Replicating Intergenerational Longevity Risk Sharing in Collective Defined Contribution Pension Plans using Financial Markets

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Abstract

Intergenerational risk sharing is often seen as a strong point of the Dutch pension system. The ability to absorb financial and actuarial shocks through the funding ratio allows for the smoothing of returns over generations. Nevertheless, it implicitly means that generations subsidize each other, which has its disadvantages, especially in the light of incomplete contracts and situations of hard regulation constraints. This paper highlights the advantages of intergenerational risk sharing as a main characteristic in certain collective pension plans, investigating if and how much of this can be replicated by individual participation in the market. Using a stylized model based on different pension plans such as "hard"/"soft" defined benefit, collective/"pure" defined contribution, this paper identifies the effects of an increase in life-expectancy as one of the most important demographic shocks. The existence of regulatory constraints modifies agents' behavior so that they tend to choose individual investment to ensure their retirement savings. In the absence of regulatory constraints, individual investment under-performs and highly replicates pension fund performance. Thus, choosing collective participation is more rational. Moreover, as the effect of the shock is decomposed, a discussion of the absorption heterogeneity by different plans is presented.

Keywords: pension plans, individual investment, intergeneration risk-sharing, longevity.

JEL Classification: H55, J26.

*Corresponding Author: E-mail address: enareta.kurtbegu@univ-lemans.fr NOTE: This study started during my visiting period in the Research and Innovation Department at CARDANO Risk Management, Rotterdam, Netherlands. The author is responsible for any errors or omissions in this paper.

1 Introduction

Diamond (1977), Gordon and Varian (1988), Ball and Mankiw (2007) and Gollier (2008) among others, theoretically showed that the inability of the current generations to share risk with those who are not yet born makes markets inefficient. Therefore, any absence of intergenerational risk sharing implies that workers face more uncertain future pension incomes. The markets' inability to efficiently allocate risk across generations has been used to argue in favor of more public interventions such as introducing sophisticated pension schemes and an appropriate use of financial instruments. Cui et al. (2008) showed that in the collective pension contract, although the pension system participation is *ex-ante* considered a zero-sum game, there exist welfare enhancing features related to the intergenerational risk sharing not only in the government pay-as-you-go ($PAYG$) but also in the funded plans.

The original defined benefit (DB) schemes completed the market for the employees by offering lifelong stable real cash flows in retirement. The Netherlands is one country which no longer provides this "hard" guaranteed pension benefit based on DB plans. However, it offers a defined contribution (DC) system that uses a DB accounting framework[1]. In contrast, most countries have opted for redistribution in the first pillar, clearing ownership rights as well as *ex-ante* fair risk sharing in the second pillar. More precisely, the Dutch pension system consists in a residence based universal first pillar, a quasi-mandatory funded second pillar (mandatory except for some specific industries) and a voluntary third pillar. The sustainability issues did not disappear despite the continuous pension reforms the country went through. The recent challenge consists in consequent attempts to improve the matching process between assets and liabilities in the second pillar. Classical asset-liability management theory shows that the more risk you take, the higher the expected return provided and the more volatile the funding ratio.

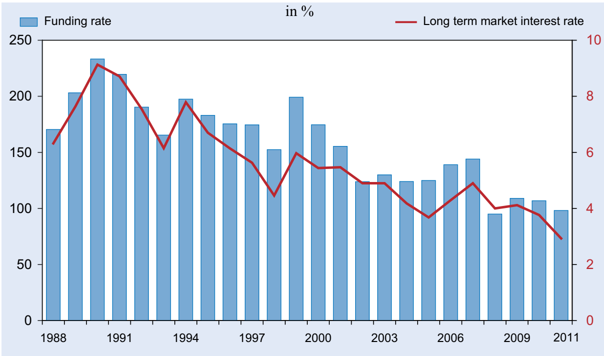
Academic studies point out the enlarging welfare potential of the Dutch pension funds, attributed to intergenerational risk sharing. It allows pension funds to take more risk in asset allocation and provides smooth consumption by stabilizing the contribution rates and pension payouts. The ability to absorb financial and actuarial shocks through the funding ratio allows for the smoothing of asset returns over generations.

The funding ratio of Dutch pension funds reached its peak at the end of the 1990s followed by a sharp drop in pension funding during the "dotcom" crisis. The Dutch government imposed supplementary funding requirements in 2002 in order to reduce the risk absorption. The funding ratio slowly recovered from the lowest levels in 2003 but fell dramatically during the global financial crisis (2008) attaining the lowest level for a high

[1] The collective defined contribution (CDC) scheme is halfway between a DB and a DC scheme. Contrary to the DB scheme, returns are not guaranteed. Unlike pure DC schemes, contributions of all cohorts are not individually but collectively invested in the market.

number of pension funds.

Figure 1: Funding Ratio and Interest Rate Evolution in The Netherlands



Source: Statistics Nederlands; De Nederlandsche Bank; Provided by: Broeders and Ponds (2012).

As a consequence, the level of trust in these *CDC* pension plans has decreased and social support for intergenerational risk sharing is not as strong as it used to be (Figure 1). Current regulation allows pension funds to cut benefits and pension-in-payments to restore solvency levels, in the case of under-funding. It is important to note that participation in a specific pension fund is still mandatory for the employee. Currently, there is a debate in the Netherlands on a new pension deal which is even more *DC* like. The pension age will be linked to systematic longevity and there will be a ceiling on the contribution level. Associated with an increase in strict constraints by the regulatory entity, this research study should be viewed within the perspective of the proposed changes to the Dutch pension system. It consists in measuring the resilient constraints implemented by the regulator and determining the impact of a continuous life-expectancy increase on the current fragile sustainability of pensions.

Focusing on studying the employer-based supplementary schemes (Pillar II), one can ask, what would happen to the support for the intergenerational risk sharing model when some of the actuarial variables do not follow a random pattern, but a trend instead? There have been several demographic changes over the last 80 years. In 1932, the average life-expectancy in the Netherlands was 64 years, while today it is 18 years higher. Fertility has decreased and not only does the average woman give birth to fewer children but she gives birth to her first child later in life. More young people today focus on getting a higher education which leads to a reduced number of years in working life. Furthermore, there is a long-term trend to earlier retirement in many countries while evidence shows that this does not induce a parallel decline in unemployment rates. Given these biometrical and societal developments, one can postulate that what we are facing is not just random shocks but social and demographic trends. Therefore, one may wonder how all these developments will affect the fairness of the current pension contracts with respect to intergenerational risk sharing (*resp.* transfers).

Recently there has been a decrease in trust in pension funds and a growing interest in the design of pension annuities, which insure against idiosyncratic longevity risks while pooling and sharing systematic risks. Because of the relatively high price for life insurance, demand still remains low. Hence, on the one hand, the literature shows that pooled funds have an advantage over life annuities (Maurer et al. (2013)) especially in pooling the life-expectancy risk on a macro level. On the other hand, a life annuity is an optimal choice for agents to share the longevity risk on the micro level. If individuals are free to choose their pension savings strategy, is it collective fund participation or individual investment that they would choose? In order to measure the value of intergeneration risk sharing this study compares the collective fund and its individualistic equivalent.

Given that the "hard" promise is no longer part of the Dutch second pillar which remains a mandatory, privately managed pillar, it is important to investigate and measure its uniqueness in providing intergenerational risk sharing. Hence, one could ask: How much of the remaining intergenerational risk sharing in the CDC can be solved by the markets? In other words, the interest of this study is to identify precisely what happens in the CDC pension schemes in terms of the remaining intergenerational risk sharing. How much of this risk sharing is unique and how much can be replicated by the markets? How can one make the pension deal fair for the younger generations and still retain some intergenerational risk sharing? If nowadays intergenerational risk sharing is no longer considered the strength of the Dutch pension system, could mandatory participation be considered as a necessary condition for youth participation or are there still incentives to do so?

The current CDC pension plans could be described as a "black box" in which redistribution takes place, but it is not really clear what happens inside. Providing answers to these questions would lead to a better understanding of the collective pension contract real value. This paper is based on the stylized pension contract methodology. Diverse contracts are constructed based on different pension plans such as the "hard" DB plan, the conditional "soft" DB plan, the CDC plan and the individual plan ("pure" DC).

The paper is organized as follows: the next section describes the data. Section 3 presents the methodology used to measure the effect of shocks on risk sharing among generations. Finally, we discuss the main results concerning the "normal" and "shock" models.

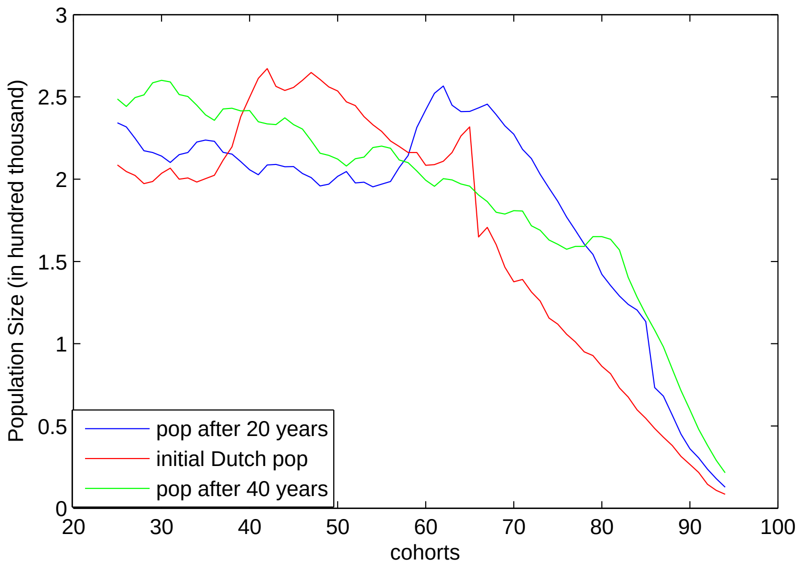
2 Data Description

This empirical study is based on simulated stylized contracts. To model each pension fund, we use real population data, financial market simulated variables and generated possible scenarios. The number of assumptions is minimized where the implemented parameters depend on the contract characteristics and represent a *proxy* of the reality.

2.1 Population characteristics

We focus our study on an open fund approach, which consists of an infinitely lived instrument with repeated loops of 70 years. Individuals' participation in the pension fund starts at age 25. They contribute for 40 years, start getting benefits at age of 65 and definitively lose the retirement benefits at the maximum age of 95 years. Therefore, since an individual can participate in the fund at most for 70 years, at any period in time there are 70 co-existing generations. The population distribution is based on the real Dutch data for the year 2012, provided by the *CBS* (Centraal Bureau voor de Statistiek). Its structure *per* cohort is a hump shape function (Figure 2). The uncertainty in agents' lives is presented in Figure 3, by the survival probability. Moreover, the population is updated each year with new entries on the labor market where the actual birth rate growth (with a time lag of 25 years) is used as a *proxy*.

