Evaluating welfare and economic effects of raised fertility[*]

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Abstract

In the context of second demographic transition many countries consider pro-natalistic policies as viable solutions to the fiscal pressure stemming from longevity and declining fertility. However, increased number of births implies immediate economic costs and delayed economic gains. Moreover, quantification of these gains remains a challenge. We develop an overlapping generations model with family structure and utilize this model to quantify the effects in the increases in birth rates. We show the overall welfare and macroeconomic effects as well as distribution of these effects across cohorts. We also show how the distribution of children across families affects those estimations for a given birth rate.

Key words: fertility, welfare, pro-natalistic policies, overlapping generations model

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1 Introduction

Many of the natalistic policies have objectives unrelated to long-term population trends: disease spread (Khan et al. 2016), reproductive health (Régnier-Loilier and Vignoli 2011, Ahmed et al. 2012, Bongaarts and Sobotka 2012, Bratti and Tatsiramos 2011, Casterline and Han 2017), use of contraception and family planning (Dereuddre et al. 2016, Singh et al. 2017), etc. However, most of the advanced and middle income economies are expected to observe a substantial decline in population due to lowering fertility (Takayama and Werding 2011). This decline in population size has multiple long-term implications on both society and economy. Among the latter, the most pronounced concern the fiscal stability of the current arrangements for the social security provision and financing health care (e.g. Lutz and Skirbekk 2005). While hotly debated as a solution, pro-natalistic policies have not been evaluated so far in terms of how effective they can be in addressing these future demographic challenges.

In order to provide means to avert fertility decline, one has to first identify them. Literature on the causes of the fertility decline is abundant and multi-directional. Naturally, substantial literature shows the correlation between access to birth control and family planning and fertility decline (Guinnane 2011). Many argue that education was an important driver (Becker et al. 2010), although evidence seems mixed. For example, recently DeCicca et al. (2016) show that education does not reduce fertility *per se*, rather it “compresses” the fertility distribution – women are more likely to have at least one child, but less likely to have multiple children. Substantial literature points to economic uncertainty (Kreyenfeld et al. 2012, Balbo et al. 2013), but when credible causal identifications are at disposal, the estimated effects are small at best (e.g. Huttunen and Kellokumpu 2016) argue that the effect is at best small. Using evidence from the beginning of the first demographic transition, Skirbekk et al. (2015) discusses the role of other societal and institutional factors which stay behind relatively low and swiftly decreasing fertility among Europeans and Fenge and Scheubel (2017) show how these trends were reinforced by some of the policy instruments, such as the introduction of old-age security at Bismark’s times. Summarizing, the overall trends in fertility, seem to be mostly driven by long-term factors, slow-moving institutions in the language of Gorodnichenko and Roland (2017) and Alesina and Giuliano (2015).

Yet, many countries introduce a variety of pro-natalistic policies from direct financial transfers, through tax incentives to public provision of child care service, these instruments are varied and often costly. In response, a substantial body of empirical literature evaluated the previous policy interventions. In a recent overview, Olivetti and Petrongolo (2017) argue that in developed economies, the previous century of family policies facilitated female labor force participation, but had typically negative or negligible effects on fertility rates (the notable exception is early childhood education and care, which had positive effects for female labor for participation as well as fertility). There appears to be a large scope of country specificity in which type of policies work better (Baizan et al. 2016, Rossin-Slater 2018). Nonetheless, the typical causal estimates are relatively small. For example, financial transfers appear to boost fertility, but at a very high cost. Drago et al. (2011) finds that an average cost of additional child in Australia amounts to roughly $A130k (studies for other countries comprise Milligan 2005, Brewer et al. 2012, Frejka and Zakharov 2013, Laroque and SalaniÃľ 2014, Garganta et al. 2017, among others). Child care availability and parental leave, by contrast, increases fertility, but mostly at the intensive margin (e.g Dehejia and Lleras-Muney 2004, Del Boca et al. 2009, Lalive and Zweimueller 2009, Rindfuss et al. 2010, Havnes and Mogstad 2011, Bauernschuster et al. 2015).

While all these evaluation attempts are very useful and often provide intuition as to the long-term effects of given policies, they are insufficient to evaluate the macroeconomic and welfare effects of these policies. First, one needs to account for the costs of the instruments and the possible benefits, which is rarely feasible in the empirical setup of evaluation models. Second, in the case of natalistic policies, costs are immediate and benefits are delayed, which necessitates the use of structural modeling. Finally, while child bearing and rearing costs are typically private, the benefits of larger future working population can only be internalized through general equilibrium effects (e.g. lower taxes or higher pension benefits to the retirees). Hence, one requires a general equilibrium framework to be able to address these effects.

