pay $v^i(N^{-1})$ for a N^{-1} permanent reduction in fatality risk. Assuming identical, linear preferences, the value of a statistical life is obtained as $\sum_{i=1}^{N} v^i(N^{-1}) = Nv(N^{-1})$, i.e. the collective willingness to pay to save one individual. Since that person cannot be identified *ex-ante*, the WTP thus obtained corresponds to a *statistical*, rather than person-specific value of life.

[5]A special issue directed by Viscusi (2010) reviews recent findings on VSL heterogeneity. A meta analysis of the revealed-preference VSL is presented in Bellavance et al. (2009). See also Doucouliagos et al. (2014) for a *meta*-meta analysis of the stated- and revealed-preferences valuations of life.

could understate true valuation of life. Moreover, because wages are an equilibrium object in the HW variant of the VSL, they encompass both labor demand and supply considerations with respect to mortality risk. Hence, a high death risk gradient in wages could reflect high employer aversion to the public image costs of employee deaths, as much as a high aversion of workers to their own death. Finally, as was the case for HK measures, wages-based estimates relate primarily to workers, and are hardly adaptable to other non-employed groups, such as young, elders, or the unemployed.

Our approach offers several advantages over standard HK and VSL alternatives to calculate the value of life. First, by emphasizing the destruction of the human capital in the willingness to pay to avoid certain death, we bridge a gap between HK and VSL literature. Unlike HK however, we do not uniquely ascribe the service flows of human capital to labor revenues, but explicitly calculate other self-insurance services provided by health. Second, and related, any endogeneity of morbidity and mortality risk exposition is fully accounted for in the model. Indeed, we explicitly ascribe an increase in fatality risk to the exogenous component in death intensity; the optimal WTP fully accounts for possible adjustments to such exogenous increases in death risk via the endogenous elements to mortality risk.

Third, by focusing on a credible money-or-life threat, the question of whose risks preferences are involved is not an issue in our setup. Furthermore, we rely on a representative panel (PSID) accounting for wide ranging consumption and health-related decisions to elicit the WTP measure. Unlike the HW variant of the VSL, our GPV approach neither involves equilibrium objects such as wages, nor does it apply uniquely to workers to elicit the WTP. This also means that the GPV reflect the value of life to a representative subset of those who are primarily concerned, i.e. the holders of the life capital. Finally, unlike VSL, our Gunpoint Value makes no assumption on how the marginal values corresponding to increases in fatality risk may be extrapolated to compute the value of life. Rather, our closed-form expressions allows us to compute the theoretical value agents would pay in a stand-and-deliver situation, and contrast it with VSL. Indeed, we show that, consistent with economic intuition, the marginal value ascribed to small increases in death intensity is positive, but falling in the latter. The direct implication is that linear extrapolation favored by VSL sharply overestimates the value of one's own life.

2 Model

We base our valuation on the theoretical model of Hugonnier et al. (2013). For completeness, we briefly reproduce its key elements here, and then discuss the corresponding indirect utility that is relied upon to compute the Hicksian EV.

2.1 The agent's problem

Hugonnier et al. (2013) consider the problem of an agent whose publicly observable health capital H follows:

$$\mathrm{d}H_t = \left[I_t^{\alpha} H_{t-}^{1-\alpha} - \delta H_{t-}\right] \mathrm{d}t - \phi H_{t-} \mathrm{d}Q_{st}, \quad H_0 > 0, \tag{1}$$

where investment $I \geq 0$ captures health expenditures, the deterministic depreciation occurs at rate δ whereas sickness is stochastic and entails additional depreciation at rate ϕ, upon occurrence of $\mathrm{d}Q_{st}$, a Poisson morbidity shock with intensity:

$$\lambda_s(H_{t-}) = \eta + \frac{\lambda_{s0} - \eta}{1 + \lambda_{s1} H_{t-}^{-\xi_s}} \in [\lambda_{s0}, \eta], \tag{2}$$

and $H_{t-} = \lim_{s \uparrow t} H_s$ is health prior to occurrence of the sickness shock $\mathrm{d}Q_{st}$.

The agent's health is also a determinant of the stochastic age at death T_m, the first occurrence of a Poisson mortality shock $\mathrm{d}Q_{mt}$ with intensity:

$$\lambda_m(H_{t-}) = \lambda_{m0} + \lambda_{m1} H_{t-}^{-\xi_m}. \tag{3}$$

Observe that both intensities for morbidity (2) and mortality (3) are strictly positive, decreasing and convex functions of the health stock. Hence, becoming healthier reduces exposure to sickness and death shocks, subject to diminishing returns, and lower bounds in self-insurance $\lambda_{k0} > 0$, for $k = m, s$. The parameters $\lambda_{k1} \geq 0$ control whether or not mortality and morbidity are endogenous, i.e. whether self-insurance is feasible or not.

The agent's budget constraint is given as:

$$dW_t = [rW_{t-} + Y_t - c_t - I_t] \, \mathrm{d}t + \pi_t \sigma_S \left[\mathrm{d}Z_t + \theta \mathrm{d}t\right] + x_t \left[\mathrm{d}Q_{st} - \lambda_s(H_{t-})\mathrm{d}t\right], \tag{4}$$

where health-dependent labor income Y is given by:

$$Y_t = Y(H_{t-}) = y + \beta H_{t-}, \tag{5}$$

and is increasing in health to capture revenue losses associated with sickness and/or poor health. The law of motion (4) defines W as financial wealth, c as consumption, and reveals that the agent invests a money value π in the risky asset characterized by a standard Brownian motion Z, and a market price of risk $\theta = \sigma_S^{-1}(\mu - r)$. The remaining balance is invested in the risk-less asset paying net rate r. Actuarially fair health insurance contracts purchased in quantity $x \geq 0$ are priced at the morbidity intensity $\lambda_s(H)$, and each pay one unit of the numeraire for every occurrence of the health shock $\mathrm{d}Q_s$.

Starting with the seminal papers of Yaari (1965), and Hakansson (1969), standard time-additive frameworks model utility as the sum of discounted period utilities up to the random time of death, at which point welfare is normalized to zero. Hugonnier et al. (2013) retain the latter assumption, but relax time-additivity along the lines of Duffie and Epstein (1992) and further allow for source-dependent risk aversion. More formally, subject to the laws of motion (1), and (4), and taking into account the distributional assumptions (2), and (3), the agent selects optimal consumption, portfolio, health insurance and health investment so as to solve:

$$V(W_t, H_t) = \sup_{(c,\pi,x,I)} U_t,$$

where the continuation utility U_t is given as:

$$\begin{aligned} U_t &= 1_{\{T_m > t\}} E_t \int_t^{T_m} \left(f(c_\tau, U_{\tau-}) - \frac{\gamma|\sigma_\tau(U)|^2}{2U_{\tau-}} - \sum_{k=m}^{s} F_k(U_{\tau-}, H_{\tau-}, \Delta_k U_\tau) \right) \mathrm{d}\tau \\ &= 1_{\{T_m > t\}} \mathcal{U}_t, \end{aligned} \tag{6}$$

and where the modified utility $\mathcal{U}_t$ solves:

$$\mathcal{U}_t = E_t \int_t^{\infty} e^{-\int_t^{\tau} \nu_m(H_v)\mathrm{d}v} \left(f(c_\tau, \mathcal{U}_{\tau-}) - \frac{\gamma|\sigma_\tau(\mathcal{U})|^2}{2\mathcal{U}_{\tau-}} - F_s(\mathcal{U}_{\tau-}, H_{\tau-}, \Delta_s \mathcal{U}_\tau) \right) \mathrm{d}\tau. \tag{7}$$

The first term in (6), and (7) is the standard Kreps-Porteus aggregator:

$$f(c_t, U_{t-}) = \frac{\rho\, U_{t-}}{1 - 1/\varepsilon} \left(\left(\frac{c_t - a}{U_{t-}} \right)^{1-\frac{1}{\varepsilon}} - 1 \right), \tag{8}$$

and encodes the generalized recursivity, with a being the survival consumption level, ε being the elasticity of inter-temporal substitution, and ρ being the subjective discount rate. The second term captures the Duffie and Epstein (1992, eq. (3), p. (416) penalty for exposure to the Brownian financial risk in Z, where

$$\sigma_t(U) = \frac{1}{\mathrm{d}t} d\langle U, Z \rangle_t,$$

is the volatility of the continuation utility induced by financial exposure, and where the aversion to financial risk is given by $\gamma \geq 0$. The third and fourth terms in (6), and (7) are given by:

$$F_k(U_{t-}, H_{t-}, \Delta_k U_t) = U_{t-}\, \lambda_k(H_{t-}) \left[\frac{\Delta_k U_t}{U_{t-}} + u(1; \gamma_k) - u\left(1 + \frac{\Delta_k U_t}{U_{t-}}; \gamma_k\right) \right], \tag{9}$$

where the expected jump in utility is:

$$\Delta_k U_t = E_{t-}[U_t - U_{t-} | \mathrm{d}Q_{kt} \neq 0],$$

for $k = s, m$, and where $u(\cdot; \gamma_k)$ is the CRRA function with curvature γ_k. The functions F_k in (9) are the Poisson analogs to the Brownian volatility term, and capture the penalties induced by exposure to the discrete health-related shocks $\mathrm{d}Q_k$. The parameters $\gamma_s \geq 0$ and $\gamma_m \in (0, 1)$ respectively encode aversion with respect to sickness and death risks. As discussed in Hugonnier et al. (2013, Fig. 2), the penalty functions F_k are positive for all relative jumps $\Delta_k U/U \neq 0$, and are zero otherwise, are increasing in risk aversion γ_k, and are asymmetric in that negative jumps are more costly than positive ones.

Finally, the health-dependent endogenous discount factor in (7) is given by:

$$\nu_m(H_{t-}, \lambda_{m0}) = \frac{\lambda_m(H_{t-}, \lambda_{m0})}{1 - \gamma_m} \geq 0, \tag{10}$$

and highlights the iso-morphism between the agent's problem with endogenous stochastic life horizon in (6), and the infinite horizon problem in (7), with endogenous discounting at rate $\nu_m(H_{t-}, \lambda_{m0})$ in (10). Put differently, an unhealthy agent faces a higher risk of dying and behaves as though he were more impatient; *ceteris paribus* higher aversion to death risk γ_m further increases discounting.