Figure 2: Population Structure *per* Cohort in Time



Source: The number of individuals per cohort at the initial time $t = 1$ (in blue), $t = 20$ (in red) and $t = 40$ (in green).

The change in population is a key issue for the pension fund since it greatly affects the management of the asset-liability balance. First, we deduce the survival probabilities $(p(x,i))$ of each cohort using the Dutch mortality rate $(q_{x,t})$ on the 2011 and 2012 population:

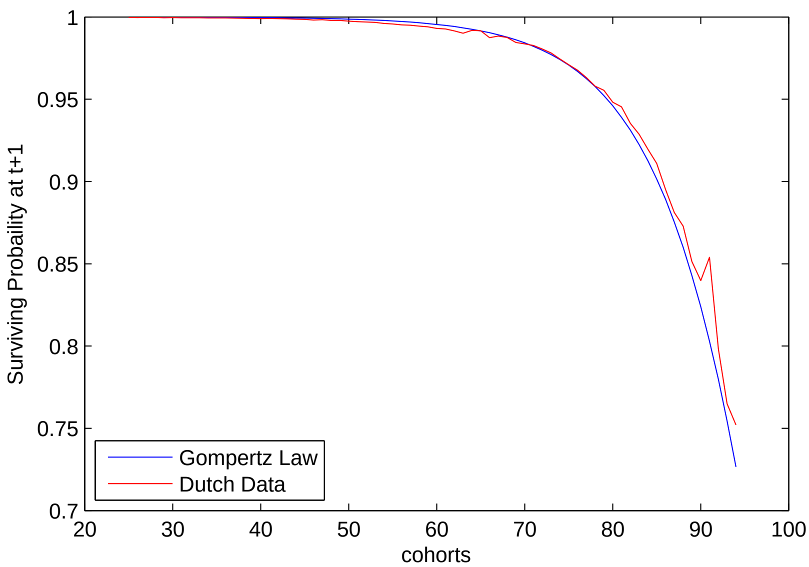
$$p(x,i) = \prod_{j=0}^{j=i-1} (1 - q_{x+j})$$

Secondly, we use the Gompertz (1825) model to *proxy* the population structure. This allows us to have a dynamic population model in which one could reproduce projections of the population considering shocks on different predefined parameters (*e.g.* life-expectancy). The Gompertz Law states that over a large part of the age range (excluding infancy and youth or very old age) the force of mortality increases with age at a steady exponential rate. Therefore, assuming that the mortality rates increase not only with age but also in time by the same amount every year, the Gompertz Law is written as follows:

$$ln(p(x,t)) = \left(1 - e^{t/b}\right) \times e^{(x-m)/b}$$

where, t denotes the survival period, x the current age of the individual, m the modal age at death and b the depression coefficient of the age at death. Parameters $(b = 8, m = 87)$ are calibrated based on the initial mortality table.

Figure 3: Conditional Surviving Probability (lag 1)



Source: The real Dutch data (red line) and the Gompertz Law (blue line); estimated parameters: $m = 87$, $b = 8$.

In a time lag of one year, there is no significant difference in the real surviving population and the one used as a *proxy*. Therefore, we can use the Gompertz Law to generate the population survival probability.

The baseline scenario assumes that the table of conditional survival probabilities is deterministic and constant in time. The survival probability matrix is constructed as follows:

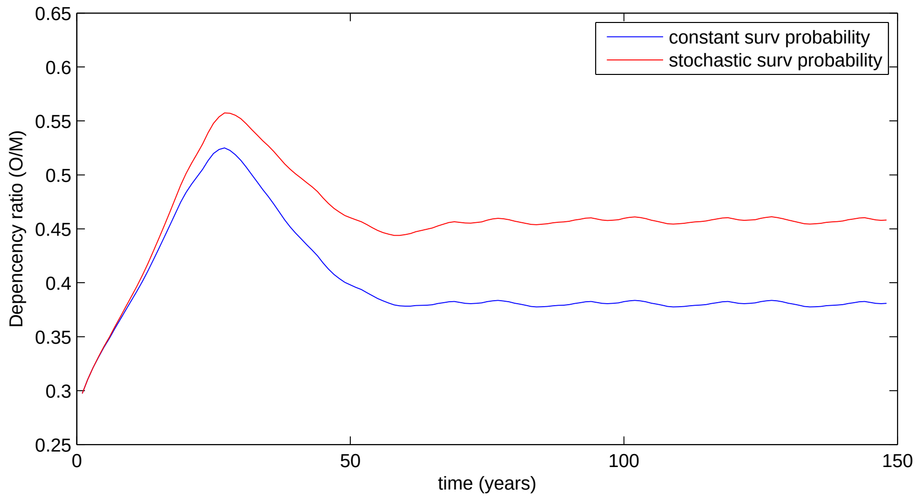
$$p^x(t+i|t) = p^x(t+i-1|t) \times p^x(t+i|t+i-1)$$

where the $p^x(t+1|t)$ represents the probability that the representative agent of cohort x would survive at time $t+1$ knowing that he was alive as an individual of cohort $x-1$ at time t.

The dependency[2] ratio (DR) of the Dutch population is 29.74%. Thus, because of a higher flux of the working force than retired people, the support[3] ratio is higher than one. As a *proxy* for the birth rate growth, we use the 25 years history of population growth already provided by the historical population data of 2012 for the cohorts zero to 25 years old[4]. To avoid the assumptions related to projections, we repeat these 25-year birth rates to provide a history of 150 years.

During the first 26 years, the dependency ratio follows a positive trend ("population aging") followed by a "youthing" population during the next 23 years (Figure 4). Although stability is reached after 50 years and the population is characterized by a dependency ratio of 38%, it still remains an older population than the one of 2012.

Figure 4: Dependency Ratio Dynamics



Source: The dependency ratio in time; Period: baseline (in blue) *vs.* "life-expectancy shock" (in red).

The population structure is a source of shock variables. In the baseline scenario, the standard deviation of the birth rate growth for the first 20 years is 1.74% and during the

[2]The aged dependency ratio is considered as:

$$DR = \frac{number\ of\ Old\ people}{number\ of\ Middle\ aged\ people}$$

[3]The support ratio presents the inverse of the dependency ratio.

[4]We assume that mortality for these cohorts during the next 25 years is negligible.

first 40 years, it is 1.79% (Table I).

Table I: Descriptive Statistics of Population Evolution

	Dependency ratio		**Mean birth rate growth**	
	After 20 years	After 40 years	During 20 years	During 40 years
Positive stoch. birth rate	49.16%	44.08%	0.58%	0.44%
Stable population	29.74%	29.74%	0%	0%

Source: The dependency ratio and the mean birth rate during the first 20 and 40 years.

2.2 Financial market characteristics

The asset-liability management model is based on Monte Carlo simulations of 1000 possible future economic scenarios for a period of 150 years. Although in real life the duration of a specific unchanged pension contract is shorter than 150 years (the pension system is often reformed), in this study, a long historic dataset was used because of information loss caused by the construction of forward-looking indicators.

The simulation is based on fixed inflation dynamics. Thus, there is no uncertainty related to price inflation (fixed at 2% *per* year) and wage inflation (fixed at 3% *per* year). The term structure is defined by Vasicek (1977)'s one-factor model, where the interest rate is derived from only one source of risk (market risk). It is known as one of the earliest non-arbitrage models of interest rates based on a mean reverting mechanism and its stochastic differential equation is given as follows:

$$dr_t = \kappa_r \left(\mu_r - r_t\right) dt + \sigma_r dW_t;$$

where W_t is a standard Wiener process under the risk-neutral framework, σ_r is the standard deviation parameter characterizing the amplitude of the instantaneous random inflow. The speed of adjustment of the interest rate (reversion) towards its long-run normal level is $\kappa_r = 0.05$, the long-term mean is $\mu_r = 0.03$ and the instantaneous volatility is $\sigma_r = 0.05$. The longer the time to maturity, the higher the interest rate. Moreover, it remains constant for a maturity above 30 years.

The return on the bank account represents the short-term stochastic risk-free instrument ($R_0^f = 1$ and $\mu(r_f) = 2.81\%$). The expected risk-free return increases in time during the first 60 years and then stabilizes at 3.1%.

The financial market is composed of two financial equities, a bond and a stock. To keep the model simple, the financial system is exogenous and is not contagious on other parameters such as the demographic structure or the learning process. Bond returns are deduced from the term-structure model. It was assumed that the fund buys the bond of maturity 6 years at the beginning of time t paying its price at maturity 6 years at t, sells it at the end of time t under the price of a bond at maturity 5 years at t, and re-buys bonds at maturity, 6 years at the beginning of time $t+1$ at a price of a bond at maturity 6 years at $t+1$.

Therefore, the bond return (r_b) is calculated as follows:

$$r_b = \frac{\left(1+r_t^{6Y}\right)^6}{\left(1+r_{t+1}^{5Y}\right)^5} - 1$$

As it concerns stock simulations, the Black and Scholes' model (Black and Scholes (1973)) is used to generate equity return scenarios with a stochastic short rate. The parameters for the simulation are volatility $\sigma_s = 0.2$, the risk-premium $\lambda = \mu_s - r_f = 0.04$ and no correlation $\rho(r, s) = 0$. The optimal portfolio is actually a very important issue in the pension industry where high liquidity, low risk and high returns represent a big challenge. Moreover, the business cycle and the demographic structure of the system justify its dynamic behavior. However, the optimal portfolio is not within the scope of this paper. Here, two constant investment strategies are implemented. On the one hand, relying on the Dutch pension fund characteristics, we consider its investment strategy being a static "constant-mix" (50% bonds and 50% stocks) unconditional on the actual fund performance (Lekniute (2011)). Some robustness checks are provided in Section 4.4. On the other hand, the individual investment strategy is based on "age-dependent" investment. Thus, based on the simulated scenarios, one could resume the risky market with the following bond, stock and "constant-mix" characteristics (Table II):

Table II: The Descriptive Statistics of the Financial Market

	Mean	**Volatility**
Bond	3.21%	5.10%
Stock	7.01%	21.72%
constant-mix (50%-50%)	5.11%	11.38%

Source: The calculations are based on 150 years historic data.