Our paper builds on some earlier work utilizing macroeconomic simulations on micro-foundations to evaluate the effects of demographic processes on economic outcomes. For example Prettner (2013) discusses the effects of longevity on economic growth in theoretical context of two endogenous growth models. On a similar note, in the context of an overlapping generation model, Hock and Weil (2012) show that longevity implies higher fertility in the long run. In the study of Taiwan, Liao (2011) provides an account of the adjustments in a rapidly growing economy with endogenously declining fertility. Georges and Seçkin (2016) give an account of possible macroeconomic outcomes for Turkey if instead of central path, an optimistic scenario of population growth occurred. However, this study looks at aggregate change in the population size, rather than family and fertility *per se.* Momota (2016) and Fehr et al. (2017) provide models with family structure and exogenous fertility. They show that indeed intensive vs extensive margin adjustment in the number of children have important consequences for the economy. For example, the capital stock seems to exhibit a non-linear, U-shaped relationship with the share of mothers in the economy. However, Momota (2016) is a highly stylized 3-period model, while the interest of Fehr et al. (2017) lies in the old-age insurance provided within family. As a consequence, they do not elaborate the fertility scenarios, nor the consequences of changed fertility. In addition to exogenous fertility models, there is also a growing body of literature with endogenous fertility, e.g. Liao (2011), Ludwig et al. (2012), Hock and Weil (2012). However, for the objective of this paper, endogenizing fertility would not be a fortunate model setup. The model results would depend crucially on the assumed response of the households to the family policies, which too would have to be stylized and assumed. Instead, we develop a tool to evaluate the macroeconomic and welfare effects of fertility changes, providing an estimate of how much may be spent at all, to achieve certain fertility targets and maintain long-term aggregate welfare unharmed.

With reference to this literature, our paper offers several important innovations. First, we provide direct identification of the costs and benefits of higher fertility. Namely, our model recognizes fully the private and immediate nature of costs. In the spirit of Fehr et al. (2017), our model has families structure. We construct an overlapping generations economy populated by families with zero, one, two or more children. Households are fully tractable, i.e. we know which parents have how many children and how many siblings had a given parent as a child. Hence, we can identify the individual costs and benefits of changes in the birth rate, thus measuring direct individual welfare effects. Namely, raising children exhibits certain economies of scale – childless individuals may benefit from a higher number of adults in the future despite bearing no immediate costs, which creates moral hazard in response to pro-natalistic policies. We are able to provide evaluation of that effect.

Second, this is a macroeconomic simulation exercise, we thus abstract from considering specific policies and taking assumptions on how effective they can be. Instead, we ask what are the economic and welfare gains from a given increase in birth rate. Since the research question in our study concerns the effects rather than the causes, fertility and fertility change are exogenous scenarios in our simulations. By contrast, endogenous are the labor supply decisions of the population, as well as consumption, saving, implied taxation as well as interest rates and wages. The outcome variables in our simulations – economic growth and economic welfare – allow to compare between the baseline scenario and the reform scenario. Specifically, in the baseline scenario we will assume that the birth rate in the economy follows a demographic projection of the Eurostat. In the reform scenarios we will introduce the changes to the number of births – permanent and transitory.

Third, a change in fertility rate may occur because of adjustments at the extensive margin (more families have children at all) or at the intensive margin (families with children have more of them). Both the costs and the benefits of increased number of birth depend on the proportion between the intensive and extensive margin adjustments, which is highly uncertain and may depend in a given policy instrument employed. To avoid arbitrariness in this stage of the model, we adopt a fairly novel approach, i.e. provide evaluation from many reform scenarios. The analyzed scenarios comprise incremental changes in intensive and extensive margins for a given fertility rate. Hence, we obtain a distribution of welfare and macroeconomic variables, conditional on the distribution of children in the population. This approach yields a sensitivity analysis of our findings, providing also conceptual "confidence intervals" for the simulated macroeconomic and welfare effects.

This paper is structured as follows. First, we present the model in section 2 and the demographic projection in section 3. Subsequently, in section 4 we analyze the results of the simulations along with the sensitivity analysis in section 5. As much as increased fertility may be desirable from the policy perspective, it will generate winners and losers, while the final effects may depend substantially on the distribution of children across households. The paper is concluded by a policy recommendation section.

2 The model

We develop a general equilibrium overlapping generations model in the spirit of Auerbach and Kotlikoff (1987). As is typical in such frameworks, our model has technological progress which augments labor productivity. We also introduce longevity. Consumers use their cohort specific mortality rates to compute expected future utility. Individuals derive utility from leisure and consumption. The intra-temporal choice between leisure and labor supply determines the life-time path of earned income. In addition to earned income, individuals may also earn interest on savings if in a given period they chose not to consume available income. This inter-temporal choice allows to accumulate savings over the working period to finance consumption at the old age. Based on a life-time sequence of endogenously chosen leisure and consumption, individuals derive utility and they chose leisure and consumption such that their utility is maximized, given the budget constraints.