2.2 Indirect utility and optimal rules

The endogeneity of the discount rate $\nu_m(H)$ in (7) implies that closed-form solutions to the agent's problem cannot be computed.[6] Instead, Hugonnier et al. (2013) calculate an approximate solution relying on a Taylor expansion. First, a zero-order solution is explicitly obtained by imposing $\lambda_{k1} = 0, k = m, s$ on the parameter controlling the endogeneity of the morbidity (2) and mortality (3) intensities. Second, a first-order solution is calculated via an order-1 expansion around the exogenous intensities benchmark $\lambda_{k1} = 0$.

In particular, under the theoretical assumptions in Appendix A, we can let the non-negative marginal-Q of health B solve $g(B) = 0$, subject to $g'(B) < 0$ in:

$$g(B) = \beta - (r + \delta + \phi\lambda_{s0})B - (1 - 1/\alpha)(\alpha B)^{\frac{1}{1-\alpha}}, \tag{11}$$

and define

$$K = \alpha^{1/(1-\alpha)} B^{\alpha/(1-\alpha)}. \tag{12}$$

Moreover, let the non-negative marginal propensity to consume and the marginal value of net total wealth be defined as:

$$A(\lambda_{m0}) = \varepsilon\rho + (1 - \varepsilon)\left(r - \frac{\lambda_{m0}}{1 - \gamma_m} + \frac{\theta^2}{2\gamma}\right), \tag{13}$$

$$\Theta(\lambda_{m0}) = \rho\left(\frac{A(\lambda_{m0})}{\rho}\right)^{\frac{1}{1-\varepsilon}}. \tag{14}$$

[6]To facilitate notation, we henceforth suppress time subscripts, i.e. we denote $G(W, H) = G(W_{t-}, H_{t-} \mid W_{t-} = W, H_{t-} = H)$ for any function G.

Finally, let the non-negative first-order adjustments associated with endogenous morbidity and mortality risks be defined as:

$$
\begin{aligned}
l_s &= \frac{\phi(\eta - \lambda_{s0})}{r - F(1-\xi_s)}, && (15)\\
l_m(\lambda_{m0}) &= \frac{1}{(1-\gamma_m)[A(\lambda_{m0}) - F(-\xi_m)]}. && (16)
\end{aligned}
$$

where

$$
\begin{aligned}
F(x) &= x(\alpha B)^{\frac{\alpha}{1-\alpha}} - x\delta - \lambda_{s0}\chi(-x), && (17)\\
\chi(x) &= 1 - (1-\phi)^{-x}. && (18)
\end{aligned}
$$

Given these elements, Hugonnier et al. (2013, Prop. 1, 2, and Thm. 1, 2) show that the optimal policy can be characterized as follows.

Theorem 1 (Indirect utility and optimal policy) *Assume that the regularity conditions* (40) *in Appendix A are verified. Then, up to a first-order approximation, the nonnegative indirect utility of an alive agent is:*

$$
V(W,H,\lambda_{m0}) = \Theta(\lambda_{m0})\left[N_1(W,H) - \lambda_{m1}H^{-\xi_m}l_m(\lambda_{m0})N_0(W,H)\right], \qquad (19)
$$

and generates the nonnegative optimal consumption

$$
c^*(W,H,\lambda_{m0}) = a + A(\lambda_{m0})\left[N_1(W,H) - (1-\varepsilon)\lambda_{m1}H^{-\xi_m}l_m(\lambda_{m0})N_0(W,H)\right], \qquad (20)
$$

as well as the other optimal policy functions:

$$
\begin{aligned}
\pi^*(W,H) =& (\theta/(\gamma\sigma_S))N_0(W,H) - \lambda_{s1}(\theta/(\gamma\sigma_S))l_sH^{-\xi_s}P_0(H)\\
x^*(W,H,\lambda_{m0}) =& \phi P_0(H) - \lambda_{m1}\chi(\xi_m)(1-1/\gamma_s)l_m(\lambda_{m0})H^{-\xi_m}N_0(W,H)\\
&- \lambda_{s1}\chi(\xi_s-1)l_sH^{-\xi_s}P_0(H), \qquad (21)\\
I^*(W,H,\lambda_{m0}) =& KP_0(H) + \lambda_{m1}(\xi_mK/(1-\alpha))l_m(\lambda_{m0})H^{-\xi_m}N_0(W,H)\\
&+ \lambda_{s1}((\xi_s-1)K/(1-\alpha))l_sH^{-\xi_s}P_0(H),
\end{aligned}
$$

where any dependence on the endowed mortality rate λ_{m0} is explicitly stated, and where the nonnegative order-0 value of human capital and nonnegative orders-0 and 1 of net *total wealth are defined as:*

$$
\begin{aligned}
P_0(H) &= BH, && (22)\\
N_0(W,H) &= W + P_0(H) + \frac{y-a}{r}, && (23)\\
N_1(W,H) &= N_0(W,H) - \lambda_{s1} H^{-\xi_s} l_s P_0(H). && (24)
\end{aligned}
$$

2.2.1 Financial, human, and total wealth

Both welfare (19), and the optimal rules (20), (21) are functions of the orders-0 and 1 values of the health capital, and of net total wealth. In particular, the order-0 value of the human capital $P_0(H)$ in (22) is added to financial wealth W and to the present value of base income y, net of subsistence consumption a to yield the order-0 net total wealth $N_0(W,H)$ in (23). When endogenous sickness risk exposure is reintroduced, allowing for $\lambda_{s1} > 0$ mechanically raises the morbidity intensity (2) and reduces the value of the human capital to $P_0(H)(1 - \lambda_{s1} H^{-\xi_s} l_s)$, and consequently lowers the first-order net total wealth $N_1(W,H)$ in (24).

Two points are worth mentioning. First, Hugonnier et al. (2013, Prop. 2) show that up to a first-order:

$$
N_1(W_t, H_t) = E_t \int_t^{\infty} m_{t,\tau}(c^*_\tau - a)\mathrm{d}\tau \tag{25}
$$

where $m_{t,\tau}$ is a stochastic discount factor induced by bond, stock, and insurance prices, and where $c^*_\tau = c^*(W_\tau, H_\tau, \lambda_{m0})$ is the consumption at the optimum in (20). Hence, the order-1 net total wealth $N_1(W,H)$ in (24) captures the net present value of consumption along the optimal path, net of minimal subsistence. Second, observe that neither $P_0(H), N_0(W,H)$ nor $N_1(W,H)$ are affected by exposure to death risk $(\lambda_{m0}, \lambda_{m1})$. This irrelevance stems directly from the solution method where an incomplete markets setup with finite horizon, and endogenous death risk exposure (6) is equivalently recast as a complete market one with infinite horizon, and endogenous discounting (7). Under completeness, the agent can sell a claim to his net labor income stream to recover human capital, and selects optimal rules subject to health-dependent discounting (see Hugonnier et al., 2013, for details), i.e. death risk alters optimal choices and welfare, but not the value of assets, be them human or financial.

2.2.2 Effects of exogenous death intensity

Next, contrasting welfare $V(W, H, \lambda_{m0})$ in (19) and consumption $c^*(W, H, \lambda_{m0})$ in (20) reveals that the exogenous death intensity λ_{m0} affects the marginal propensity to consume $A(\lambda_{m0})$ in (13), through which both the marginal value $\Theta(\lambda_{m0})$ in (14), and the endogenous dealth risk adjustment $l_m(\lambda_{m0})$ in (16) are also affected. In particular the MPC $A(\lambda_{m0})$ is increasing in the death intensity λ_{m0} for elastic inter-temporal substitution ($\varepsilon > 1$), is independent at unit elasticity, and is decreasing for inelastic preferences ($0 < \varepsilon < 1$).

To understand these conflicting effects of death risk on the MPC, recall from (10) that an increase in exogenous death risk exposure induces heavier discounting at rate $\nu_m(H, \lambda_{m0})$ of future utility flows, and leads to two opposite outcomes on the marginal propensity to consume. First, more discounting makes future consumption less desirable and shifts future towards current consumption (i.e. by increasing the MPC); that effect is dominant at high elasticity of inter-temporal substitution $\varepsilon > 1$. Second, higher discounting of future consumption requires shifting current towards future consumption to maintain utility (i.e. by lowering the MPC); that effect is dominant at low elasticity of inter-temporal substitution $\varepsilon \in (0, 1)$. Unit elasticity implies exact cancellation of the two effects and no changes in the MPC.

Moreover, when the EIS is low, declining marginal propensity requires that an upper bound on death intensity be imposed to maintain non-negative MPC. That upper bound is given by:

$$\lambda_{m0} \leq \bar{\lambda}_{m0} = (1 - \gamma_m) \left[\left(\frac{\varepsilon}{1 - \varepsilon} \right) \rho + \left(r + \frac{\theta^2}{2\gamma} \right) \right], \quad \text{when } \varepsilon \in (0, 1). \tag{26}$$

We note that when preferences are inelastic, we have $A(\bar{\lambda}_{m0}) = 0$ implying that $c^*(W, H, \bar{\lambda}_{m0}) = a$ in (20), while $\Theta(\bar{\lambda}_{m0}) = 0$ implies that $V(W, H, \bar{\lambda}_{m0}) = 0$ in (19). Put differently, when the EIS is low, and mortality risk is evaluated at maximal admissible intensity $\bar{\lambda}_{m0}$, the agent consumes minimal subsistence level, and is indifferent between life and death.