3 Methodology

This study presents the detailed structure of each pension contract, including both collective participation and individual investment. Each representative agent is considered to obtain wage flow w during his 40 years of working life. The initial remuneration level is normalized $w_{x,t=1} = 1$ for all generations $x \in [25:64]$. It evolves in time homogeneously for each existing cohort $w_{x,t} = w_{x-1,t-1} \cdot g_t$, for $x \in [25:64]$ where g_t is the wage growth. In this model, the rate of wage growth is considered constant in time and could be broken down into the inflation and the real wage growth component. This study excludes the stochastic dimension and considers constant inflation growth to avoid all other sources that would affect the complexity of the model. Since the aim of the paper is to capture only the effect of demographic shocks, it tries to eliminate macroeconomic business cycles and other financial effects provided by the optimality of investment portfolios. Nevertheless, differences in experience, depreciation of knowledge or age-related trends in physical and mental capabilities of a representative worker's productivity could systematically differ over his active lifetime. This case was omitted[5] since it does not influence the results of this study.

The population data correspond to the active population, since unemployment is supposed to be neglected. The contribution level that the active population should pay depends on the type of pension contract they signed up for. Full-time contributions are supposed to be uniform across generations but variable in time and dependent on the state of nature.

Considering the Netherlands reform of 2003, the plan's benefit distribution rule has been transformed from final salary to average salary. Moreover, each contributing year translates to an accrual rate of $\epsilon = 2\%$ of the average wage. Thus, the representative agent accrues his pension rights corresponding to 80% of the average wage indexed to inflation. The agents make it possible to get their accrued benefit paid during 40 past years translated to the real value in the year of their retirement. In addition, the pension benefit is considered as being contagious to the inflation indexation during the retirement years. Pension rights are indexed to inflation while the pension benefit indexation can be full or partial, conditional on the pension fund's performance.

The pension plans are modeled as being Projected Benefit Obligations where liabilities are calculated as the present value of claims about the accrued benefits. The fund is initially considered balanced ($FR_0 = 1$). As long as, the accrued benefits *per* cohort are the same, the discount elements for each cohort can be summed up to determine the discount element for a given cohort.

$$D^x_{t,s} = \sum_{i=\max(65-x,0)}^{95-x} \frac{p^x(i|t)}{\left(R^{(i)}_{t,s}\right)^i}$$

where $R^{(i)}_{t,s}$ is the yield to maturity i at time t in scenario s, and $p_x(i|t)$ is the survival probability of cohort x, i years later conditional on the fact that he/she is alive at time t. The discounting coefficient is a hump shape function of age. The present value of accrued benefits' claims is higher for the middle-aged cohorts because they have just started collecting benefits or will start doing so. The youngest cohorts are those who have not contributed much to the pension plan yet and they expect to receive the payments quite late in time. The oldest cohorts are the ones who do not have more benefits left to receive and the survival probabilities are quite low, that is why the present value of accrued

[5]Results based on the Mincer Wage Profile can be provided upon request.

benefit is low. In line with the existing literature, we calculate target liability, as in Cui et al. (2008). In general, the impossibility of forward looking for several variables such as inflation as well as pension benefit which is indexed on inflation, makes it impossible to deduce the exact future accrued benefits.

$$L_{t,s} = \sum_{x=25}^{94} \epsilon \times \min(x-24;40) \times \bar{w}_{x,t} \times (1+\pi_{t,s})^{\tau}$$

where $\bar{w}_{x,t}$ is the average working age, $\pi_{t,s}$ inflation and τ the time at which the cohort x was 64 years old.

The assets are calculated as the remuneration of a "constant-mix" investment strategy (R_t^{inv}) of net asset balance.

$$A_{t,s} = \left(A_{t-1,s} + \sum_{x=25}^{64} C_{t,s}^{x} \cdot Pop_t^x - \sum_{x=65}^{94} B_{t,s}^{x} \cdot Pop_t^x \right) \cdot R_t^{inv}$$

At the end of each year, despite the determination of the funding ratio FR_t, variables such as population, survival probabilities, wages and price inflation level are updated.

3.1 The Characteristics of Collective Pension Contracts

We will focus on three distinct collective pension contracts. They differ in the rules for both the collection of contributions (variable/fixed) and the distribution of benefits (fixed/variable). Pension funds are highly regulated entities. The constraints on funding performance greatly affect the funds' strategy in distributing benefits or collecting contributions. It has recently become one of the main issues for the Dutch pension system. Because of the consequences to the pension plan's participants, to avoid these extreme events the extra policy safety constraints (PSC) and two distinguish frameworks were implemented.

- If $FR < FR_{forbidden}$: no pension benefit is paid out directly by the plan to its individuals. However, during this year, the pension paid to the individuals is only 80% of the final wage and is issued not by the fund itself but by the insurance company covering the pension fund.

- If $FR > FFR_{max}$: no contributions (or to a c_{bound} level) are collected by the working individuals during that year and the pension fund redistributes the excess of funding ratio to the $FR_{surplus}$. The smoothing factor is considered $\theta = 10\%$ which stays in line with the supervisory recommendation.

$$redistrib_{t,s} = \begin{cases} 1 & if\ FR_{forbidden} < FR_{t-1,s} < FR_{max} \\ 1 + (FR_{t-1,s} - FR_{surplus}) \times \theta & if\ FR_{t-1,s} \geq FR_{max}. \end{cases}$$

Contrary to the baseline model, the introduction of the PSC allows for cuts and surplus redistribution (Table III).

Table III: Summary of Pension Plan Details

	Contribution	**Benefit 80%**	**Indexation**	**PSC cuts**
Plan DB-hard	Variable	Promise	Full	No & Yes
Plan DB-soft	Variable	Promise	Ladder	No & Yes
Plan CDC	Fixed	Promise	Ladder	No & Yes
Plan pure DC	Fix/Var	Unsure	No	No

Source: information on cuts/surplus related to regulatory rules.

3.1.1 Variable Contribution

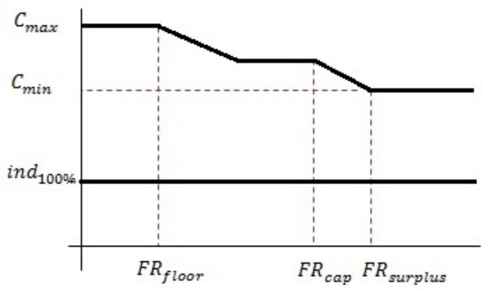
The variable contribution on a collective pension contract coincides with the Defined Benefit (DB) pension plans (conditional/unconditional indexation).

Plan DB**-"hard"** : This contract consists of a DB scheme with full pension promise where full indexation is provided regardless of the funding performance (Figure 5). Its unsustainability under demographic shocks, among others, has limited its usage.

The contribution rate ($c_{t,s}$) is homogenous among workers. Nevertheless, there is heterogeneity in time for price inflation and the lifetime average wage.

$$c_{t,s} = \begin{cases} c_{max} & if\ FR_{t-1,s} < FR_{floor} \\ c_{max} - \frac{FR_{t-1,s} - FR_{floor}}{FR_{cap} - FR_{floor}} (c_{max} - c_{min}) & if\ FR_{floor} \leq FR_{t-1,s} \leq \frac{FR_{floor} + FR_{cap}}{2} \\ \frac{c_{min} + c_{max}}{2} & if\ \frac{FR_{floor} + FR_{cap}}{2} < FR_{t-1,s} < FR_{cap} \\ \frac{c_{min} + c_{max}}{2} - \frac{FR_{t-1,s} - FR_{cap}}{FR_{surplus} - FR_{cap}} \left(\frac{c_{max} - c_{min}}{2}\right) & if\ FR_{cap} \leq FR_{t-1,s} \leq FR_{surplus} \\ c_{min} & if\ FR_{t-1,s} > FR_{surplus} \end{cases}$$

Figure 5: DB-"hard" Pension Contract Characteristics



Source: full unconditional indexation and adjusted contribution level.

The pension benefit indexation is:

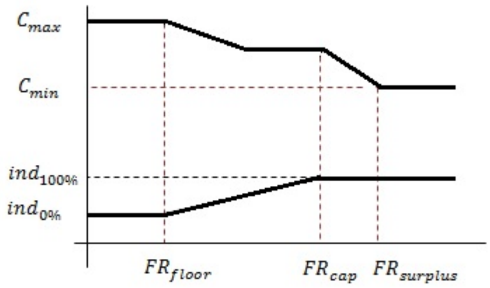
$$index_{t,s} = 100\%, \ \forall t \in [1:T] \ \ \forall s \in [1:S]$$

Plan 1: ***DB*-"soft"** : The attempts to increase the system's sustainability and to avoid continuous divergence of fund performance incited many countries to reform their pension plans. The *DB*-"soft" pension contract consists of a defined benefit where the pension rights are kept as promised but the indexation is contagious to the fund performance (Figure 6).

$$index_{t,s} = \begin{cases} 0 & if \ FR_{t-1,s} < FR_{floor} \\ \frac{FR_{t-1,s} - FR_{floor}}{FR_{cap} - FR_{floor}} & if \ FR_{floor} \leq FR_{t-1,s} \leq FR_{cap} \\ 1 & if \ FR_{t-1,s} > FR_{cap} \end{cases}$$

This ladder policy was introduced in the Netherlands in 2005 (Ponds and van Riel (2007)). There will be no indexation if the fund's funding ratio is under-performing (below FR_{floor}), while there will be full benefit indexation if the fund is over-performing (above FR_{cap}). Nevertheless indexation remains non negative, and there is no possibility for surplus redistribution.

Figure 6: *DB*-"soft" Pension Contract Characteristics



Source: both adjusted conditional indexation and contribution level.

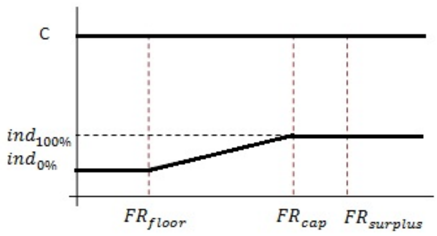
3.1.2 Fixed contribution

There exist several derivative plans related to the fixed contribution characteristic. We consider the collective defined contribution (CDC) plan as a "hybrid" contract where contributions are predefined and kept constant in time.