The economy has a production sector (which utilizes labor and private savings to produce output consumed by the individuals). The economy has also government, which collects taxes and provides pension benefits to the retirees. The pension system is a defined contribution one, i.e. individuals receive pension benefits as annuity from their contributions over the working years.

This standard model is enriched to comprise families. To keep the model tractable, households are formed when agents are young and the number of children is the main characteristic which distinguishes household of a given birth cohort from one another (Fehr et al. 2017). The number of children is exogenous: households may have zero, one, two or more children. Households consist of women and men. Following empirical evidence, if there is children in a given household, labor supply of a woman is temporarily reduced, to reflect the asymmetric nature of the costs of child bearing and rearing (e.g. Attanasio et al. 2015, Erosa et al. 2016, Adda et al. 2017).[1]

2.1 Consumers

The economy is populated by agents forming κ classes of households with a differentiated household structure, but preferences drawn from the same function family. Households consists of two adults, the class of the household κ denotes the number of children to be born and raised in that household: $\kappa = 0, 1, 2, 3+$. Agents live live for $j = 1, 2..., J$ periods facing a time and age specific mortality rate $\pi_{j,t}$. Agents have no bequest motive, but since survival rates until the age of j at time t – i.e. $\pi_{j,t}$ – are lower than one, in each period t certain fraction of subcohort (κ, j) leaves unintended bequests, which are distributed within their subcohort. Hence, the subcohort (κ, j) is identified by the number of children to be raised and the year of birth and characterized by survival probabilities, same for all agents born in a given year.

Until adult ($j \leq 20$), agents live in the household of type κ to which they were born at time $t - j$. After reaching adulthood at $j = 21$ agents form a new household and observe the

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[1] In the interest of clarity and tractability, we abstract from the quantity-quality problem formulated in the literature on fertility, e.g. Baudin (2011).

realization of κ i.e. how many children are born in their household. Once born, agents do not die until they reach the age of $j = 40$ (children are raised).

After reaching adulthood, at each point in time t an individual of age $j \geq 21$ and subcohort κ born at time $t-j+1$ consumes a non-negative quantity of a composite good $c_{\kappa,j,t}$ and allocates $l_{\kappa,j,t}$ time to work. We assume a collective decision making within a household.[2] Consequently, households lifetime utility is as follows:

$$\sum_{j=21}^{J} \beta^{j-21} \pi_{j,t+j-21} [u_j(\tilde{c}_{\kappa,j,t+j-21}, \tilde{l}_{\kappa,j,t+j-21})] \tag{1}$$

where β denotes time preference used by agents to discount future utility. In this notation $\tilde{c}_{\kappa,j}$ denotes total consumption of the household, whereas $\tilde{l}_{\kappa,j}$ denotes joint labor supply of the adults in the household.

Households maximize utility subject to the budget constraint, which consists of (net) earned income, interest on savings, pension benefits and (net) social transfers. Earned income $w_t l_{\kappa,j,t}$ is subject to labor income tax τ_l. Agents also pay mandatory social security contributions τ. The two are additive, yielding disposable labor income. Labor income tax is also deducted from pension benefits. Unspent earnings from the previous period are saved, with a interest rate on savings r_t. Interest earned is subject to capital income tax τ_k. Households of subcohort $\{\kappa, j\}$ receive unintended bequests $beq_{\kappa,j,t}$. Households may receive lump sum transfers or pay a lump sum tax Υ equal for all subcohorts (and used to close the model in the initial steady state). Hence, budget constraint of household type κ at age j in time t is given by:

$$\begin{aligned}(1+\tau_c)\tilde{c}_{\kappa,j,t} + \tilde{s}_{\kappa,j+1,t+1} \quad = \quad & (1-\tau-\tau_l) w_{j,t} \tilde{l}_{\kappa,j,t} \\ & + (1 + r_t(1-\tau_k))\, \tilde{s}_{\kappa,j,t} \\ & + (1-\tau_l)\tilde{b}_{\kappa,j,t} \\ & + beq_{\kappa,j,t} + \Upsilon_t \end{aligned} \tag{2}$$

Consumption is taxed with τ_c.

Total time endowment is normalized to one for men and childless women. For women of type $\kappa = 1, 2, 3$ time endowment is reduced by child bearing and rearing time $\forall_j \leq 40 : \ \varphi(\kappa) > 0$. Once children reach adulthood, women in each type of the household have total time endowment normalized to one. Agents work until $j \geq \bar{J}$, when they retire.