Two final elements are worth discussing. First, the net effect of exogenous death risk on the endogenous death risk adjustment $l_m(\lambda_{m0})$ in (16) is the inverse of that on the MPC, i.e. l_m is decreasing (resp. increasing) at high (resp. low) EIS. Second, whereas a higher risk of dying can have an increasing or depressing effect on the marginal propensity to consume, strict preference for life implies that it always reduces the marginal value of total wealth $\Theta(\lambda_{m0})$ in (14) regardless of how elastic is inter-temporal substitution. By non-negativity, it follows that $\Theta(\infty) = V(W, H, \infty) = 0$, i.e. the agent's welfare converges to the indirect utility at death as the latter becomes certain.

To summarize, the exogenous death intensity λ_{m0} conditions welfare and consumption through the the marginal propensity to consume $A(\lambda_{m0})$. How the latter is affected depends on whether preferences are elastic or not with respect to inter-temporal substitution. As we show next, the EIS also is a key determinant for the willingness to pay to avert death.

3 Willingness to pay, Gunpoint and Statistical values of life

Relying on the indirect utility function (19), we first compute the Hicksian Equivalent Variation to avert an arbitrary increase in the exogenous death risk. Under a similar reasoning, we then evaluate the Gunpoint value of life as the maximal willingness to pay to avert certain death. Finally, both the indirect utility and the WTP can be relied upon to calculate the Value of a Statistical Life.

3.1 Willingness to pay to avoid a finite increase in death risk

Consider a permanent change of $\Delta \geq 0$ in the exogenous death intensity $\lambda^*_{m0} = \lambda_{m0} + \Delta$. We resort to standard principles to compute the Hicksian Equivalent Variation (EV) as the maximum amount $v(\lambda^*_{m0}) \geq 0$ that an individual is willing to pay to avoid the unfavorable change from λ_{m0} to λ^*_{m0}:

$$V\left(W - v(\lambda^*_{m0}), H, \lambda_{m0}\right) = V\left(W, H, \lambda^*_{m0}\right). \tag{27}$$

We can substitute the indirect utility $V(W, H)$ given by (19) in the implicit EV (27), and solve for $v(\lambda^*_{m0})$ through a first-order approximation as follows:

Theorem 2 (willingness to pay) *Assume that the regularity conditions* (40) *in Appendix A are verified. Then, up to a first order approximation, the maximal willingness* *to pay to avoid a permanent change Δ in the exogenous death intensity λ_{m0} is given by:*

$$v(W,H,\lambda^*_{m0}) = \left[1 - \frac{\Theta^*}{\Theta}\right] N_1(W,H) + \frac{\Theta^*}{\Theta}\lambda_{m1}H^{-\xi_m}\left[l^*_m - l_m\right] N_0(W,H), \tag{28}$$

and where the order-0 and order-1 values of total wealth $N_0(W,H)$, and $N_1(W,H)$ are given in (23)*, and* (24)*, and where we have set $\Theta = \Theta(\lambda_{m0})$, $\Theta^* = \Theta(\lambda^*_{m0})$ in* (14) *and $l_m = l_m(\lambda_{m0})$, $l^*_m = l_m(\lambda^*_{m0})$ in* (16)*.*

We saw earlier that a change in λ_{m0} affects welfare $V(W,H,\lambda_{m0})$ through the $\Theta(\lambda_{m0})$, and the $l_m(\lambda_{m0})$ channels, both of which transit through the MPC, $A(\lambda_{m0})$. This is naturally reflected in the WTP (28) via its effects on Θ^*, and l^*_m. Second, it was shown earlier that the marginal value of total wealth $\Theta(\lambda_{m0}) \geq 0$ in (14) is a decreasing function $\forall\varepsilon \neq 1$. Consequently, the weights $\Theta^*/\Theta \in [0,1]$, and the WTP $v(W,H,\lambda_{m0}) = 0$ (i.e. when $\Delta = 0$), and is otherwise a weighted average of two components: the first-order net total wealth $N_1(W,H)$, and the change in the endogenous mortality adjustment that is induced by a change in the endowed intensity $\lambda_{m1}H^{-\xi_m}\left[l^*_m - l_m\right] N_0(W,H)$. Interestingly, we saw that unit elasticity cancels out the two conflicting effects on the MPC $A(\lambda_{m0})$, and consequently implies that $\Theta = \Theta^*$, and $l_m = l^*_m$. It follows $v(\lambda^*_{m0}) = 0, \forall\lambda^*_{m0}$, i.e. that the agent is indifferent to an increase in the risk of death, regardless of the magnitude of the change, and therefore his willingness to pay to avoid it is zero.

Third, since l_m is declining in the MPC, we have that $l^*_m - l_m < 0$ when $\varepsilon > 1$, i.e. agents with elastic preferences substitute more consumption when faced with a shorter horizon, and are therefore willing to pay less to avert it. Finally, the weights Θ^*/Θ are falling as exogenous death risk λ^*_{m0} increases. This implies that the elements capturing attitudes towards mortality risk γ_m, and the endogeneity of death risk (λ_{m1}, ξ_m) that are encoded in (Θ^*, l^*_m) gradually lose any relevance as the endowed death intensity λ^*_{m0} becomes large. This is formalized in the following result.

Corollary 1 (limiting WTP) *Assume that the regularity conditions* (40) *in Appendix A are verified. Then, up to a first order approximation,*

1. *For high elasticity of inter-temporal substitution $\varepsilon > 1$:*

$$\lim_{\lambda^*_{m0}\to+\infty} v(W,H,\lambda^*_{m0}) = N_1(W,H). \tag{29}$$

2. For low elasticity of inter-temporal substitution $\varepsilon \in (0,1)$:

$$v(W, H, \bar{\lambda}_{m0}) = N_1(W, H), \tag{30}$$

where the order-1 value of total wealth $N_1(W, H)$ is given in (24)*, and where the maximal admissible death intensity $\bar{\lambda}_{m0}$ is given in* (26)*,*

Hence, when preferences are sufficiently elastic with respect to inter-temporal substitution ($\varepsilon > 1$), the willingness to pay converges to the morbidity-adjusted net total wealth $N_1(W, H)$ as death becomes certain. It also converges to $N_1(W, H)$ when preferences are inelastic, and the exogenous death risk intensity attains its maximal admissible value $\bar{\lambda}_{m0}$.

For the other cases of finite Δ, the shape of the willingness to pay $v(W, H, \lambda^*_{m0})$ in function of the death risk increment crucially depends on the EIS ε, as well as on the other parameters, and the health level, and is difficult to establish ex-ante.[7] We will instead perform an empirical evaluation below. As will be seen shortly, the monotone increasing, and concave WTP function $v(W, H, \lambda^*_{m0})$ that is inferred from our estimates has important implications for the relative magnitude of the Gunpoint versus VSL estimates.

3.2 Gunpoint Value of Life

To calculate the value of life at gunpoint, we again resort to the Hicksian EV, this time computing the WTP to avert *certain*, rather than possible death. From preferences (6), and (7) the utility at death is normalized at zero, such that the gunpoint value v_g is implicitly defined from:

$$V(W - v_g, H, \lambda_{m0}) = 0. \tag{31}$$

Again relying on the welfare function (19), and resorting to a first-order approximation reveals the following result.

Theorem 3 (Gunpoint value of life) *Assume that the regularity conditions* (40) *in Appendix A are verified. Then, up to a first order approximation, the maximum value an*

[7] Rosen (1988) stresses the importance of inter-temporal substitution in valuing longevity. See also Córdoba and Ripoll (2013) for the importance of the EIS in value of life calculations, as well as Huggett and Kaplan (2016) for EIS effects on human capital valuation.

agent is willing to pay to avoid certain death is given by:

$$v_g(W,H) = N_1(W,H) \tag{32}$$

where the order-1 value of total wealth $N_1(W,H)$ is given in (24)*.*

Relying on (22), (23), and (24) allows us to re-write the gunpoint value of life (32) as:

$$v_g(W,H) = W + \frac{y-a}{r} + P_1(H) \tag{33}$$

where the first-order value of the human capital is given as:

$$\begin{aligned} P_1(H) &= HB\left[1 - \lambda_{s1} l_s H^{-\xi_s}\right], \\ &= Hp_1(H). \end{aligned} \tag{34}$$

In the absence of a bequest motive, and under perfect markets, the agent who is forced to evaluate life at gunpoint is thus willing to pay his total financial wealth W, plus the capitalized value of his fixed income endowment y. This total wealth measure has been used extensively in the HK literature (e.g. Huggett and Kaplan, 2016). However, as (33) makes clear, it provides an incomplete proxy to the value of a human life.

First, the previous discussion of net total wealth in (25) showed that the minimal consumption level a is required at all periods for subsistence, and must be deducted from the Gunpoint value in (33). Second, since human capital is non-transferable, and is entirely destroyed at death, the agent is also willing to give up the shadow value of his health capital $P_1(H)$, i.e. his stock of current health H, times its shadow price $p_1(H)$. From (34), the latter can be expressed as the health capacity to generate labor revenues B, adjusted for the endogeneity of the agent's exposure to health shocks. As discussed earlier, the health shock intensity (2) is higher for $\lambda_{s1} > 0$, i.e. the health stock is more subject to stochastic depreciation. Consequently, its value must be discounted accordingly, and is lower than under exogenous exposure to sickness $\lambda_{s1} = 0$.

Furthermore, the value of human capital $P_1(H)$ in (34) is positive only for sufficiently high levels of health:

$$H > H_{min} = (\lambda_{s1} l_s)^{\frac{1}{\xi_s}} \geq 0.$$

Hence, the model predicts a threshold health level under which agents are willing to pay less than their total financial wealth to ward off certain death. We note further that the shadow price of health $p_1(H)$ is monotone increasing and concave, such that healthier agents face lower sickness risks, and thus value more highly their health capital that is destroyed at death. This property provides a natural background for the quality of life adjustment in life value calculations (QALY).

In addition, sufficient health $H > H_{min}$, and curvature $\xi_s > 1$ jointly imply that the value of life is increasing in the order-zero marginal value of health B. Using (11) reveals that the latter is increasing in the marginal income value of health β, whereas it decreases in the interest rate r, as well as the deterministic and stochastic depreciation $\delta, \phi\lambda_{s0}$. Hence, a higher health gradient in labor income, and/or lower discount, or depreciation rates all contribute to increasing the shadow price of the health capital, and therefore the Gunpoint value of life.