Plan ***CDC*** : The *CDC* contract stands in between the *DB* and the *DC*. It inherits from the *DB* pension contract the pension benefit distribution, which is promised but contagious to the fund performance. Moreover, the benefit indexation is represented by the same ladder equation as in the DB-"soft" pension plan (Figure 7). The contribution level equalizes all contributions present value to the present value of accrued benefits

$$c_{t=1,s} \times \sum_{x=25}^{64} w_{t=1}^{x} \times Pop_{t=1}^{x} = \epsilon \times \sum_{x=25}^{64} D_{t=1,s}^{x}$$

Figure 7: CDC Pension Contract Characteristics



Source: adjusted conditional indexation and fixed contribution level.

The summary of the calibrated parameters is presented in Table IV.

Table IV: The Main Calibrated Parameters

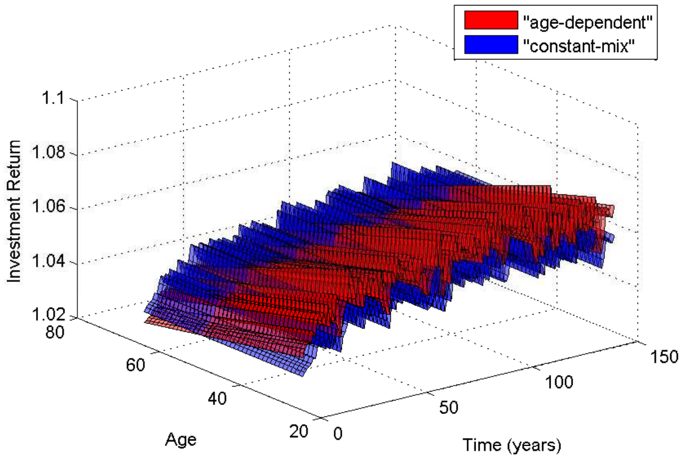
type	**Demographic**		**Financial**								**Utility**
name	m	b	π	g_w	κ_r	μ_r	σ_r	λ	σ_s	$\rho(r,s)$	γ
parameters	87	8	2%	3%	5%	3%	5%	4%	20%	0%	1.5
Pension Plans	**Forbidden**	**Min**	**Floor**	**Cap**	**Surplus**	**Max**					
C_i	0%	10%	-	-	-	25%					
FR_i	0.6	-	1.00	1.30	1.60	1.65					

Source: Parameters of the calibration of the pension plans (Lekniute (2011)).

3.2 Individual Investment Characteristics

To compare fund participation with individual investment, we avoid all other sources of the distinction between these pension savings. Thus, we consider the savings share of agents' wages as being identical to the one they would have contributed by participating in the fund. Moreover, the individual investment strategy is kept identical to the one the pension fund applies. The empirical studies (Ameriks and Zeldes (2004) and Campbell (2006))) which contest the theory of more risk-taking at a young age compared to an older age, argued that an investment position in the market is related to the human capital level and the long-run labor risk exposure. Because the results are robust to the individual investment being 100-age, to make a better *proxy* of the reality, we present the results of this strategy here. The "constant-mix" and the age-dependent strategy reflect different levels of risk undertaken (Figure 8).

Figure 8: Expected Investment Return "Constant-mix" *versus* "Age-dependent"



Source: Individual investment is age-dependent; Fund investment is "constant-mix".

Similarly to the "pure" DC plan, the individual portfolio of each representative individual at the age of 64 equals his total future retirement income.

$$\begin{cases} Ptf_{t,s}^{x} = C_{t,s}^{x} \times R_{t,s}^{inv}, & for \ \ t = 1 \\ Ptf_{t+1,s}^{x+1} = \left(Ptf_{t,s}^{x} + C_{t+1,s}^{x+1}\right) \times R_{t,s}^{inv}, & for \ \ t > 1 \end{cases}$$

Because of consumption smoothing during retirement, the 64-year-old representative agent buys a level annuity[6]($Annuity_{t,s}^{65}$), whose expected present value ($EPV_{t,s}^{64}$) equals the individual's bucket at age 64.

$$EPV_{t,s}^{64} = \sum_{time=t+1}^{t+R} \frac{Annuity_{t,s}^{65} \cdot p^{64}\left(time - t|t\right)}{\prod_{k=t}^{time}\left(1 + r_{t,s}^{(k-t)}\right)}$$

where $r_{t,s}^{(k-t)}$ is the nominal spot yield corresponding to the scenario s at time t maturity $(k-t)$, T is the maximum length of pension based on the assumption that no one lives beyond the age of 94.

3.3 Value based pension deals

It is important for the fund manager to measure *ex-ante* the value of each generation. Let us consider V the value of the contingent claim and $\mathbb{E}_t^{\mathbb{Q}}$ the risk neutral expectation

[6]The level annuity is calculated for individuals at age 65 and provides constant annuities in time.

under the $\mathbb{Q}$-measure which is calculated as the expectations of the outcomes of all future cash flows under risk-neutral scenarios, discounting them under risk-free rates (Cochrane (2001)). The value of the generational account varies in three dimensions: time, scenario and cohort. Depending on individuals' age, there are two profiles called the contributors (agents between 25 and 64 years) and the retirees (agents older than 65). Hence,the value based general account for each cohort (x) at time t is given as follows:

$$V_{t,s}^{x} = \begin{cases} \mathbb{E}_t^{\mathbb{Q}}\left(\sum_{i=x}^{95}\left(p^x\left(t+(i-x)\,|t\right)B_{t+(i-x),s}^{i}\prod_{j=t}^{t+(i-x)}\left(R_j^f\right)^{-1}\right)\right) & for \;\; x \geq 65 \\ \mathbb{E}_t^{\mathbb{Q}}(\sum_{i=x}^{64}\left(-p^x\left(t+(i-x)\,|t\right)C_{t+(i-x),s}^{i}\prod_{j=t}^{t+(i-x)}\left(R_j^f\right)^{-1}\right)+... & \\ ...+\sum_{i=65}^{95}\left(p^x\left(t+(i-x)\,|t\right)B_{(t+(i-x)|t),s}^{i}\prod_{j=t}^{t+(i-x)}\left(R_j^f\right)^{-1}\right)) & for \;\; 25 \leq x < 65 \end{cases}$$

where $R_j^f = 1 + r_j^f$ and represents the return on investment in a short-term risk-free bank account. Because of the heterogeneity absence among the same cohort agents, the value based generation account only for one representative individual was considered.

The study analyzes the changes between participation in a collective pension scheme *versus* individually investing in the market while using the same financial instruments. Hence, the difference between the value based generational account of each retirement saving option was evaluated ($V_{t,s}^{x,collec.}$ and $V_{t,s}^{x,indiv.}$).

$$\Delta V_{t,s}^{x} = V_{t,s}^{x,collec.} - V_{t,s}^{x,indiv.}$$

The higher the value-based generation account for a given cohort, the more expensive this cohort is for the pension fund. The higher the value of the individual investment case, the more profitable this cohort is from the swap between their final portfolio at age 65 and the proposed annuity at that time.

3.4 Utility as a measure of a pension contract

The value based account provides important information from the fund's but not from the individual's perspective. Thus, we evaluate not only the expected net benefit (gain/loss) of each participant but also the utility level. The first reason is that the aim of pension systems in general is not just to offer individual benefits during their retirement but also to realize consumption smoothing. Secondly, utility allows us to measure agents' well-being. This paper aims to measure the utility reached by fund participation, which could be replicated by individual investment in the market. Hence, the remaining non-replicated part is a property of the contract *via* intergeneration risk sharing. Utility captures not only the actual value of the pension contract but also its price and the ability to smooth consumption when switching to retirement. Finally, because of the absence of full initial information on individual investment, we focus our analysis on the 25-year-old representative agent. The constant relative risk aversion ($CRRA$) utility function:

$$u(w) = \frac{w^{1-\gamma}}{1-\gamma}$$

γ is the concavity degree of $u(\cdot)$, inter-temporal smoothing (savings precaution). The inverse of inter-temporal elasticity of substitution $\gamma = 1.5$, as the aim of the representative agent to smooth consumption, was considered. The relative risk aversion parameter is considered the same for both pension strategies. Because of its complex demographic structure, the pension fund contains contributions from different people with distinct risk preferences. Although it invests on a constant-mix strategy, on average it is considered as being a "patient", risk-averse global entity (increasing dependency ratio in time). Thus, using $\gamma = 1.5$ is compatible not only for the individual investment but also for the pension fund strategy. The lifetime utility of the 25-year-old representative agent is calculated *ex-ante* to capture the price paid for the contract (pension contribution) and the benefit obtained.

$$U_{t,s}^{x} = \mathbb{E}_t^{\mathbb{Q}}\left(\sum_{i=x}^{64}\frac{p^x\left(t+(i-x)\,|t\right)}{\left(1+r^f{}_{t,s}\right)^{t+i-25}}U\left(W_{t+(i-25),s}^{i-24} - C_{t+(i-25),s}^{i-24}\right) + \sum_{i=65}^{94}\frac{p^x\left(t+(i-x)\,|t\right)}{\left(1+r^f{}_{t,s}\right)^{t+i-25}}U\left(B_{t+(i-25),s}^{i-24}\right)\right)$$

We calculate the utility underfund participation and individual investment. The replicating coefficient ($coef_{replic}$) expresses the share of the fund participation utility replicated by the individual investment. Moreover, we emphasize the components of the shock effect on both the fund participation and on an individual investment.

4 Results

This paper aims to analyze the effect of demographic shocks on both individual and commective retirement savings, by running the three distinct contracts[7] (DB-"hard", DB-"soft" and CDC) and presenting each of them in two different frameworks:

- First step: policy security constraints are omitted (no PSC).
- Second step: policy security constraints are introduced (yes PSC).

For both the baseline and the shock case, this paper presents the fund performance characteristics, the generation account values and the *ex-ante* lifetime utility representation of a 25-year-old representative individual.

4.1 Baseline Results

This paper considers the "normal" (baseline) model as being the benchmark of this study and compares the results related to shocks in "life-expectancy".

[7] We used Matlab to program the pension system and to manipulate the results.

4.1.1 Fund performance

Pension plan characteristics do not change in time and despite the long time duration, we decide to focus on presenting the statistics of two particular years such as year 20 and year 40 (Table V). The retirement of the first 25-year-old agents coincides with the year 40. Because the long history and the difficulty of keeping the funding ratio constantly converging, the focus was placed on an intermediary moment in time (year 20).

Among the represented variables, some of the distribution quantiles of average funding ratio (FR) were highlighted, the probability of being underfunded ($P(FR) < 1$), the probability of overstepping either the upper bound limit FR ($P(FR > FR_{max})$) or the lower bound one ($P(FR < FR_{forbidden})$), the probability of having applied pension cuts because of policy regulation ($PSC\ cuts$) and finally the replacement rate (RPR)[8].