Individual consumption for each adult in a household depends on the number of children in this household and can be defined as follows:

$$c_{\kappa,j,t} = \frac{1}{(2+\vartheta\kappa)^{\varpi}} \tilde{c}_{\kappa,j,t} = \Xi_\kappa \tilde{c}_{\kappa,j,t} \tag{3}$$

where ϑ is a scaling factor which adjusts consumption for the number of children and ϖ is a consumption scaling factor, that jointly capture scale effects (Ξ_κ) in household consumption.

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[2] With such assumption, weights for both adults may be equal or display a disparity (e.g. Gray 1998, Agarwal 1997).

The instantaneous utility function for agents in a household is defined as follows:

$$\begin{aligned}
\text{men in age} \quad j < \bar{J}: &\quad u_j(c_{\kappa,j,t}, l_{\kappa,j,t}) = \log c_{\kappa,j,t} + \phi \log(1 - l_{\kappa,j,t}) && (4)\\
\text{women in age} \quad j < 41: &\quad u_j(c_{\kappa,j,t}, l_{\kappa,j,t}) = \log c_{\kappa,j,t} + \phi \log(1 - l_{\kappa,j,t} - \varphi(\kappa)) && (5)\\
\text{women in age} \quad 41 \leq j < \bar{J}: &\quad u_j(c_{\kappa,j,t}, l_{\kappa,j,t}) = \log c_{\kappa,j,t} + \phi \log(1 - l_{\kappa,j,t}) && (6)\\
\text{men and women in age} \quad j \geq \bar{J}: &\quad u_j(c_{\kappa,j,t}, l_{\kappa,j,t}) = \log c_{\kappa,j,t} && (7)
\end{aligned}$$

2.2 Production

Individuals supply labor (time) to the firms. The amount of effective labor of age j in subcohort κ used at time t by a production firm is $L_t = \sum_{j=21}^{\bar{J}-1} \sum_{\kappa=1}^{4} N_{\kappa,j,t} \tilde{l}_{\kappa,j,t}$, where $N_{\kappa,j,t}$ is the size of a $\{\kappa, j\}$ subcohort at time t.

Perfectly competitive producers supply a composite final good with the Cobb-Douglas production function $Y_t = K_t^{\alpha}(z_t L_t)^{1-\alpha}$ that features labor augmenting exogenous technological progress denoted as $\gamma_t = z_{t+1}/z_t$. Standard maximization problem of the firm yields the return on capital $r_t = \alpha K_t^{\alpha-1}(z_t L_t)^{1-\alpha} - d$ and real wage $w_t = (1-\alpha) K_t^{\alpha} z_t^{1-\alpha} L_t^{-\alpha}$, where d denotes the depreciation rate of capital.

2.3 Pension system

We consider a pay-as-you-go defined contribution system, with a mandatory contribution rate τ. The DC pension system collects contributions and uses them to cover for contemporaneous benefits, but pays out pensions computed on the basis of accumulated contributions, as given by the equation:

$$b_{\kappa,\bar{J},t} = \frac{\sum_{s=1}^{\bar{J}_t - 1} \left[\prod_{\iota=1}^{s} (r^I_{t-j+\iota-1})\right] \tau w_{t-j+s-1} l_{\kappa,s,t-j+s-1}}{\prod_{s=\bar{J}}^{J} \pi_{s,t}} \quad \text{and} \quad r^I_t = \gamma_t \frac{w_t L_t}{w_{t-1} L_{t-1}} \tag{8}$$

where r^I_t denotes the rate of the payroll growth. The benefits are indexed annually, with the same rate of payroll growth r^I_t. In the interest of brevity, survival rates are common to men and women throughout the model. This implies that there are no survivor pensions within households. In childless households, contributions and thus pension benefits of men and women will be equal. In households with children, women work less temporarily, hence contributing less to the pension system. We denote by $\tilde{b}_{\kappa,j}$ the joint pension benefits in a household.

The DC system is by construction balanced in a sense that each cohort collects exactly the contributions accumulated. However, some individuals die before reaching the retirement age, and hence before their accumulated funds are converted to an annuity, following equation (8). Government balances the pension system.

$$subsidy_t = (1-\tau) w_t L_t - \sum_{j=\bar{J}}^{J} \sum_{\kappa=1}^{4} N_{\kappa,j,t} \tilde{b}_{\kappa,j,t} \tag{9}$$

The *subsidy*$_t$ enters directly into the government budget.