Interestingly, the shadow value of the health stock $P_1(H)$ in (34), and therefore the value of life v_g in (32) are both independent of the attitudes towards death risk γ_m and of the endogenous components in the mortality intensity (λ_{m1}, ξ_m). Consequently, neither aversion to death risk nor the shadow value of health attributed to its ability to ward off death determine the value of life. This result stems from the way the latter is evaluated. Because the outcome of death is certain when life is evaluated at gunpoint, both the attitudes towards death *risk* and the ability to marginally alter exposure to that risk become irrelevant. Indeed, this element could already be inferred from our previous discussion of equations (29), and (30) which showed that the willingness to pay $v(W, H, \lambda^*_{m0})$ is converging to $N_1(W, H)$ as the increment in exogenous death risk λ^*_{m0} becomes large. The willingness to pay to avert a finite exogenous death risk increase thus converges to the Gunpoint Value of Life as death becomes certain.

Unlike the WTP $v(W, H, \lambda_{m0})$ however, the Gunpoint value is independent of the elasticity of inter-temporal substitution ε. This again stems from the way $v_g(W, H)$ is computed, i.e. as a willingness to pay to avert certain death regardless of *how* death occurs. Put differently, the specific mechanism – be it through increases in λ^*_{m0} or not – is irrelevant, only the outcome is. Since the elasticity of inter-temporal substitution was identified as the key driver for the effects of the exogenous death intensity on life valuation, the EIS is irrelevant as well in the Gunpoint value.

3.3 Value of a Statistical Life

3.3.1 Permanent infinitesimal changes in death intensity

From standard definitions, the Value of a Statistical Life is the willingness to pay to avoid an increase in death risk Δ, divided by the latter. Since the WTP $v(W, H, \lambda_{m0}^*) = 0$ for $\Delta = 0$, the VSL is thus equal to a slope through the origin when the WTP is plotted against the increment. As Δ falls towards zero, this slope converges to the marginal willingness to pay $\partial v(W, H, \lambda_{m0}^*)/\partial \lambda_{m0}^*$, evaluated at $\lambda_{m0}^* = \lambda_{m0}$, which corresponds to the theoretical measure of the VSL. Moreover, as is well known, the VSL is also equal to the marginal rate of substitution (MRS) between the probability of life and wealth (e.g. Aldy and Smyth, 2014; Andersson and Treich, 2011; Bellavance et al., 2009). Using the value function $V(W, H, \lambda_{m0})$, the VSL can thus also be computed as the (negative of) the MRS between λ_{m0}, and W. Both approaches yield the following expression for the VSL.

Theorem 4 (value of statistical life) *Assume that the regularity conditions* (40) *in Appendix A are verified. Then, up to a first order approximation, the Value of a Statistical Life is:*

$$
\begin{aligned}
v_s(W, H, \lambda_{m0}) &= \lim_{\Delta \to 0} \frac{v(W, H, \lambda_{m0}^*)}{\Delta} = \left. \frac{\partial v(W, H, \lambda_{m0}^*)}{\partial \lambda_{m0}^*} \right|_{\Delta=0}, \\
&= \frac{-V_{\lambda_{m0}}(W, H, \lambda_{m0})}{V_W(W, H, \lambda_{m0})}, \\
&= \frac{-\Theta'(\lambda_{m0})}{\Theta(\lambda_{m0})} N_1(W, H) + \lambda_{m1} H^{-\xi_m} l_m'(\lambda_{m0}) N_0(W, H). \qquad (35)
\end{aligned}
$$

First, as explained earlier, the marginal value of total wealth $\Theta(\lambda_{m0})$ is a decreasing function for all levels of EIS. It follows that the VSL is an increasing function of first-order net total wealth $N_1(W, H)$. However, the marginal effects of λ_{m0} on the endogenous death risk factor l_m depends on the elasticity of inter-temporal substitution. The VSL is consequently lower ($l_m'(\lambda_{m0}) < 0$) when the agent's preferences are sufficiently elastic with respect to time (i.e. $\varepsilon > 1$) and higher otherwise. Finally, unit elasticity again entails that $\Theta'(\lambda_{m0}) = l_m'(\lambda_{m0}) = 0$ and therefore that $v_s(W, H, \lambda_{m0}) = 0$.

3.3.2 Finite changes per time period in death intensity

The previous calculations of the VSL (35) featured permanent infinitesimal changes in the death intensity. In the spirit of the empirical VSL literature, the value of a statistical life can also be computed as the maximal willingness to pay to avoid an exogenous increase Δ in the probability of death over a given time interval (e.g. a change $\Delta = 0.1\%$ per one year period), divided by Δ. This calculation involves two steps. First, we compute the new value of the endowed intensity $\lambda^*_{m0}(H, \Delta, T)$ corresponding to a change in death risk Δ occurring over a duration of T. This calculation reveals the following result.

Lemma 1 *A higher likelihood of death of Δ per time interval of $s \in [0, T]$ corresponds to a permanent increase in the endowed intensity to $\lambda^*_{m0}(H, \Delta, T) > \lambda_{m0}$ given by:*

$$\lambda^*_{m0}(H, \Delta, T) = \frac{-1}{T} \log \left[e^{-\lambda_{m0} T} - \frac{\Delta}{1 - \lambda_{m1} k(H, T)} \right], \tag{36}$$

where,

$$\begin{aligned} k(H, T) &= H^{-\xi_m} \left(\frac{e^{\psi T} - 1}{\psi} \right) \geq 0, \\ \psi &= \xi_m \left[\delta - (\alpha B)^{\frac{\alpha}{1-\alpha}} \right] + \lambda_{s0} \left[(1 - \phi)^{-\xi_m} - 1 \right] \geq 0. \end{aligned}$$

Second, given $\lambda^*_{m0}(H, \Delta, T)$ in (36), the value of a statistical life associated with a finite increase in death risk of Δ over interval $s \in [0, T]$ is:

$$v_s(W, H, \lambda^*_{m0}(H, \Delta, T)) = \frac{v(W, H, (\lambda^*_{m0}(H, \Delta, T))}{\Delta}, \tag{37}$$

where $v(W, H, \lambda^*_{m0}(H, \Delta, T))$ is the willingness to pay (28), evaluated at $\lambda^*_{m0}(H, \Delta, T)$.

4 Structural estimation

4.1 Econometric model

The econometric model that we rely upon assumes that agents are heterogeneous with respect to their health, and wealth statuses, and are homogeneous with respect to the distributional, revenue, and preference parameters. To structurally estimate the latter, we use the quadri-variate closed-form expressions for the optimal rules (20), and (21), to which we append the exogenous income equation (5). Specifically, let $\mathbf{Y}_j = [c_j - a, \pi_j, x_j, I_j]'$ denote the vector of optimal excess consumption, portfolio, health insurance, and health spending for agents $j = 1, 2, \ldots, n$. The estimated optimal rules are:

$$\mathbf{Y}_j = \mathbf{B}(H_j)\,[N_0(W_j, H_j), P_0(H_j)]' + \mathbf{u}_j, \tag{38}$$

where the $\mathbf{u}_j$'s are (potentially correlated) Gaussian error terms. The value of human capital $P_0(H_j)$ is given in (22), and the net total wealth $N_0(W_j, H_j)$ is given in (23), where the price of health B is implicitly given in (11). The health-dependent loadings matrix $\mathbf{B}(H_j)$ is restricted as follows:

$\mathbf{Y}_j$	$N_0(W_j, H_j)$	$P_0(H_j)$
$c_j - a$	$A - \lambda_{m1} A\,(1-\varepsilon) l_m H_j^{-\xi_m}$	$-\lambda_{s1} A\, l_s H_j^{-\xi_s}$
π_j	$\theta/(\gamma\sigma_S)$	$-\lambda_{s1}\theta/(\gamma\sigma_S) l_s H_j^{-\xi_s}$
x_j	$-\lambda_{m1}\chi(\xi_m)(1-1/\gamma_s) l_m H_j^{-\xi_m}$	$\phi - \lambda_{s1}\chi(\xi_s - 1) l_s H_j^{-\xi_s}$
I_j	$\lambda_{m1}\xi_m K/(1-\alpha) l_m H_j^{-\xi_m}$	$K + \lambda_{s1}(\xi_s - 1)K/(1-\alpha) l_s H_j^{-\xi_s}$

(39)

where the marginal propensity to consume A is given in (13), the endogenous morbidity and mortality adjustments l_s, l_m are obtained in (15), (16), with $\chi(x)$ given in (18), and K is given in (12). To ensure theoretical consistency, we estimate the structural parameters in (38)–(39) imposing the full set of regularity conditions (40) in Appendix A. In light of the strong nonlinearities not all the deep parameters can be identified, and a subset are calibrated. We resort to a two-stage, iterative Maximum Likelihood procedure. In stage one, we fix the curvature parameters in the Poisson intensity functions ξ_k, for $k = m, s$, then estimate the remaining deep parameters. In stage two, we condition on the latter to re-estimate the ξ_k. We iterate on this procedure until a fixed point is reached.

4.2 Data

We use a sample of 8,378 individuals taken from the 2013 wave of the Institute for Social Research's Panel Study of Income Dynamics (PSID). The data construction is detailed in Appendix G. We proxy the health variables through the polytomous self-reported health status that is linearly converted to numeric values from 1 to 4. The financial wealth comprises risky, and riskless assets. Using the method in Skinner (1987), we infer the unreported total consumption by extrapolating the food, transportation, and utility expenses reported in the PSID. Finally, health expenditures and insurance are respectively the out-of-pocket spending, and premia paid by agents. All nominal values are scaled by 10^{-6} for the estimation.

Tables 1, and 2 present descriptive statistics, as well as mean values (in \$) for the main variables of interest, per health status, and per wealth quintiles. Table 2.a shows that financial wealth remains very low for the first three quintiles (see also Hubbard et al., 1994, 1995; Skinner, 2007, for similar evidence). Moreover no clear effects of the health status on wealth levels can be deduced. The level of consumption in panel b is clearly increasing in financial wealth. However, the effects of health remain ambiguous, except for the least healthy who witness a significant drop in consumption.