The unconstrained pension DB-"hard" plan (no PSC) is greatly influenced by the demographic structure. The dependency ratio increase indicates the population aging phenomenon (Figure 4). The low FR level during the first 20 years and its continuous decrease during the first 40 years especially because of the full pension guarantees promised by this plan is the source of unsustainability (see Figure 9).

Thus, there is a decrease in the FR especially in the scenarios lower than the median. The median result itself shows that the funding ratio decreases from 97.48% (during the first 20 years) to 90.18% (during the first 40 years) on average.

The higher quantiles (because of the surpluses already stocked) are not much affected by the increase in the dependency ratio. Figure 9 represents the expected funding ratio dynamics for the no PSC framework. As a consequence of the decreasing funding ratio, an increase in the fund probability to be underfunded and for the funding ratio probability to go past the limit bounds. Finally, because of the guaranteed pension benefit, the replacement rate remains constant.

The statistics related to plan DB-"soft" are in line with the ones discussed for the DB-"hard" pension plan. The decrease of fund sustainability in time is reflected by the decrease in both the funding ratio (FR) and the replacement rate (RPR). Furthermore, time positively impacts the probability of being underfunded, the probability of exceeding the predefined bounded limits and the share of pension benefit cuts (in the case of implemented PSC).

The CDC plan is defined by fixed contributions ($c_{t,s} = 19.39\%, \quad \forall t, \ \forall s$) which is in between the allowed extreme possible values for the contribution c_{min} and c_{max} and it is calculated based only on the information available at time $t = 1$. The absence of forward-looking variables such as population, discounting rate, future wages and inflation history, makes it impossible to calculate an expected value of the contribution levels.

[8] RPR is the ratio between the pension benefit (indexation included) and the average wage.

Table V: Collective Plans in "Normal" Model

Plan DB-**"hard"**												
no PSC	**Q2.25%**	**Q25%**	**Q50%**	**Q75%**	**Q97.5%**	$P(FR<1)$	$P(FR>max)$	$P(FR<Forbidden)$	$P(index<0)$	$P(index>1)$	RPR	**PSC cuts**
FR_N 40y	0,5234	0,7840	0,9748	1,2304	1,9414	0,5065	0,0793	0,1293	0,0000	0,0000	0,8160	0,0000
FR_N 20y	0,3279	0,6312	0,9018	1,2445	2,7973	0,5637	0,1266	0,2587	0,0000	0,0000	0,8160	0,0000
yes PSC												
FR_N 40y	0,6314	0,8093	0,9820	1,2236	1,8314	0,5024	0,0730	0,0868	0,0000	0,0000	0,6941	0,0783
FR_N 20y	0,5875	0,7673	0,9456	1,2370	2,4185	0,5469	0,1144	0,1443	0,0000	0,0000	0,6306	0,1382

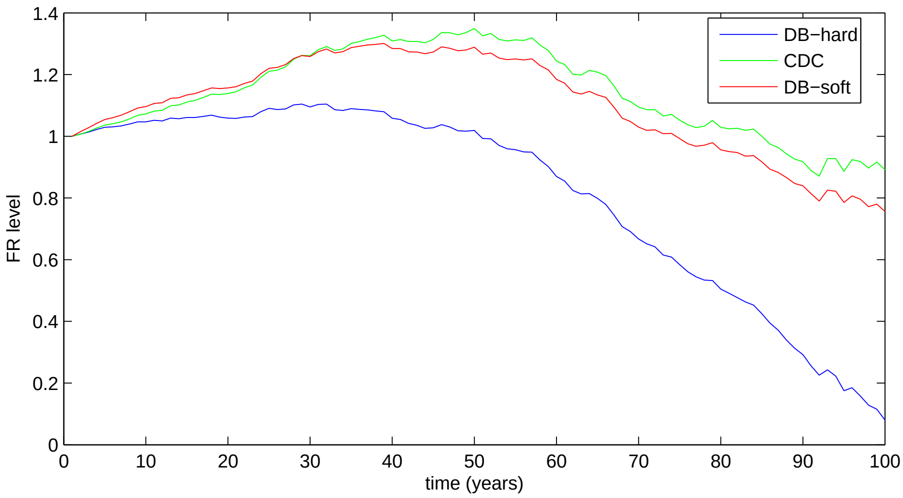
Plan DB-**"soft"**												
no PSC	**Q2.25%**	**Q25%**	**Q50%**	**Q75%**	**Q97.5%**	$P(FR<1)$	$P(FR>max)$	$P(FR<Forbidden)$	$P(index<0)$	$P(index>1)$	RPR	**PSC cuts**
FR_N 40y	0,5736	0,8426	1,0313	1,2763	1,9814	0,4542	0,0896	0,0878	0,0000	0,0000	0,8064	0,0000
FR_N 20y	0,4304	0,7605	1,0150	1,3424	2,8962	0,4935	0,1477	0,1755	0,0000	0,0000	0,8058	0,0000
yes PSC												
FR_N 40y	0,6507	0,8499	1,0357	1,2688	1,8751	0,4516	0,0830	0,0639	0,0000	0,0000	0,7224	0,0577
FR_N 20y	0,6083	0,8223	1,0253	1,3086	2,4883	0,4820	0,1332	0,1082	0,0000	0,0000	0,6767	0,1035

Plan CDC												
no PSC	**Q2.25%**	**Q25%**	**Q50%**	**Q75%**	**Q97.5%**	$P(FR<1)$	$P(FR>max)$	$P(FR<Forbidden)$	$P(index<0)$	$P(index>1)$	RPR	**PSC cuts**
FR_N 40y	0,5239	0,7879	0,9925	1,2761	2,0722	0,4936	0,0996	0,1269	0,0000	0,0000	0,8060	0,0000
FR_N 20y	0,3343	0,6437	0,9448	1,3710	3,3165	0,5373	0,1617	0,2432	0,0000	0,0000	0,8054	0,0000
yes PSC												
FR_N 40y	0,5004	0,7838	0,9912	1,3090	2,1909	0,4913	0,1140	0,1300	0,0000	0,0000	0,6030	0,1300
FR_N 20y	0,1480	0,6152	0,9766	1,5693	3,8702	0,5127	0,1993	0,2453	0,0000	0,0000	0,5311	0,2453

Source: Pension fund statistics, baseline model, during 20 and 40 years; Type of contract: DB-"hard", DB-"soft" and CDC; Framework: no & yes PSC.

The contribution cuts induced by exceeding the upper limit bound are more difficult to reduce because of the surplus stock asset value. The safety policies causing benefit cuts help to increase the fund's buffer. Thus, the probability of being underfunded and exceeding the PSC boundaries decreases. During the first 20 years, there have been almost 8% of cases when the policy cuts are applied and this number rises to 14% during the 40 years. Finally, the replacement rate is negatively affected by both, the introduction of constraints and time.

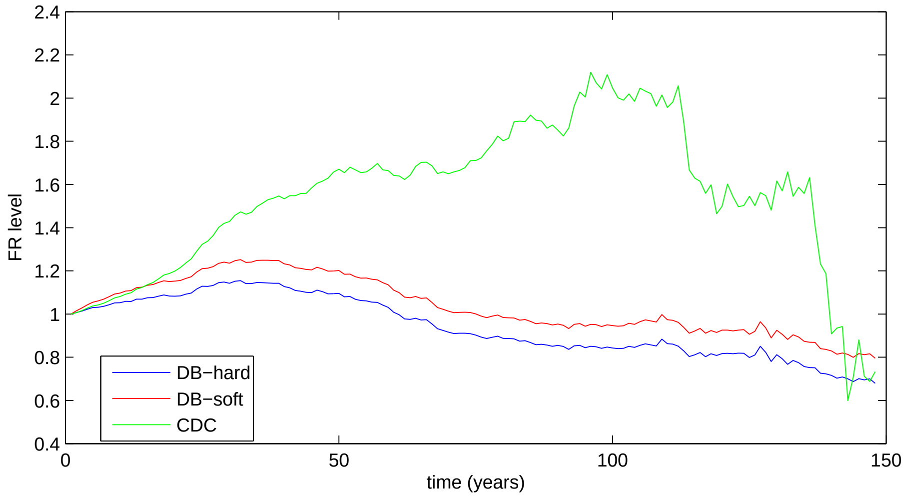
Figure 9: Expected Funding Ratio Dynamics (no PSC)



Source: Normalization to one; The DB-"hard" plan diverges; Model: "normal"; Framework: no PSC.

The PSC stabilizes the variable contribution plans and positively affects the CDC plan (Figure 10). The contribution level is high enough to keep the fund over-funded by overstepping the upper limit for about 30 years in a row. Contrary to the fund participation, the individual investment keeps the funding ratio constant to one.

Figure 10: Expected Funding Ratio Dynamics (yes PSC)



Source: Normalization to one; Model: "normal"; Framework: yes PSC.

4.1.2 Generation account results

In this study, a value based model was used to calculate the generation account (GA) level of co-existing cohorts during both fund participation and individual investment. Because of individual homogeneity in a given cohort, it considers one representative agent for each cohort. To show the results, three moments in time were considered:

- Year 20 (as an intermediate moment);
- Year 40 (coincides with the active age for each agent);
- Year 70 (maximum number of years the agent is in the system).

Contrary to the open fund whose data allows us to calculate the value of each generation account at each moment in time, the individual investment does not because of its incompleteness. Its matrix during the first 70 years is an upper triangular one. Thus, during the first 70 years ($t \leq 70$) one can calculate the GA corresponding to only t first co-existing cohorts. Hence, we use the value-based model to have a broader vision for all existing cohorts at a given moment in time and focus on the 25-year-old representative agent when using the utility measure. The generation account representation value for each pension plan in time and the individual investment corresponding values are given in Appendix A.

The GA corresponding to individual investment is represented by a hump shape function with respect to the co-existing cohorts (Figure 20 in Appendix A). For the collective fund participation (DB-"hard"), the GA corresponds to a decreasing function with respect to the age of the co-existing cohorts (in a no PSC framework). First, this is highly related to the generosity of this collective plan. Secondly, pension benefits are variable when individuals participate in collective pension plans. However, they are considered constant when individual investment is used for pension savings.