2.4 The government

The government collects taxes: τ_k on capital, τ_l on labor and τ_c on consumption, as well as a lump-sum tax/transfer Υ_t. A fixed share of GDP is spent every year on unproductive yet necessary consumption $G = g \cdot Y$. Given that the government is indebted, it also services the debt outstanding.

$$\begin{aligned} T_t &= \tau_l w_t L_t + \tau_l \sum_{j=\bar{J}}^{J} \sum_{\kappa=1}^{4} N_{\kappa,j,t} \tilde{b}_{\kappa,j,t} + \tau_c \sum_{j=21}^{J} \sum_{\kappa=1}^{4} N_{\kappa,j,t} \tilde{c}_{\kappa,j,t} + \tau_k r_t K_t + \Upsilon_t & (10) \\ T_t &= G_t + subsidy_t + r_t D_{t-1} - (D_t - D_{t-1}) & (11) \end{aligned}$$

In the initial steady state and final steady state D_t is set at 55% share in GDP and it is kept constant throughout the simulation. This value is set in concordance to the data. We calibrate Υ_t in the steady state to match the deficits and debt to maintain the long run debt/GDP ratio fixed and keep it unchanged throughout the whole path. On the transition path the values of Υ and G are held fixed *per capita* at the level from the initial steady state.

2.5 Market clearing, equilibrium and model solving

In the equilibrium the goods market clearing condition is defined as

$$\sum_{j=1}^{J} \sum_{\kappa=1}^{4} N_{\kappa,t,j} \tilde{c}_{\kappa,j,t} + G_t + K_{t+1} = Y_t + (1-d) K_t. \tag{12}$$

This equation is equivalent to stating that at each point in time the price for capital and labor would be set such that the demand for the goods from the consumers, the government and the producers would be met. This necessitates clearing in the labor and capital markets. Labor is supplied according to: $L_t = \sum_{j=21}^{\bar{J}-1} \sum_{\kappa=1}^{4} N_{\kappa,j,t} \tilde{l}_{\kappa,j,t}$ and capital accumulates according to $K_{t+1} = (1-d)K_t + \sum_{j=21}^{J} \sum_{\kappa=1}^{4} N_{\kappa,t,j} \tilde{s}_{\kappa,j,t}$, where $\tilde{s}_{\kappa,j,t}$ denotes joint private voluntary savings of the adults in the household in *per capita* terms.

An equilibrium is an allocation $\{(c_{\kappa,21,t}, ..., c_{\kappa,J,t}), (s_{\kappa,21,t}, ..., s_{\kappa,J,t}), (l_{\kappa,21,t}, ..., l_{\kappa,J,t}), K_t, Y_t, L_t\}_{t=1}^{T}$ and prices $\{w_t, r_t\}_{t=1}^{T}$ such that:

- for all $t \in [1,T]$, for all $j \in [21,J]$, for all $\kappa \in [1,4]$ $((\tilde{c}_{\kappa,j,t}, ..., \tilde{c}_{\kappa,J,t+J-j}), (\tilde{s}_{\kappa,j,t}, ..., \tilde{s}_{\kappa,J,t+J-j}),$ $(\tilde{l}_{\kappa,j,t}, ..., \tilde{l}_{\kappa,J,t+J-j})$ solves the problem of an agent at the age of j from subcohort κ in period t, given prices;
- prices are given by: $r_t = \alpha K_t^{\alpha-1}(z_t L_t)^{1-\alpha} - d$ and $w_t = (1-\alpha) K_t^{\alpha} z_t^{1-\alpha} L_t^{-\alpha}$
- government sector is balanced, i.e. equation (11) is satisfied and markets clear.

The solution procedure follows the Gauss-Seidel method. In the steady states, we start with guesses on capital which are enough to compute aggregates in the economy. Perfect foresighted households take them as given and solve their maximization problem. Aggregated variables are employed to produce a new guess in the next iteration. The procedure is repeated until the difference between the initial aggregate capital and the capital aggregated from household savings is numerical, i.e. 10^{-8}.

Along the transition path, we produce a path of guessed aggregate variables based on the results of the initial and final steady states. The solution procedure is then analogous to the one used to compute the steady states. The model is solved multiple times. First, the baseline scenario is computed keeping fertility rate constant, with the value as in the first steady state. Second, the model is solved for every simulation scenario of changes in the fertility, as described in detail in Section 3.

2.6 Measuring macroeconomic and welfare effects of fertility changes

In the baseline scenario we follow the fertility path as projected by the Central Statistical Office. In the reform scenario we consider alternative paths of fertility. To measure the welfare and macroeconomic effects of e.g. higher fertility propose the following. Define by G^R the government spending after TFR change and by G^B the baseline government spending. Then

$$\lambda_G = \frac{\sum_{t=2}^{T}\left[(G_t^R - G_t^B)\frac{\prod_{t=2}^{T}(1+\gamma_t)}{\prod_{t=2}^{T}(1+r_t)}\prod_{t=2}^{T}(1+\nu_t)\right]}{\sum_{t=2}^{T}\left[G_t^B\frac{\prod_{t=2}^{T}(1+\gamma_t)}{\prod_{t=2}^{T}(1+r_t)}\prod_{t=2}^{T}(1+\nu_t)\right]} - 1 \tag{13}$$

where ν_t is the rate of change in the number of born children, year-on-year. Indeed, TFR would have been a more intuitive measure to consider, but the indexing of TFR in equation (13) would require acknowledging that children are born to a cohort 20 years after this cohort is born.