In panel c, stock holdings remain very low for all but the fourth, and fifth quintiles, illustrating the non-participation puzzle (e.g. Friend and Blume, 1975; Mankiw and Zeldes, 1991). Again, a clear positive wealth gradient is observed, whereas health effects are weakly positive. The health insurance expenses in panel d are modest relative to consumption. They are increasing in wealth, and devoid of clear health gradients. Finally, health spending in panel e is of the same order of magnitude as insurance. It is strongly increasing in wealth, and also sharply decreasing in health status.

5 Results

5.1 Structural parameters

Table 3 reports the calibrated (with subscript ^{c}), and estimated (standard errors in parentheses) deep parameters. Overall, the latter are precisely estimated, and are close to other estimates for this type of model (e.g. Hugonnier et al., 2013, 2017).

First, the health law of motion parameters in panel a are indicative of significant diminishing returns in adjusting health status ($\alpha = 0.70$). Although depreciation is relatively low ($\delta = 1.09\%$), additional depletion brought upon by sickness is consequential ($\phi = 1.36\%$). Second, the sickness and death intensities parameters in panel c are consistent with endogeneous morbidity, and mortality ($\lambda_{k1}, \xi_k \neq 0$ for $k = s, m$). Moreover, high convexity parameters ($\xi_k > 1$) indicate strongly diminishing returns in adjusting exposure to death and sickness risks. Furthermore, mortality risk is more difficult to adjust than morbidity risk ($\lambda_{s1}, \xi_s > \lambda_{m1}, \xi_m$). Finally, a large calibrated value for η entails that sickness risks increase very steeply as health falls.

Third, the income parameters in panel c are consistent with a significant positive effect of health on labor income ($\beta = 0.0095$), as well as a realistic calibrated value for base income ($y \times 10^6 = 12.2$ K\$).[8] The returns process parameters (μ, r, σ_S) are calibrated at standard values. Finally, the preference parameters in panel d indicate realistic aversion to financial risk ($\gamma = 3.52$), and to mortality risk, where the latter is less than one as required ($\gamma_m = 0.29$), as well as a high calibrated value for aversion to morbidity risk ($\gamma_s = 7.4$). Importantly, as for other cross-sectional estimates using survey data (Gruber, 2013; Hugonnier et al., 2017), the elasticity of inter-temporal substitution is larger than one ($\varepsilon = 1.67$). Observe that the inverse of the EIS is nonetheless larger than the mortality risk aversion ($1/\varepsilon = 0.60 > 0.29 = \gamma_m$), an issue to which we will return shortly. The minimal consumption level is realistic, and larger than base income ($a \times 10^6 = 14.4$ K\$).

5.2 Estimated valuations

Gunpoint Value Using the point estimates of the deep parameters, Table 4 reports the Gunpoint values $v_g(W, H)$ in (32), by wealth, and health status. The GPV are ranging between 88 K\$, and 729 K\$. Contrasting these valuations with the low observed financial wealth in Table 2.a, reveals that the bulk of the Gunpoint value captures human wealth, with $(y - a)/r + P_1(H)$, ranging between 88 K\$ (Poor health), and 607 K\$ (Excellent health), and corresponding to 4 to 12 times annual revenues. These human capital are realistic, yet somewhat lower than other HK estimates.[9] Part of the difference relates to the absence of subsistence consumption in human capital models (e.g. Rosen, 1988; Huggett and Kaplan, 2016). As discussed earlier, the agents need to maintain subsistence consumption $c_t \geq a$ for survival. Consequently, the capitalized value $(y - a)/r$ they are willing to pay to survive is net of subsistence. Since the latter is higher than base income ($a = .0146 > .0122 = y$), the human capital value is lowered by -50 K\$. Another

[8] For example, U.S. Census Bureau (2017) poverty thresholds for single-agent households under age 65 were 12.5 K\$ in 2016.

[9] For example, Huggett and Kaplan (2016, benchmark case, Fig. 7.a, p. 38) find human capital starting at about 300 K\$ at age 20, peaking at less than 900 K\$ at age 45, and falling steadily towards zero afterwards.

explanation relates to the absence of endogenous morbidity risk in alternative HK models. Indeed, the adjustment for exposure to sickness $P_0(H) - P_1(H)$ lowers human capital by -1.7 K$ (Excellent health) and as much as -26 K$ (Poor health).

Second, the Gunpoint value is increasing in both wealth, and health. Unsurprisingly, rich agents are thus willing to pay more to protect their own life, since all financial wealth is paid out in the absence of a bequest motive. The higher GPV for healthy agents reflects their higher human wealth. From the health capital $P_1(H)$ in (34), healthy agents have a lower exposure to sickness risks $\lambda_{s1} H^{-\xi_s}$, as well as more human capital HB at stake, and are thus willing to spend more in (33) in order to remain alive.

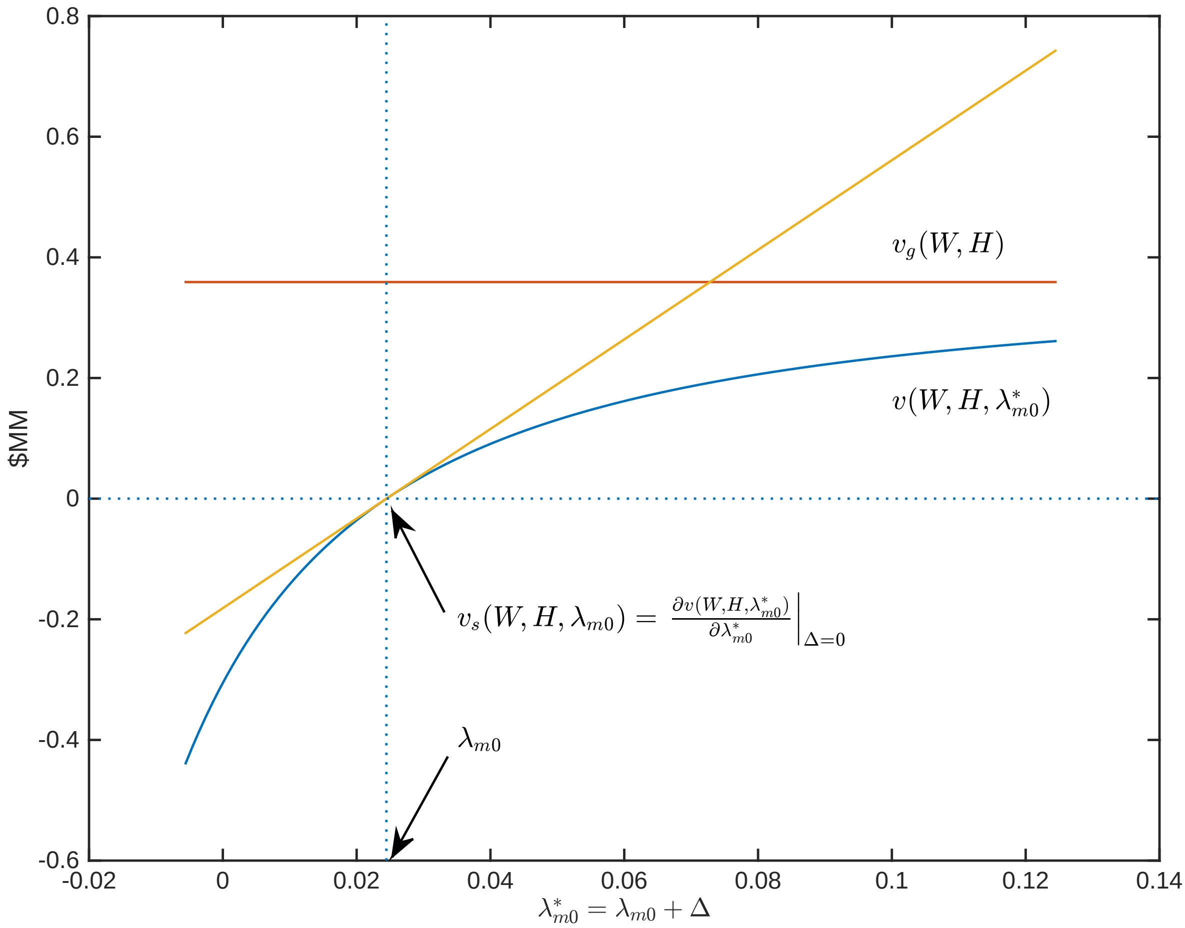
Value of Statistical Life Again relying on the estimated structural parameters, Table 5 reports the Values of Statistical Life by health, and wealth statuses, where the VSL is estimated from $v_s(W, H, \lambda_{m0})$ in (35). First, the calculated values are between 2.06 MM$, and 15.82 MM$, and are well within the ranges usually found in the empirical VSL literature.[10] Overall, the concordance of these values with previous findings provides additional evidence that our structural estimates are well grounded.

Second, the VSL is increasing in both wealth, and especially health. Positive wealth gradients have been identified elsewhere in the literature (Bellavance et al., 2009; Andersson and Treich, 2011; Adler et al., 2014) whereby diminishing marginal value of wealth and higher financial values at stake both imply that richer agents are willing to pay more to improve survival probabilities. The literature has been more ambivalent with respect to the health effect (e.g. Andersson and Treich, 2011; Robinson and Hammitt, 2016; Murphy and Topel, 2006). On the one hand better health increases the value of life that is at stake, on the other hand, healthier agents face lower death risks, and are willing to pay less to attain further improvements (or prevent deteriorations). Our estimates unambiguously indicate that the former effect is dominant and that better health raises the VSL.