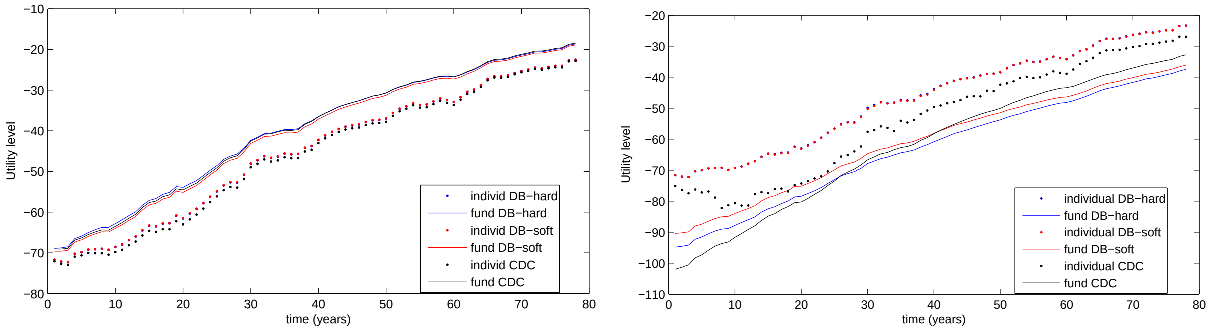
Inasfar as policy rules are implemented, severe cuts may occur. The generation account while participating in collective DB funds is transformed into a hump shape function (Figure 20 and 21 in Appendix A). The CDC plan is in line with the no PSC framework because of the trade-off contribution-benefit that this plan offers (Figure 22 in Appendix A).

The differences in generation account value between the fund participation and the corresponding individual investment are positive and increase in time when no PSCs are implemented. The contrary happens when policy rules are applied. The difference is negative for the DB collective plans. On the one hand, one could conclude that it is better to individually invest and be in a "pure" defined contribution scheme when such policy rules related to fund performance are applied. On the other hand, collective fund participation is more generous when no such safety constraints are taken into consideration. The CDC plan with constraints stays in the middle since the fund participation is optimal for the majority but not for all cohorts. Therefore, to take a decision on whether fund participation or individual investment is a better pension investment choice, the argument was based on the utility measure study. Hence, the value of the generation account was not only measured but there need to determine whether the agent is better off over a lifetime by individually investing for retirement or by participating in the pension plan while focusing on individual well-being.

4.1.3 Utility outcome

There are two reasons for considering the 25-year-old representative agent. First, it is the generation for whom we can have a wider view of the data. Secondly, it is the first generation joining the labor market. Hence, in a context where participation is not mandatory, they could refuse to participate in the fund if fairness is not ensured. Contrary to the generation account, the price of the signed contract and its future benefits are taken into consideration. Figure 11 shows that under no PSC the fund participation outperforms the individual investment in terms of utility to the agent. The inverse happens when policy rules are applied Figure 11. The differences in the amplitude between each respective couple depend on the plan.

Figure 11: *Ex-ante* Lifetime Utility

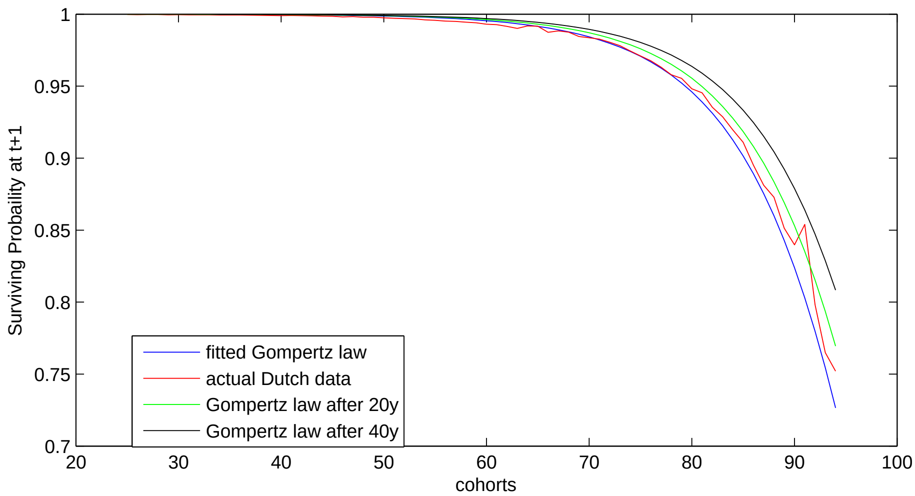


Source: Representative 25-year-old individual in time; Model: "normal"; Framework: no *PSC* (on the left), yes *PSC* (on the right).

4.2 Results of demographic shock

Life-expectancy, fertility and migration are the three most important factors affecting population dynamics. Aging is obviously a positive development, as it means that people on average live longer. However, there are also worries that it may negatively impact the economy in general and the pension system in particular.

Figure 12: Surviving Probability (Life-expectancy Shock)



Source: The evolution of the surviving probability; Period: "life-expectancy shock".

This study focused on the effects of an increase in life-expectancy as a phenomenon which has been widely present during the last decade. The estimated modal age at death is 87. Introducing a shock in which there is an increase of one month each year (until the age of 92) of this variable. This upward demographic shock implies an increase in the survival probability (Figure 12), even when for simplicity in modeling the system, the maximum age is considered constant (94 years). Therefore, the surviving probability matrix will be no longer deterministic and constant in time, but stochastic and time contingent. This is the only source of difference with respect to the baseline.

The population presents an increase in the dependency ratio up to 55.5% after 25 years and a stabilization 50 years later (Figure 4).

4.2.1 Fund performance

The population aging caused by the increase in life-expectancy reduces fund sustainability faster than the possible downward demographic shock. The retirement benefits have to be insured longer on average for more retirees. Compared to the baseline results, the funding ratio distribution of all plans is lower. Moreover, the limits are difficult to maintain under control, especially for the first 40 years. The replacement rate decreases and the probability of applying cuts provided by the PSC increases (Table VI).

4.2.2 Generation account results

The positive life-expectancy shock influences both the fund and the individual performance. The latter is affected as the pension annuity decision has been taken and the swap contract signed. Hence, by rule of thumb, the generation account valuation is contingent. The increase in the survival probability and the corresponding conditional survival probability increases the fund liability which it is necessary to balance by an increase in the asset side. As long as the contribution is modeled as being bounded (or fixed), the maximum share of wage that is contributed by the agents is $c_{max} = 20\%$. Despite the increase of the liability level, there is a bounded limit for the assets to cover it. Under these circumstances the fund loses liquidity balance and tends towards lower funding ratios.

Nevertheless, the situation is optimistic for agents who contribute not much more (sometimes the same c_{max}) and get more during retirement, by increasing the probability of being alive in the system. This is all explained by an increase in the generation account under "life-expectancy" shock[9] for each pension contract and framework, compared to the "normal" model results. Therefore, one could give the same conclusions in terms of value based valuation as in the baseline model since the shape of these functions does not change, only their amplitude.

[9]The concrete generation account values for all co-existing cohorts for each plan under constrained and non-constrained frameworks can be provided on request.

Table VI: Collective Plans under Positive Life-expectancy Shock

Plan DB-"hard"												
no PSC	**Q2.25%**	**Q25%**	**Q50%**	**Q75%**	**Q97.5%**	$P(FR<1)$	$P(FR>max)$	$P(FR<Forbidden)$	$P(index<0)$	$P(index>1)$	RPR	**PSC cuts**
FR_N 40y	0,5089	0,7612	0,9471	1,1928	1,8739	0,5302	0,0682	0,1475	0,0000	0,0000	0,8160	0,0000
FR_N 20y	0,3037	0,5805	0,8405	1,1684	2,5688	0,6007	0,1058	0,3003	0,0000	0,0000	0,8160	0,0000
yes PSC												
FR_N 40y	0,6214	0,7874	0,9563	1,1914	1,7806	0,5265	0,0627	0,1008	0,0000	0,0000	0,6685	0,0909
FR_N 20y	0,5658	0,7348	0,8924	1,1657	2,2456	0,5853	0,0960	0,1732	0,0000	0,0000	0,5768	0,1659

Plan DB-"soft"												
no PSC	**Q2.25%**	**Q25%**	**Q50%**	**Q75%**	**Q97.5%**	$P(FR<1)$	$P(FR>max)$	$P(FR<Forbidden)$	$P(index<0)$	$P(index>1)$	RPR	**PSC cuts**
FR_N 40y	0,5569	0,8195	1,0051	1,2399	1,9092	0,4811	0,0789	0,1036	0,0000	0,0000	0,8058	0,0000
FR_N 20y	0,3997	0,7090	0,9583	1,2564	2,6553	0,5356	0,1258	0,2101	0,0000	0,0000	0,8048	0,0000
yes PSC												
FR_N 40y	0,6344	0,8310	1,0070	1,2347	1,8168	0,4784	0,0728	0,0758	0,0000	0,0000	0,6980	0,0679
FR_N 20y	0,5820	0,7895	0,9774	1,2405	2,3075	0,5240	0,1128	0,1327	0,0000	0,0000	0,6338	0,1268

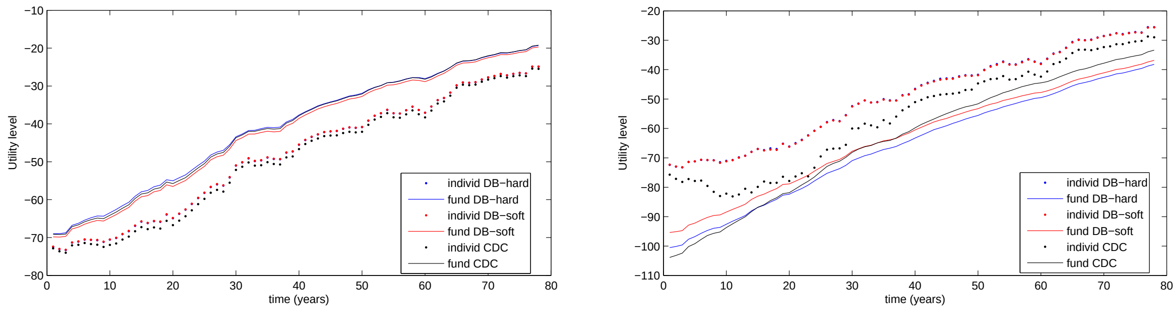
Plan CDC												
no PSC	**Q2.25%**	**Q25%**	**Q50%**	**Q75%**	**Q97.5%**	$P(FR<1)$	$P(FR>max)$	$P(FR<Forbidden)$	$P(index<0)$	$P(index>1)$	RPR	**PSC cuts**
FR_N 40y	0,5097	0,7641	0,9632	1,2296	1,9957	0,5168	0,0856	0,1443	0,0000	0,0000	0,8054	0,0000
FR_N 20y	0,3149	0,5966	0,8788	1,2684	3,0318	0,5726	0,1351	0,2814	0,0000	0,0000	0,8046	0,0000
yes PSC												
FR_N 40y	0,4878	0,7665	0,9674	1,2738	2,1319	0,5095	0,1021	0,1424	0,0000	0,0000	0,5793	0,1424
FR_N 20y	0,1675	0,5952	0,9294	1,4798	3,6088	0,5347	0,1821	0,2637	0,0000	0,0000	0,5033	0,2637

Source: Pension fund statistics, shock model, during 20 and 40 years; Type of contract: DB-"hard", DB-"soft" and CDC; Framework: no & yes PSC.