This measure has several important advantages. First, it allows to capture the actual difference in the economy balance due to a larger or smaller number of children – once prices and the quantities adjust to the new population, the net effect of all changes exhibits in the net surplus of the government budget. It also accounts for the general equilibrium effects. Second, it may be conveniently expressed in terms of the output (GDP *per capita*), hence being intuitive measure of how much could in principle be spent on a given change in TFR. Finally, it shows the social, rather than private, gain or loss from changed fertility.

In addition to the macroeconomic measure, we also propose to use the welfare measure. Here, the unit of observation is the individual (or the household) and the metric is the utility. For each household we compute utility in the baseline scenario of fertility as in the CSO projections. We also compute utility of these households in the reform scenario of the changed fertility. The difference between the two is the net welfare for each cohort. To keep them comparable, we discount utility to $j = 21$. We convert it into a consumption equivalent, expressed as a permanent percentage change in lifetime consumption.

2.7 Calibration

The calibration replicates micro- and macroeconomic features of the Polish economy in 2014. Table 1 reports the central parameters of the model.

Production function The path of the TFP growth is constant throughout the projection. Given that Poland is a catching up economy, setting it at par with the data for the starting year would impose on the model persistently high rate of productivity growth. The 1990s and 2000s were characterized by TFP growth rate of 3.8% per year on average. According to the European Commission and AWG Aging Report, we assume the constant TFP growth of 1.4% throughout the simulation.

The capital share of income is assumed at the standard $\alpha = 0.33$ level, and the annual depreciation d rate is calibrated to replicate to yield a share of investment in GDP at the level of 21% which is the average investment rate over the past two decades.

Taxes and the government The share of government expenditure in GDP is assumed to be constant both in the steady states as well as along the transition at the level of 20%. The initial debt to GDP ratio is set at the level of 55%, corresponding to the data. We set the capital income tax rate at the *de iure* level of 19% as there are no exemptions. The labor income tax τ_l was set at 6.5%, which matches the rate of revenues from this tax in GDP (on average over the past two decades). The social security contribution rate was calibrated to replicate the resulting pensions to GDP ratio of 7%. The consumption tax rate was calibrated to match the share of VAT revenues in GDP, yielding.

Preferences The leisure preference parameter $\phi = 0.28$ was calibrated to replicate the labor market participation rate of 56.5 in 2014, reported by the World Bank. To translate household consumption into the consumption realized by the adult family members we use scaling parameters. We choose $\vartheta(\kappa)$ and ϖ parameters in order to replicate the equivalence scale used. Note that even in childless households, there are scale effects of the second adult. Using the formula from equation (3), we find the parameters $\vartheta(\kappa)$ and ϖ to replicate the values of equivalence scaling factors, which yields the value of $\varpi = 0.51$ and a vector of $\vartheta(\kappa) = \{0.31, 0.27, 0.23\}$. These values are consistent with values assumed by in the earlier literature (e.g. Fehr et al. 2017).

We calibrate the time preference parameter to replicate the interest rate of app. 6.5% in the initial steady state. While this value may seem high, note that it applies to the economy still in transition with high TFP growth. Nishiyama and Smetters (2007) find the corresponding interest rate of 6.2% for the US.[3] With economy converging towards lower TFP, interest rate also converges to substantially lower levels.

Mortality We start from the demographic projections of the Eurostat. The age structure of the mortality rates is recovered from the size of each birth cohort in subsequent years. The projection is time varying, which allows to capture longevity. To avoid the problem of survivor pensions and consistent with the lack of intentional bequests, men and women have equal mortality rates.

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[3] The average real annual rate of return at the level of 7.5% was achieved by the open pension funds with a balanced portfolio strategy in the period 1999-2009.

Since in our model, for the purpose of tractability, parents cannot die until they complete child rearing, we adapt the initial age structure of the mortality rates. We fix the mortality rates to be zero until the age of 40. To minimize the departure from the actual projected population structure, we raise the mortality rates at $j > 40$ to compensate for the lack of mortality until that age. The additional mortality is spread equally across all ages, see Figure A2.