[10] A meta-analysis by Bellavance et al. (2009, Tab. 6, p. 452) finds mean values of 6.2 MM$ (2000 base year, corresponding to 8.6 MM$, 2016 value). Survey evidence by Doucouliagos et al. (2014) ranges between 6 MM$, and 10 MM$. Robinson and Hammitt (2016) report values ranging between 4.2, and 13.7 MM$. Finally, guidance values published by the U.S. Department of Transportation were 9.6 MM$ in 2016 (U.S. Department of Transportation, 2016), whereas the Environmental Protection Agency relies on central estimates of 7.4 MM$ (2006$), corresponding to 8.8 MM$ in 2016 (U.S. Environmental Protection Agency, 2017).

Understanding the differences between VSL and GPV Our estimated values show that the Statistical Life value is much larger than the Gunpoint Value of Life. To understand these differences, it is useful to characterize the willingness to pay in function of the change in death risk, and to contrast what the two values are effectively measuring.

Figure 1: Estimated willingness to pay, Statistical Life and Gunpoint Values



Notes: At estimated parameter values, for third quintile of wealth and Good health levels. $v(W, H, \lambda^*_{m0})$ in blue is the maximum willingness to pay to avoid an increase of Δ in exogenous death intensity λ_{m0}; $v_g(W, H)$ in red is the Gunpoint value of life; $v_s(W, H, \lambda_{m0})$ is the Value of statistical life, and the slope of the yellow tangent evaluated at λ_{m0}. In MM$.

Figure 1 plots the estimated willingness to pay $v(W, H, \lambda^*_{m0})$ to avoid an increment Δ in function of the death intensity $\lambda^*_{m0} = \lambda_{m0} + \Delta$. These valuations are calculated from (28) at the estimated parameters, and relying on the third wealth quintile, and Good health status in Table 2.a ($W = 1,802\$ \times 10^{-6}, H = 2.50$). First, the estimated WTP in blue is an increasing, and concave function that equals zero at $\lambda^*_{m0} = \lambda_{m0} = 0.0244$, is negative[11] for $\Delta < 0$, and positive for positive increments. The pronounced curvature of the WTP is consistent with standard economic intuition of diminishing marginal valuation of exposure to death (e.g. Philipson et al., 2010; Córdoba and Ripoll, 2016). Concavity of the WTP is also expected when the reciprocal of the EIS is larger than mortality risk aversion (as was found in Table 3.c) in other life valuation literature using Non-Expected Utility (see Córdoba and Ripoll, 2016, for discussion).

Second, we saw from Corollary 1 that for $\varepsilon > 1$, the limit of the willingness to pay when death becomes certain – i.e. when λ^*_{m0} tends to infinity – is the morbidity-adjusted net total wealth $N_1(W, H)$. From Theorem 3, this limiting value is also the gunpoint value $v_g(W, H)$ plotted in red. Third, as explained in Theorem 4, the VSL $v_s(W, H, \lambda_{m0})$ is the value of the slope of the yellow tangent of $v(W, H, \lambda^*_{m0})$ evaluated at $\Delta = 0$ or, equivalently, the value of the yellow tangent evaluated as $\lambda^*_{m0} = 1 + \lambda_{m0}$. The pronounced curvature of the WTP in Figure 1 is informative as to why the VSL is much larger than the Gunpoint value (7.88 MM\$ vs 359 K\$). Put differently, the linear extrapolation of marginal values that is relied upon in the VSL calculation overstates the willingness to protect one's own life when the WTP is very concave in the death risk increment.

Perhaps more fundamentally, as famously pointed out by Schelling (1968), and widely recognized by the literature, the VSL does *not* measure a value ascribed to a particular human life, but instead gauges the aggregate willingness to pay for infinitesimal changes in the risk of dying indiscriminately affecting entire populations.[12] Conversely, the Gunpoint value measures the willingness to pay to avoid a large change in death risk (i.e. life versus certain death), and affecting a single individual. There is therefore no ex-ante reason why the Statistical Life, and Gunpoint values should be equal.

Indeed, Pratt and Zeckhauser (1996) argue that concentrating the costs, and benefits of death risk reduction leads to two opposing effects on valuation. On the one hand, the *dead anyway* effect leads to higher payments on identified (i.e. small groups facing

[11] A negative willingness to pay to *avoid* a change to $\lambda^*_{m0} < \lambda_{m0}$ thus corresponds to a positive WTP to *attain* a lower death intensity.

[12] In his opening remarks, Schelling (1968, p. 113) writes

> "This is a treacherous topic and I must choose a nondescriptive title [*The life you save may be your own*] to avoid initial misunderstanding. It's not the worth of a human life that I shall discuss, but of 'life saving', of preventing death. And it's not a particular death, but a statistical death. What it is worth to reduce the probability of death – the statistical frequency of death – within some identifiable group of people, none of whom expects to die except eventually. "

large risks), rather than statistical (i.e. large groups facing small risks) lives. In the limit, they contend that an individual might be willing to pay infinite amounts to save his own life from certain death. On the other hand, the wealth or *high payment* effect has an opposite impact. Since resources are limited, the marginal utility of wealth increases with each subsequent payment to avoid increases in risk, thereby reducing the WTP as risk increases.[13] Although the net effect remains uncertain, Pratt and Zeckhauser (1996, Fig. 2, p. 754) argue that the wealth effect is dominant for larger changes in death risk, i.e. for those cases that naturally extend to our Gunpoint Value.

Their conjecture is warranted in our calculations. When faced with certain death, an individual is willing to pay much less than what can be inferred from the VSL. Indeed, while total financial wealth, plus the value of the human capital, net of subsistence costs, are paid out in the Gunpoint value, these resources are limited, and much less than what society might collectively be willing to pay to save one unidentified life.

As a final note, two caveats of our approach are worth mentioning. A first limitation is the absence of bequest motives. This omission is related to the technical difficulty in solving the model when bequeathed wealth is optimally chosen. Although it remains unclear how our results would be affected, we can however conjecture that a likely effect would be to reduce the GPV even further. Indeed, *warm glow* effects of bequest would attenuate the cost of dying, and consequently also the WTP to avert death. Moreover, bequeathed wealth is illiquid, to the extent that it is set aside for surviving heirs, and not to ensure one's own survival. Without affecting human capital, the amount of disposable financial resources that can be pledged in a money-or-death threat would therefore be reduced, and consequently so would the GPV.

A second limitation is the absence of aging in our valuation. A fair treatment would involve time varying parameters, which are made possible in the original Hugonnier et al. (2013, Appendix B) paper, yet are technically more involved. Although a complete derivation is again beyond the scope of this paper, we can conjecture that aging should also reduce the GPV. Indeed, biological limits to life expectancy would generate optimal dis-saving of financial wealth, lowering even further the Gunpoint value for elders. Moreover, as mentioned earlier, the marginal Q of health, B, is a declining function of

[13]Pratt and Zeckhauser (1996, p. 753) point out that whereas a community close to a toxic waste dump could collectively pay \$1 million to reduce the associated mortality risk by 10%, it is unlikely that someone would be willing to pay that same amount when confronted with that entire risk.

the health depreciation parameters (which would likely increase in age) and an increasing function of health gradient of income (which should fall in age). A lower value of human capital for elders would then lead to less resources being paid out to survive a credible death threat.

6 Conclusion

Computing the monetary value of a human being has generated a profound, and continued interest, with early records dating back to the late XVIIth century. The two main valuation frameworks have centered on the marginal rate of substitution between the probability of living, and wealth (VSL), and on the human capital value of a person (HK). The VSL is appropriate for the valuation of unidentified lives, i.e. a collective willingness to pay to avert diffuse risks of death among a given population, but its usefulness for a WTP to avert a concentrated death risk affecting a single person has been debated. The HK is useful to measure the net present value of the labor income stream that is foregone upon death, but measurement issues related to non-workers, and appropriate rate of discounting limit its relevance.

We have proposed a third method based on the maximum an individual would be willing to pay to avert certain death, with that amount being referred to as the Gunpoint Value of Life. Whereas it also relies on the Hicksian Equivalent Variation, it gauges an intrinsically different WTP than the VSL, i.e. a value a person would ascribe to his *own*, rather than *someone's* life in a credible money-or-life situation. Similar to HK frameworks, this GPV also encompasses a human wealth component. However this human value relates to the value of the health capital that is destroyed upon death. As such it gauges the services procured by health, i.e. the capacity to work (or equivalently to procure utility flows), to produce future health, and to ward off sickness, all of which are endogenously determined in our model. Moreover, we directly addressed the issue of what discounting process to use by fully internalizing the effects of health on exposure to death risk.

Our structural estimation of the closed-form expressions for the GPV, and for the VSL has highlighted a long-suspected feature of the WTP to avert death risks, i.e. that it is strongly concave in the latter. Consequently, the VSL – which extrapolates the marginal

WTP – leads to much larger values of life than the GPV – which computes the limiting values of the WTP corresponding to certain death. These discrepancies where shown to be directly linked to the diminishing marginal value of additional expected life, as well as to the finite capacity to pay to save oneself, compared to the collective capacity to pay to save someone.

The initial question that is implicit in the title of this paper is which of the GPV or the VSL should be relied upon to measure the value of a human life? To the extent that they measure different objects, we contend that *both* should be used. Put differently, and as hinted by Schelling (1968) the VSL, and GPV are complements, rather than substitutes, and their relevance should depend on the underlying motivation for computing a life value.

All in all, the VSL is more appropriate in issues involving collective choices that involve changes in death probabilities affecting large subsets of the population, and for which society is the ultimate payer of the associated costs. Well-known examples identified in the literature include general safety measures with respect to transportation, or public health. The GPV is inappropriate for such cases in that it gauges what someone would be willing to pay to survive, not what a society is collectively willing to pay to indiscriminately save someone in the group. The GPV on the other hand should be relied upon in situations where the risk of dying concerns a single individual. One such application could be the continued life support decisions, wrongful death litigation, or life insurance where the GPV would gauge the value of life ascribed by its main beneficiary, with full adjustments for his health, financial, and net total wealth statuses.