4.2.3 Utility outcome

To better understand the effect of this shock from the agent's perspective and able to compare an individual to fund participation, this paper is based on the *ex-ante* lifetime utility. This lifetime utility depends on the type of pension plan that the representative agent belongs to. It is obvious that the unconstrained plans, contrary to constrain ones, make greater strong promises. Hence, it is more optimal to be part of a collective plan for a 25-year-old agent, especially under such a demographic shock. Equivalently to the results presented in the generation account section, the consequences of the shock are presented by decreasing the agent's utility level.

Figure 13: *Ex-ante* Lifetime Utility under Life-expectancy Shock



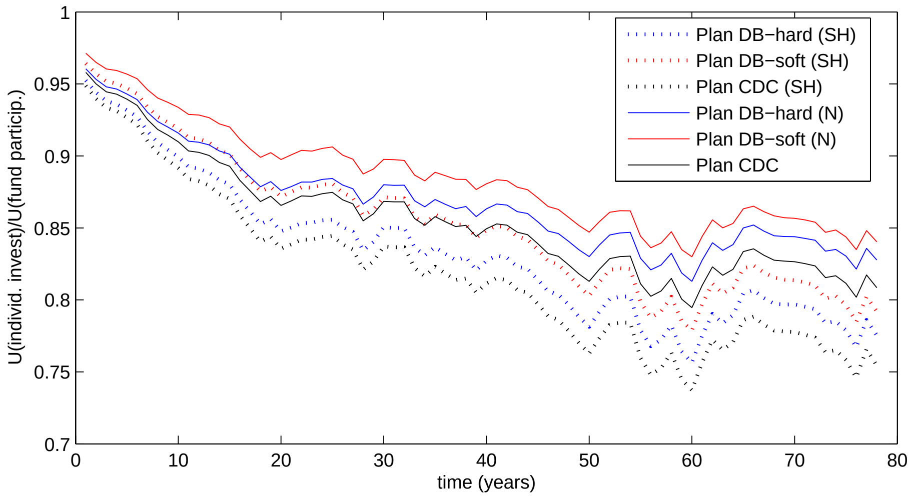
Source: Generation: 25-year-old participant; Period: "life-expectancy shock"; Framework: no *PSC* (on the left), yes *PSC* (on the right).

Despite the fact that the results stay in line with the baseline model, as one compares fund participation with individual investment, the amplitude of the shock effects is reduced.

4.3 Replicating Coefficient

We construct the variable $coef_{replic}$ to capture the proportion of the utility of the fund participation replicated by the representative agent *via* individual investment. Figure 14 presents these proportions and their dynamics in time during the normal and shock model. The individual investment replicates 95% of the fund participation utility in the baseline model during the first years after the contract is proposed. It reaches 85% 50 years later and remains stable.

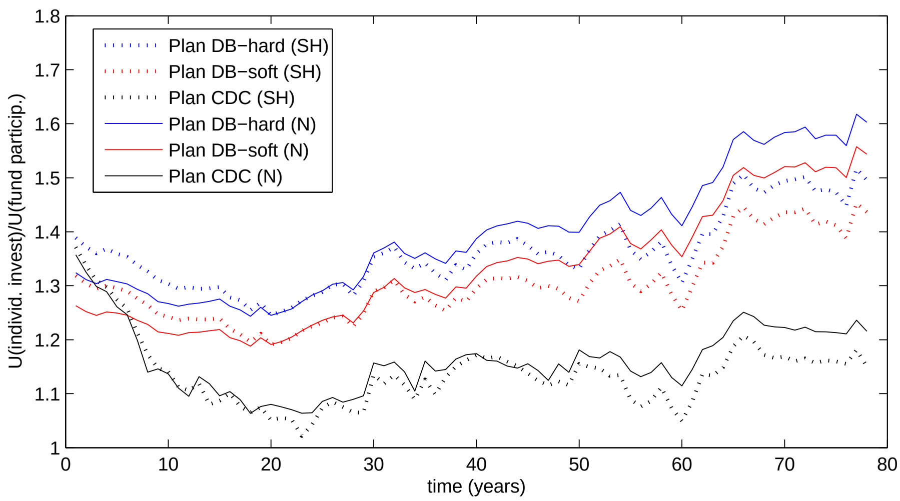
Figure 14: Utility Replication of the Fund Particip. by Individual Invest. (no PSC)



Source: The $coef_{replic}$ dynamics; Model: "normal" (N) and "life-expectancy shock" (SH); Framework: no PSC.

When policy security constraints are applied, the utility provided by pension participation is lower than that provided by individual investment (Figure 13).

Figure 15: Utility Replication of the Fund Particip. by Individual Invest. (yes PSC)



Source: The $coef_{replic}$ dynamics; Model: "normal" (N) and "life-expectancy shock" (SH); Framework: yes PSC.

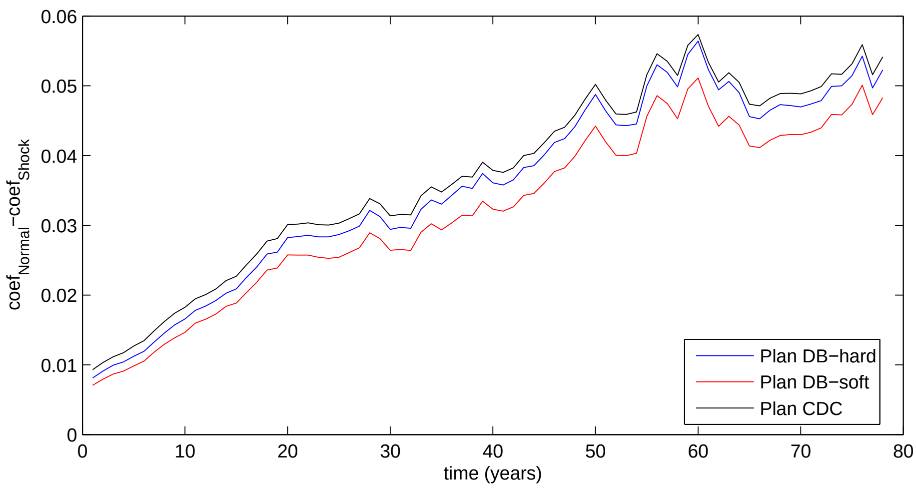
Hence, under the same contribution level, the 25-year-old agent would prefer to invest individually rather than to participate in the fund, regardless of the pension plan. When the demographic shock is applied, individuals do not change their choice compared to the baseline model. Moreover, there is a drop in the replication coefficient and its amplitude depends on the considered plan.

Assuming that the agent invests using the same investment strategy as the fund, the difference between the utility provided by the individual and the fund investment quantifies the risk sharing characteristic of the pension plan. Although here agents' investment is slightly different, the differences are not significant and without loss of generality one could consider them identical.

$$\Delta coef = coef_{replic}^{Normal} - coef_{replic}^{Shock}$$

Therefore, Figure 16 and Figure 17 present $\Delta coef$, respectively with and without implementing PSCs. The smaller and flatter the difference between the baseline model and the shock replication coefficient (in absolute value), the more one can state that the shock is well amortized. Therefore, when the PSC framework is implemented, the CDC contract best amortizes the shock. On the contrary, when the no PSC is considered, the difference between the replicating coefficients in the baseline model and under shock has a positive trend in time. Nevertheless, it stabilizes 50 years later[10]. The lowest effect of the shock is seen on the "soft" DB plan while the CDC is the plan with the highest consequences in the "life-expectancy shock" model.

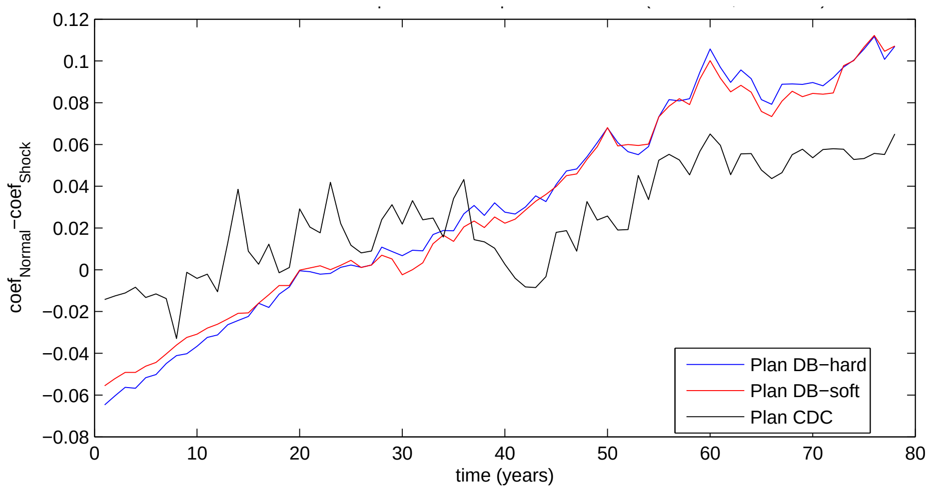
Figure 16: The Shock Effect on the Replicating Coefficient in Different Plans (no PSC)



Source: The $\Delta coef$ dynamics; Model: baseline *vs.* "life-expectancy shock"; Framework: no PSC.

[10]The population structure (DR) stabilizes starting from the years 50.

Figure 17: The Shock Effect on the Replicating Coefficient in Different Plans (yes PSC)



Source: The $\Delta coef$ dynamics; baseline *vs.* "life-expectancy shock"; Framework: yes PSC.

Not only does this shock change the rational choice of agents in selecting between fund or individual investment, it affects these strategies differently. It is important to analyze the effect of the shock and determine which strategy is less affected (figure Figure 18 and 19).

$$Units\ of\ utility(type) = utility(type)^{Normal} - utility(type)^{Shock}, type = \{fund; individ.\}$$

Based on the type of the utility function, a negative (*resp.* positive) value of $Units\ of\ utility(\cdot)$ induces a positive (*resp.* negative) effect of the shock on the agent's utility.