Table 1: Economic parameters calibration

Macroeconomic parameters		Calibration	Target	Data	Model
ϕ	preference for leisure	0.28	average hours	56.8%	56.8%
δ	discounting rate	0.97	interest rate	6.5%	6.5%
d	one year depreciation rate	0.14	investment rate	21%	21%
τ_l	labor tax	0.065	revenue as % of GDP	4.6%	4.5%
τ_c	consumption tax	0.18	revenue as % of GDP	11.2%	11%
τ	social security contr.	0.12	benefits as % of GDP	6.78%	7%
Preference parameters					
ϖ	consumption scaling factor	0.51	equivalence scale	0.7	
$\vartheta(\kappa)$	children scaling factor	$\{0.31, 0.27, 0.23\}$	equivalence scale	$\{0.65, 0.62, 0.6\}$	

Note: OECD equivalence scales adopted. Average hours based on OECD, averaged for 2000-2010, computed as share of hours worked in the economy over the hours available 16 hours a day, 250 days a year. Interest rate calibrated to the real net rate of return reported by the investment rate, averaged over 2000-2010. Investment rate based on national accounts data, averaged over the analogous period. The labor tax, consumption tax and social security contribution calibration matched to national accounts, averaged for 2015-2005.

3 Demographics

In the model, the initial steady state is calibrated to the data. We describe below procedures employed to obtain the initial population structure. Subsequently, we utilize the mortality rates as projected by the Central Statistical Office until 2060 and stable thereafter. We also use the projected fertility as our baseline scenario. Fertility projections by the CSO are in terms of total fertility rate, whereas our model structure operates with the changes in the size of the birth cohort. We explain below the procedure to replicate the demographic assumptions in our model.

Our model starts from the number of children born in every year. Applying the mortality rates, the model is able to reconstruct size of each age group in every year. Hence, knowing the current size of each age group and the observed mortality rates in the past, we could have obtained the size of each birth cohort. However, detailed mortality data are not readily available beyond 1990. Moreover, the legacy of the World War II, 1968 and subsequent waves of emigration and return migration confound the regular, age specific mortality patterns with other population flows. Moreover, household structure in terms of the number of children is only available for the recent years. Prior to 1990, the data may be recovered from the census, hence at low frequency. Given these constraints, we simplify the population in the initial steady state in the following manner.

Table 2: Calibration of the population in the initial steady state

	Data	Model
TFR	1.38-1.52	1.44
Share of cohorts at $j < 21$	0.23	0.23
Share of cohorts at $20 < j < 41$	0.31	0.30
Share of cohorts at $j \geq \bar{J}$	0.18	0.19
Life expectancy at $j = 1$	73.47	73.83
Life expectancy at $j = \bar{J}$	15.41	15.42
Proportion of childless women	0.36	0.35
Proportion of women with one child	0.16	0.16
$s_1 : s_2 : s_{3+}$	0.16 : 0.28 : 0.2	0.16 : 0.29 : 0.2

Note: Data on TFR and proportions relate to completed fertility (measured as realized fertility for women aged 45 years or older, data averaged over 2006-2014. Shares of age groups based on population structure data, averaged over 2006-2014. Data from Eurostat.

The initial steady state We obtain the fertility data for the period 1964-2014. We obtain the age-specific mortality rates for the period 1964-2014. We compute the average fertility for the years 2006-2014 and average age-specific mortality rates for the same period. We adjust these measures to reflect the constraints on $\pi_{j,t}$ as described earlier, notably in our model individuals do not die until the age of $j = 41$, which is the age their children are raised. Subsequently, we take the simplifying assumption that this was the average fertility rate that generated the population in the initial steady state. We obtain the data about the household structure in terms of the number of children for the period 2006-2014. We define $s_i \; \forall_i \in \{0, 1, 2, 3+\}$ to denote the share of households with zero, one, two and three or more children, respectively. We compute these averages in the data, based on complete fertility measures. We allocate children to households based on these averages.

Note that the population is not stationary in the initial steady state. Neither is there a replacement of subsequent cohorts. In fact, we calibrate the model to reflect the $TFR = 1.44$, see Figure A1. Table 2 summarizes the assumptions in the model and the fit between the model and the actual population structure.

Fertility projections The fertility projections are published in terms of total fertility rate, which is a year specific measure of the number of children born in a given period relative to women within certain age brackets. In our model, the population is obtained using the completed fertility. Hence, we utilize the age specific fertility from the data to recalculate the completed fertility from the total fertility rates for the initial year of our simulation. We assume that the age specific fertility patterns will not change to the extent to which they define a relationship between the total fertility and the completed fertility. This assumption may easily be relaxed in our setup, depending on the scenario of interest.

Once the projection is reformulated in terms of completed fertility rather than TFR, it is straight forward to obtain the size of each birth cohort at time t, born to mothers born at time $t - 20$. This is the measure of population growth we utilize in our model.

Simulation scenarios We depict two scenarios: the baseline scenario and the reform scenario. In the baseline scenario we utilize the demographic projection provided by the Central Statistical Office of Poland. This projection provides the number of births until 2060. We convert the number of births to the fertility rate in our model, which is the equivalent of the completed fertility in the projections. We assume that the economy converges to the last value in the projection in the long run. This assumption is only needed for computational purposes (in order to obtain a transition path, we need to provide an end point of this path).