Future research should address some of the important elements that are omitted from our framework. In particular, we conjectured – but did not prove – that including bequests could lower the GPV by reducing the cost of dying (and therefore the WTP to avert it) through a warm glow effect, as well as by rendering the bequeathed part of financial resources illiquid. Moreover, we did not include the aging process in our valuations, although the original Hugonnier et al. (2013) model is fully amenable to such time variation. We again conjectured that aging would likely reduce the GPV through biological limits to life expectancy, and ensuing effects on optimal dis-savings, as well as through age-increasing depreciation of health and declining capacity to work. Finally, our Gunpoint Value of Life is intrinsically individualistic. The genuine costs of one's death that are associated with grief or financial hardship, and that are borne by surviving family or friends are completely abstracted from in our analysis.

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A Regularity conditions

The model of Hugonnier et al. (2013) is solved and estimated under the following transversality conditions:

$$\begin{aligned}
&\beta < (r+\delta+\phi\lambda_{s0})^{\frac{1}{\alpha}}\,, \\
&0 < A(\lambda_{m0}) - \max\left(0, r - \frac{\lambda_{m0}}{1-\gamma_m} + \theta^2/\gamma\right), \\
&0 < \min\left(\frac{\lambda_{m0}}{1-\gamma_m}, r\right) - F(1-\xi_s), \\
&0 < A(\lambda_{m0}) - \max\left(0, r - \frac{\lambda_{m0}}{1-\gamma_m} + \theta^2/\gamma\right) - F(-\xi_m),
\end{aligned} \tag{40}$$

where A is given in (13), and $F(x)$ in (17).

B Proof of Theorem 2

Following Hugonnier et al. (2013), we can define $L_k(H) = H^{-\xi_k} l_k$, for $k = s, m$, and set $\lambda_{k1} = \epsilon\bar{\lambda}_{k1}$ for some strictly positive constants $\bar{\lambda}_{k1}$, and for $k = m, s$, such that the value function in (19) can be written as:

$$\begin{aligned}
V(W,H,\lambda_{m0},\epsilon) &= \Theta(\lambda_{m0})\left[N_0(W,H) - \epsilon\bar{\lambda}_{s1}L_s(H)P_0(H)\right] \\
&\qquad - \Theta(\lambda_{m0})\epsilon\bar{\lambda}_{m1}L_m(H,\lambda_{m0})N_0(W,H), \\
&= \Theta(\lambda_{m0})\left[N_1(W,H,\epsilon) - \epsilon\bar{\lambda}_{m1}L_m(H,\lambda_{m0})N_0(W,H)\right],
\end{aligned} \tag{41}$$

where the first-order total wealth $N_1(W, H, \epsilon)$ is implicitly defined. The indirect utility (41) is obtained by Hugonnier et al. (2013) through a first-order Taylor expansion of the agent's problem around small deviations $\epsilon \approx 0$. By a similar reasoning, the first-order approximation to the Hicksian compensating value $v(\epsilon) = v(W, H, \lambda^*_{m0}, \epsilon) \geq 0$ in (27) to prevent any increase in the endowed death intensity $\lambda^*_{m0} > \lambda_{m0}$ is given as:

$$\begin{aligned}
0 &= V(W - v(\epsilon), H, \lambda_{m0}, \epsilon) - V(W, H, \lambda^*_{m0}, \epsilon) \\
&= \nabla V(W, H, \lambda^*_{m0}, \epsilon) \\
&\approx \nabla V(W, H, \lambda^*_{m0}, 0) + \epsilon\nabla V_\epsilon(W, H, \lambda^*_{m0}, 0).
\end{aligned}$$

Straightforward calculations using the indirect utility (41) reveal that:

$$\begin{aligned}\nabla V(W,H,\lambda_{m0}^*,0) &= V(W-v(0),H,\lambda_{m0},0) - V(W,H,\lambda_{m0}^*,0)\\ &= -\Theta v(0) + (\Theta-\Theta^*)N_0(W,H)\end{aligned}$$

where $\Theta = \Theta(\lambda_{m0})$, and $\Theta^* = \Theta(\lambda_{m0}^*)$ are given in (14). Setting $\nabla V(W,H,\lambda_{m0}^*,0) = 0$ uniquely solves for $v(0)$ as:

$$v(0) = \left(1 - \frac{\Theta^*}{\Theta}\right) N_0(W,H). \tag{42}$$

Similarly, we obtain:

$$\begin{aligned}\nabla V_\epsilon(W,H,\lambda_{m0}^*,0) =& - V_W(W-v(0),H,\lambda_{m0},0)v'(0)\\ &+ V_\epsilon(W-v(0),H,\lambda_{m0},0) - V_\epsilon(W,H,\lambda_{m0}^*,0),\\ =& - \Theta v'(0) + \bar{\lambda}_{m1}\Theta L_m(H)v(0)\\ &- \bar{\lambda}_{m1}\left[\Theta L_m(H) - \Theta^* L_m^*(H)\right] N_0(W,H)\\ &- \bar{\lambda}_{s1}\left[\Theta - \Theta^*\right] L_s(H)P_0(H),\end{aligned}$$

where $L_m(H) = L_m(H,\lambda_{m0})$, and $L_m^*(H) = L_m(H,\lambda_{m0}^*)$ are given in (16). Again setting $\nabla V_\epsilon(W,H,\lambda_{m0}^*,0) = 0$ uniquely solves for $v'(0)$ as:

$$v'(0) = -\bar{\lambda}_{m1}\frac{\Theta^*}{\Theta}\left[L_m(H) - L_m^*(H)\right] N_0(W,H) - \bar{\lambda}_{s1}\left[1-\frac{\Theta^*}{\Theta}\right] L_s(H)P_0(H) \tag{43}$$

The corresponding Hicksian value $v(\epsilon)$ obtains by substituting the solutions (42) and (43) in the first-order expansion of the compensating value:

$$\begin{aligned}v(\epsilon) \approx& v(0) + \epsilon v'(0)\\ =& \left[1-\frac{\Theta^*}{\Theta}\right]\left[N_0(W,H) - \epsilon\bar{\lambda}_{s1}L_s(H)P_0(H)\right]\\ &- \epsilon\bar{\lambda}_{m1}\frac{\Theta^*}{\Theta}\left[L_m(H) - L_m^*(H)\right] N_0(W,H)\\ =& \left[1-\frac{\Theta^*}{\Theta}\right] N_1(W,H,\epsilon) - \epsilon\bar{\lambda}_{m1}\frac{\Theta^*}{\Theta}\left[L_m(H) - L_m^*(H)\right] N_0(W,H).\end{aligned}$$

Substituting back $\lambda_{k1} = \epsilon\bar{\lambda}_{k1}$, and using total wealth (24) yields (28). ■

C Proof of Corollary 1

When the agent's preferences are sufficiently elastic with respect to time (i.e. $\varepsilon > 1$), the marginal propensity to consume $A(\lambda_{m0})$ in (13) is a linear increasing function, such that $l_m^* < l_m$ is decreasing and convex. Since $\Theta(\lambda_{m0})$ was found to be increasing and convex for all ε it follows that

$$\lim_{\lambda_{m0}^* \to +\infty} v(W, H, \lambda_{m0}^*) = N_1(W, H), \quad \text{if } \varepsilon > 1,$$

Conversely, when the elasticity is low, i.e. $\varepsilon \in (0, 1)$, the marginal propensity to consume $A(\lambda_{m0})$ is a linear decreasing function. To maintain non-negativity of the MPC, the maximal admissible increase in the death intensity is:

$$\bar{\lambda}_{m0} = (1 - \gamma_m) \left[\left(\frac{\varepsilon}{1 - \varepsilon} \right) \rho + \left(r + \frac{\theta^2}{2\gamma} \right) \right].$$

Using $\bar{\lambda}_{m0}$ in the regularity conditions (40) simplifies to:

$$\frac{\varepsilon \rho}{1 - \varepsilon} \geq \frac{\theta^2}{2\gamma} + F(-\xi_m),$$

under which condition, it is then straightforward to show that $\Theta(\bar{\lambda}_{m0}) = 0$, whereas $l_m(\bar{\lambda}_{m0})$ is finite, such that:

$$v(W, H, \bar{\lambda}_{m0}) = N_1(W, H), \quad \text{if } \varepsilon \in (0, 1)$$

as stated. ■

D Proof of Theorem 3

Again by a similar reasoning, the first-order approximation to gunpoint value of life $v_g(\epsilon) = v_g(W, H, \epsilon)$ in (31) is implicitly given as:

$$\begin{aligned} 0 &= V(W - v_g(\epsilon), H, \lambda_{m0}, \epsilon) \\ &\approx V(W - v_g(0), H, \lambda_{m0}, 0) + \epsilon V_\epsilon(W - v_g(0), H, \lambda_{m0}, 0). \end{aligned}$$

Straightforward calculation indicate that:

$$V(W - v_g(0), H, \lambda_{m0}, 0) = \Theta\left[N_0(W,H) - v_g(0)\right]$$

whereas,

$$V_\epsilon(W - v_g(0), H, \lambda_{m0}, 0) = \Theta\left[-v_g'(0) - \bar{\lambda}_{s1}L_s(H)P_0(H) - \bar{\lambda}_{m1}L_m(H)\left(N_0(W,H) - v_g(0)\right)\right].$$

Again equating each terms to zero uniquely solves for $v_g(0), v_g'(0)$ and reveals that:

$$\begin{aligned} v_g(\epsilon) &\approx v_g(0) + \epsilon v_g'(0) \\ &= N_0(W,H) - \epsilon\bar{\lambda}_{s1}L_s(H)P_0(H). \end{aligned}$$

Substituting back $\lambda_{s1} = \epsilon\bar{\lambda}_{s1}$, and using total wealth (24) yields (32). ■