In the reform scenario we allow the fertility in the model to deviate from the baseline scenario of the demographic projection. Implicitly, we assume that this deviation is a consequence of some unmodeled change in family related policy. We thus calibrate the scope of the deviation to the values available in the empirical literature. Frejka and Gietel-Basten (2016) provide an overview of the fertility and family policies in Poland, as well as the rest of the Central and Eastern Europe, arguing that the decline in fertility observed in these countries is partly a fault of deficient welfare state, failing to provide necessary public services. Hence, it appears plausible that with a substantial change in policy some increase in fertility is possible. In the context of some countries, the changes of fertility in the advent of family policies were occasionally high. For example, generous policies in Russia implied a 30% spike of TFR in only few years (Frejka and Zakharov 2013). Some positive effects of pro-natalistic policies were also observed in Australia (Sinclair et al. 2012), Austria (Lalive and Zweimueller 2009) and Norway (Rindfuss et al. 2010). By contrast in Canada (Milligan 2005) and Argentina (Garganta et al. 2017) costly policies observed almost no change in TFR. Moreover, the negative effect of reverting the policy is stronger than the positive effect of introducing it (Milligan 2005, Lalive and Zweimueller 2009).

Taking these empirical insights as a starting point, we consider two possible dimensions to analyze in our model: size of change in fertility and prevalence of this change. We consider small and large change in fertility, 30% and 60%, respectively. On top of the size, we consider the case when the change is transitory and permanent. The transitory case observes an increase in fertility over 20 years, with a gradual return to the initial steady state. The final steady state is equivalent to the baseline scenario in the long run. The permanent case will observe a permanent shift in fertility, i.e. the final steady state too will have a higher fertility than the baseline scenario.

The proportion of children to be born to households of type κ The literature has emphasized the relevance of the intensive margin adjustment and small effects for the extensive margin. For example in Germany, the provision of child care facilities had noticeable effects for families with children, who increased fertility, but no effects for families without children (Bauernschuster et al. 2015). Indeed, for any change in the fertility, there is an infinite number of combinations between the types of families that yields that change in fertility. Holding the share of household types constant from the initial steady state one cannot achieve a change in fertility, hence the combination of households with zero, one and two or more children has to change even in the baseline scenario. Notably, given the setup of our model, any change in the composition of households is likely to have macroeconomic and welfare effects of its own.

Given this feature of our approach, we develop the following way to avoid arbitrary choices. First, we develop simulation scenarios, which are distinguished marginally by the changes in the shares of household with a given number of children. These scenarios are generated randomly from a distribution. To be completed.

There is a rich body of literature on childlessness, arguing that trends in childlessness are even less persistent and predictable than the trends in fertility in the short and medium run, whereas in the long run the share of household with no children has been increasing over the past several decades and may continue to do so (e.g. a volume by Inhorn and Van Balen 2002). Moreover, some policies may address specifically the "first child" (e.g. first child bonus in Australia), whereas others may encourage families to have more children (e.g. child income support for families with two children or more in Poland). Given the lack of clear empirical suggestions and a potential policy relevance, we will adjust the share s_i in the scenarios in the following manner. We will hold constant one of the two parameters: (i) share of childless couples s_0; or (ii) the proportion between households with 3+, 2 and 1 child $s_1 : s_2 : s_{3+}$. For the former, increase in fertility will imply a higher number of children in families with children, i.e. intensive margin adjustment. For the latter, we permit also some extensive margin adjustment.

To be completed.

4 Results

To be completed.

5 Sensitivity analysis

To be completed.

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A Appendix

Figure A1: Completed fertility in data and in model

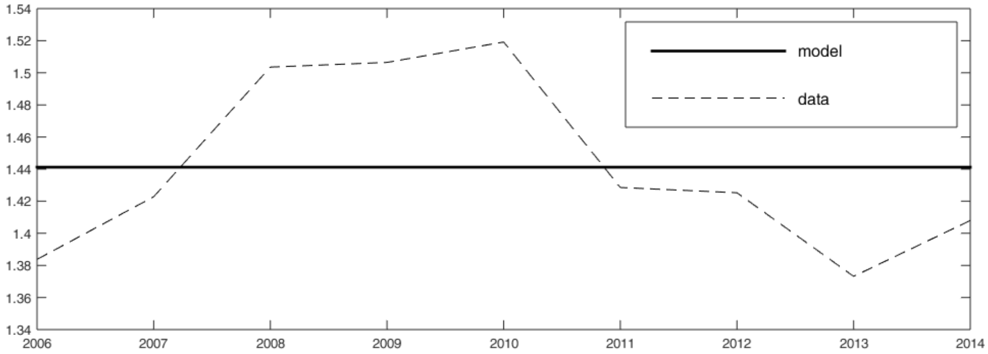

Figure A2: Survival rates comparison between data and model