E Proof of Theorem 4

Using a similar reasoning and standard principles, the VSL can be calculated as the negative of the MRS between death intensity λ_{m0}, and wealth:

$$\begin{aligned} v_s(W,H,\lambda_{m0},\epsilon) &= \frac{-V_{\lambda_{m0}}(W,H,\lambda_{m0},\epsilon)}{V_W(W,H,\lambda_{m0},\epsilon)} \\ &\approx v_s(W,H,\lambda_{m0},0) + \epsilon \left.\frac{\partial v_s(W,H,\lambda_{m0},\epsilon)}{\partial\epsilon}\right|_{\epsilon=0}, \end{aligned}$$

where

$$\begin{aligned} v_s(W,H,\lambda_{m0},0) &= \frac{N_0(W,H)\Theta'(\lambda_{m0})}{\Theta(\lambda_{m0})} \\ \left.\frac{\partial v_s(W,H,\lambda_{m0},\epsilon)}{\partial\epsilon}\right|_{\epsilon=0} &= \frac{-\bar{\lambda}_{s1}BHL_s(H)\Theta'(\lambda_{m0})}{\Theta(\lambda_{m0})} - \bar{\lambda}_{m1}N_0(W,H)\frac{\partial L_m(H,\lambda_{m0})}{\partial\lambda_{m0}}. \end{aligned}$$

Re-arranging terms, using the definition of $N_1(W,H)$ in (24), and substituting for $\lambda_{k1} = \epsilon\bar{\lambda}_{k1}$ yields the VSL in (35). Note that the alternative calculation through the marginal willingness to pay

$$v_s(W,H,\lambda_{m0},\epsilon) = \left.\frac{\partial v(W,H,\lambda^*_{m0},\epsilon)}{\partial \lambda^*_{m0}}\right|_{\Delta=0}$$

yields the same value of statistical life (35). ■

F Proof of Lemma 1

A higher likelihood of death of Δ over a time interval of $s \in [0,T]$ corresponds to an increase in the endowed intensity to $\lambda^*_{m0}(\Delta,H) > \lambda_{m0}$:

$$\begin{aligned}\Pr\left[T_m \le T \mid \lambda^*_{m0}\right] &= \Pr\left[T_m \le T \mid \lambda_{m0}\right] + \Delta, \\ &= 1 - E\left[e^{-\int_0^T \lambda^*_m(\Delta,H_s)\mathrm{d}s}\right],\end{aligned}$$

where we have set $\lambda^*_m(\Delta,H) = \lambda^*_{m0}(\Delta,H) + \lambda_{m1}H^{-\xi_m}$ in (3). Solving for λ^*_{m0} through a first-order expansion around benchmark $\lambda_{k1} = 0, k = m, s$ reveals that the latter is as:

$$\lambda^*_{m0}(\Delta,H) = \frac{-1}{T}\log\left[e^{-\lambda_{m0}T} - \frac{\Delta}{1-\lambda_{m1}k(H)}\right],$$

where,

$$\begin{aligned}k(H) &= E\int_0^T H_s^{-\xi_m}\mathrm{d}s = H^{-\xi_m}\left(\frac{e^{\psi T}-1}{\psi}\right) \ge 0, \\ \psi &= \xi_m\left[\delta - (\alpha B)^{\frac{\alpha}{1-\alpha}}\right] + \lambda_{s0}\left[(1-\phi)^{-\xi_m} - 1\right] \ge 0.\end{aligned}$$

as stated. ■

G Data

The data construction follows the procedure in Hugonnier et al. (2013). We rely on a sample of 8,378 U.S. individuals obtained by using the 2013 wave of the Institute for Social Research's Panel Study of Income Dynamics (PSID, http://psidonline.isr.umich.edu/). All nominal variables in per-capita values (i.e., household values divided by household size), and scaled by 10^{-6} for the estimation. The agents' wealth and health which are constructed as follows:

Health H_j Values of 1.0 (Poor health), 1.75 (Fair), 2.5 (Good), 3.25 (Very good) and 4.0 (Excellent) are ascribed to the self-reported health variable of the household head.

Wealth W_j Financial wealth is defined as risky (i.e. stocks in publicly held corporations, mutual funds, investment trusts, private annuities, IRA's or pension plans) plus riskless (i.e. checking accounts plus bonds plus remaining IRA's and pension assets) assets.

The dependent variables are the observed portfolios, consumption, health expenditure and health insurance, and are constructed as follows:

Portfolio π_j Money value of financial wealth held in risky assets.

Consumption c_j Inferred from the food, utility and transportation expenditures that are recorded in PSID, using the Skinner (1987) method with the updated shares of Guo (2010).

Health expenditures I_j Out-of-pocket spending on hospital, nursing home, doctor, outpatient surgery, dental expenditures, prescriptions in-home medical care.

Health insurance x_j Spending on health insurance premium.

H Tables

Table 1: PSID data statistics

	Mean	Std. dev.	Min	Max
Health (H)	2.58	0.80	1	4
Wealth (W)	38 685	122 024	0	1 430 000
Consumption (c)	9 835	11 799	1.047	335 781
Risky holdings (π)	20 636	81 741	0	1 367 500
Insurance (x)	247	718	0	17 754
Health investment (I)	721	2 586	0	107 438
Income (Y)	21 838	37 063	0	1 597 869

Notes: Statistics in 2013 $ for PSID data used in estimation (8 378 observations). Scaling for self-reported health is 1.0 (Poor), 1.75 (Fair), 2.50 (Good), 3.25 (Very good), and 4.0 (Excellent).

Table 2: PSID data statistics (cont'd)

Health	H_j	Wealth quintiles 1	2	3	4	5
		a. Wealth W_j (\$)				
Poor	1.00	0	139	2 063	11 831	152 151
Fair	1.75	0	145	1 741	12 027	123 083
Good	2.50	0	168	1 802	11 908	120 467
Very good	3.25	0	199	1 823	12 197	118 738
Excellent	4.00	0	192	1 823	12 099	122 135
		b. Consumption c_j (\$)				
Poor	1.00	3 281	4 906	6 558	10 052	7 752
Fair	1.75	4 095	6 888	8 795	11 196	13 368
Good	2.50	5 086	6 526	9 745	11 269	13 336
Very good	3.25	5 989	7 517	10 181	11 131	13 626
Excellent	4.00	5 276	6 897	10 002	12 099	14 628
		c. Stocks π_j (\$)				
Poor	1.00	0	0	0	725	46 497
Fair	1.75	0	5	279	2 309	76 721
Good	2.50	0	1	268	4 320	55 379
Very good	3.25	0	5	192	4 756	68 768
Excellent	4.00	0	0	334	5 801	90 147
		d. Insurance x_j (\$)				
Poor	1.00	165	191	503	723	856
Fair	1.75	181	196	497	775	1 095
Good	2.50	206	219	401	564	852
Very good	3.25	190	284	313	522	797
Excellent	4.00	203	254	366	429	807
		e. Investment I_j (\$)				
Poor	1.00	549	552	2 341	2 936	6 003
Fair	1.75	400	468	968	621	1 250
Good	2.50	243	238	383	500	962
Very good	3.25	276	226	275	435	596
Excellent	4.00	151	192	230	307	451

Notes: Statistics in 2013 $ for PSID data used in estimation. Means per quintiles of wealth, and per health status

Table 3: Estimated and calibrated structural parameter values

Parameter	Value	Parameter	Value
	a. Law of motion health (1)		
α	0.7045	δ	0.0109
	(0.1799)		(0.0048)
ϕ	0.0136^c		
	b. Sickness and death intensities (2), (3)		
λ_{s0}	0.0316	λ_{s1}	0.0088
	(0.0152)		(0.0042)
ξ_s	2.9802	η	50^c
	(1.0262)		
λ_{m0}	0.0244	λ_{m1}	0.0045
	(0.0087)		(0.0022)
ξ_m	1.0686		
	(0.4497)		
	c. Wealth, and income (4), (5)		
y	0.0122^c	β	0.0095
			(0.0045)
μ	0.108^c	r	0.048^c
σ_S	0.20^c		
	d. Preferences (6), (8)		
γ	3.5242	ε	1.6699
	(1.3316)		(0.5911)
a	0.0146	γ_m	0.2862
	(0.0037)		(0.1212)
γ_s	7.4^c	ρ	0.05^c

Notes: Estimated structural parameters (standard errors in parentheses); c: calibrated parameters. Econometric model (38)–(39) estimated by iterative 2-stages ML, subject to the regularity conditions (40).

Table 4: Estimated Gunpoint Value of Life ($)

Health level	Wealth quintile				
	1	2	3	4	5
Poor	87 800	87 900	89 800	99 600	239 900
Fair	229 200	229 300	230 900	241 200	352 300
Good	357 300	357 400	359 100	369 200	477 700
Very good	482 600	482 800	484 400	494 800	601 400
Excellent	607 100	607 300	608 900	619 200	729 200

Notes: At estimated parameter values, using $v_g(W, H)$ in (32), multiplied by 1 MM$ to correct for scaling used in estimation.

Table 5: Estimated Value of Statistical Life ($)

Health level	Wealth quintile				
	1	2	3	4	5
Poor	2 061 200	2 064 400	2 108 600	2 333 000	5 557 100
Fair	5 102 800	5 106 000	5 141 500	5 370 100	7 838 700
Good	7 840 400	7 844 100	7 879 900	8 101 600	10 483 200
Very good	10 515 500	10 519 800	10 555 200	10 781 200	13 102 300
Excellent	13 169 800	13 174 000	13 209 300	13 432 200	15 819 100

Notes: At estimated parameter values, using $v_s(W, H, \lambda_{m0})$ in (35), multiplied by 1 MM$ to correct for scaling used in estimation.

Table 6: Estimated Value of Statistical Life for Discrete Changes ($)

Health level	Wealth quintile				
	1	2	3	4	5
Poor	1 494 144	1 496 565	1 530 008	1 699 840	4 139 523
Fair	4 096 371	4 098 962	4 127 538	4 311 633	6 299 301
Good	6 462 782	6 465 821	6 495 384	6 678 266	8 642 695
Very good	8 782 648	8 786 261	8 815 828	9 004 636	10 943 626
Excellent	11 087 366	11 090 873	11 120 661	11 308 341	13 317 979

Notes: For increment $\Delta = 0.01$, over time interval $T = 1$. At estimated parameter values, using $v_s(W, H, \lambda_{m0}(\Delta, H))$ in (36), and (37), multiplied by 1 MM$ to correct for scaling used in estimation.