First, the shock has a negative impact on the utility regardless of its type (*i.e.* fund or individual). On the one hand, based on the no PSC framework, the effect of the shock on the fund is smaller than that on an individual investment. Furthermore, the CDC is the contract that better amortizes the shock while keeping fund sustainability[11]. Because of the higher paid contributions, the DB-"hard" contribution strategy used for the individual investment is more shock amortizing[12] (Table VII).

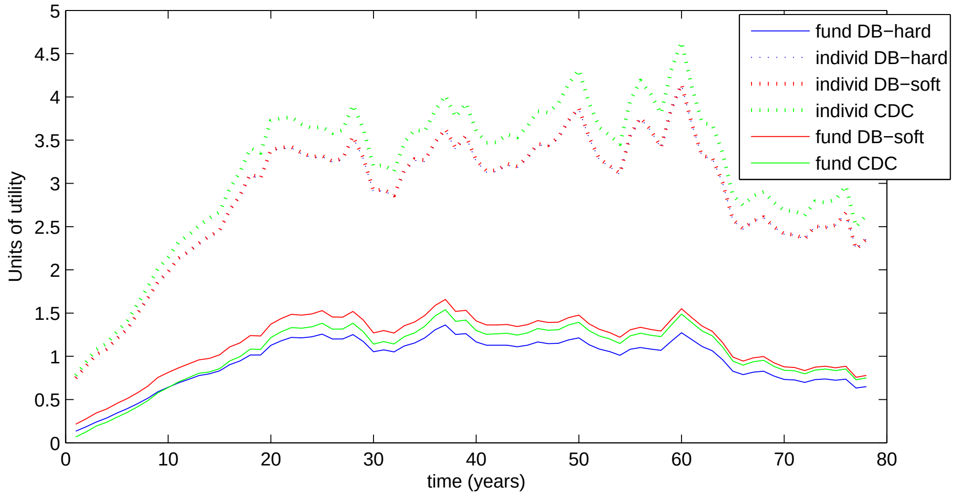
On the other hand, based on the PSC framework, the shock affects the collective funds more only during the first 35 years (because of the PSC measures). The collective fund best amortizing the shock is the CDC with a minimum variance of 17% and almost no trend. It represents the lowest mean and the lowest extreme values, expressed in units of utility. Concerning the corresponding individual plans, the CDC presents high

[11]The lowest variance (8.5%) and the lowest maximum difference (1.36) is offered by the DB-"hard" but this is an unsustainable fund and we do not take into consideration its unrealistic results.

[12]Nevertheless, it is not of interest to discuss the DB-hard because of its unsustainability.

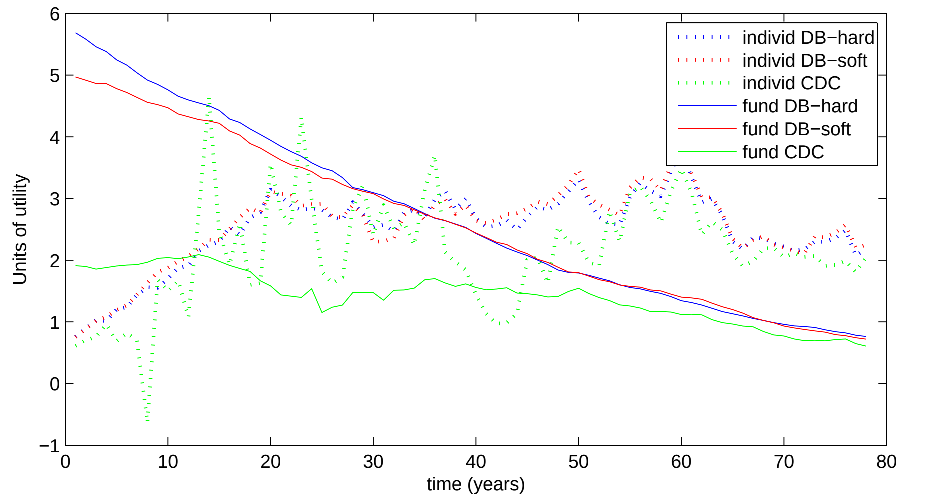
volatility; however, the mean is only 2.12 and is often the better plan in amortizing the shock.

Figure 18: The Effect of Life-expectancy Shock on Fund/Individual Invest. (no PSC)



Source: The effect of the shock is considered separately in each pension contract; Framework: no PSC.

Figure 19: The Effect of Life-expectancy Shock on Fund/Individual Invest. (yes PSC)



Source: The effect of the shock is considered separately in each pension contract; Framework: yes PSC.

Table VII: Statistics on the effects of Life-expectancy Shock

Fund Participation					
NO PSC NO MWP	**Mean**	**Variance**	**MaxDrawdown**	**Min**	**Max**
Plan DB-hard	0.9466	0.0850	0.5350	0.1336	1.3630
Plan DB-soft	1.1526	0.1181	0.5431	0.2158	1.6583
Plan CDC	1.0473	0.1228	0.5272	0.0677	1.5397
YES PSC NO MWP					
Plan DB-hard	2.7105	2.1823	0.8655	0.7646	5.6864
Plan DB-soft	2.6074	1.7545	0.8551	0.7203	4.9697
Plan CDC	1.4148	0.1669	0.7095	0.6070	2.0897
Individual Investment					
NO PSC NO MWP	**Mean**	**Variance**	**MaxDrawdown**	**Min**	**Max**
Plan DB-hard	2.8556	0.5678	0.4593	0.7522	4.1393
Plan DB-soft	2.8632	0.5753	0.4602	0.7490	4.1476
Plan CDC	3.1616	0.7328	0.4620	0.7782	4.6377
YES PSC NO MWP					
Plan DB-hard	2.4939	0.3709	0.4526	0.7585	3.7937
Plan DB-soft	2.5437	0.3835	0.4526	0.7532	3.8527
Plan CDC	2.1271	0.7692	0.7910	-0.6308	4.6466

Source: Statistics calculated based on the utility of the 25-year-old agent; Framework: no & yes PSC.

4.4 Robustness Checks

We consider the pension fund's portfolio composition as being predetermined and constant in time. Stylized facts based on the DNB (De Nederlandsche Bank) show that the equity exposure of Dutch pension funds has increased since 1995 (bounded by 60%, for the 90% quantile). The median equity exposure tripled from 1995 to 2007. However, the reinforcement of supervisory regulation *via* the policy constraints has decreased the allowed risk exposure for pension funds.

Since pension/individual portfolio optimization is not within the scope of this paper, we keep it exogenous and constant as a way of avoiding the extra effects provided by portfolio investment. Nevertheless, running the model for other portfolio compositions shows robust results[13]. They are completely in line with the presented ones, except for high-risk investments (100%investment in stocks). In this case, the result changes: the implementation of the PSCs incites rational individuals to sometimes choose individual investment and sometimes choose fund participation, depending on the moment in time when they enter the system. Despite the results of the equity premium puzzle in favor of a 100% equity portfolio, that would not be the optimal investment strategy for the pension fund. Pension funds need high returns, and a considerable high level of risk hedging measures to control the necessity for liquidity. Moreover, pension funds are under tight supervisory regulations which do not allow investments of high risk.

[13] Results are available upon request.

5 Conclusions

This paper aims to shed some light on the expression much used in recent years: "*the collective defined contribution pension contract is the best choice for intergeneration risk sharing*". The Netherlands was one of the first countries to adopt the CDC contract as part of the privately managed and mandatory second pillar. This paper questioned whether it would be possible to replicate this collective pension performance by individually investing in the market. Moreover, how does this contract react to demographic shocks? Using stylized pension contract analysis, the paper constructs three basic collective contracts and study them in two main frameworks (when PSC are implemented and when when they are absent). It further compares the collective fund and its individualistic equivalent to measuring the value of intergeneration risk sharing especially when life-expectancy increases continuously.

This paper concludes that pension fund participation insures risk sharing, although its performance could be replicated by individually investing in the market at the level of $80\% - 90\%$ in the case of no PSC. Hence, the remaining $10\% - 20\%$ is dedicated to the intergeneration specificity of the collective plans[14]. This result raises the question of the pillar's fairness and its mandatory application. The CDC plan appears to be the best choice among the sustainable collective plans for better amortizing the risk. The results state that when the regulation of pension systems is harsh, individual investment is the optimal choice for pension saving. These arguments are in favor of a voluntary or partially mandatory collective scheme. Nevertheless, this paper emphasizes that during individual investment financial literacy is important. The PSCs not only affect the welfare negatively but their presence has a distinct impact on the choice of individual and fund investment for retirement. Robustness checks on different portfolio compositions confirm the results.

However, the structure of the data could be ameliorated. More sophisticated simulation methods could be used to better *proxy* the market (van den Goorbergh et al. (2011)). A supplementary uncertainty on inflation brings the model closer to reality. Furthermore, this paper focus on the effects of demographic shock which is not the only existing shock threatening pension systems. Hence one could measure the effect of macro and financial shocks. Finally, it is of great interest to be able to analyze the risk shared inter and intra generations by introducing heterogeneity within cohorts.

[14]Without loss of generality, the agent's distinct investment strategy compared to the fund does not affect our results.

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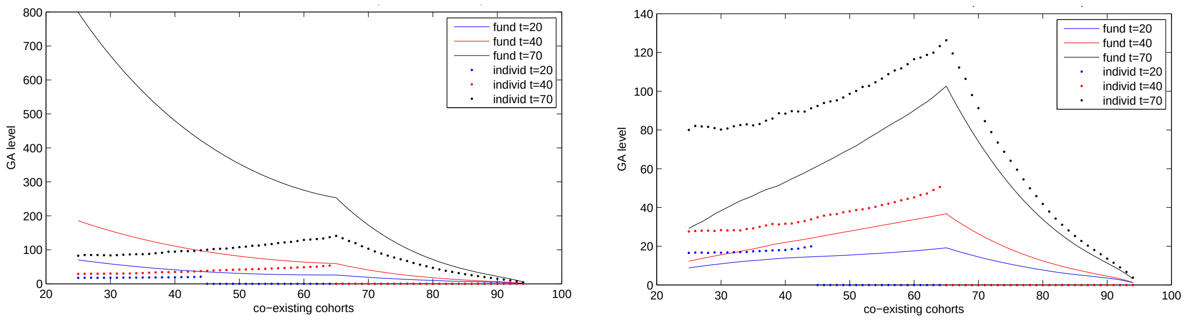
Appendices

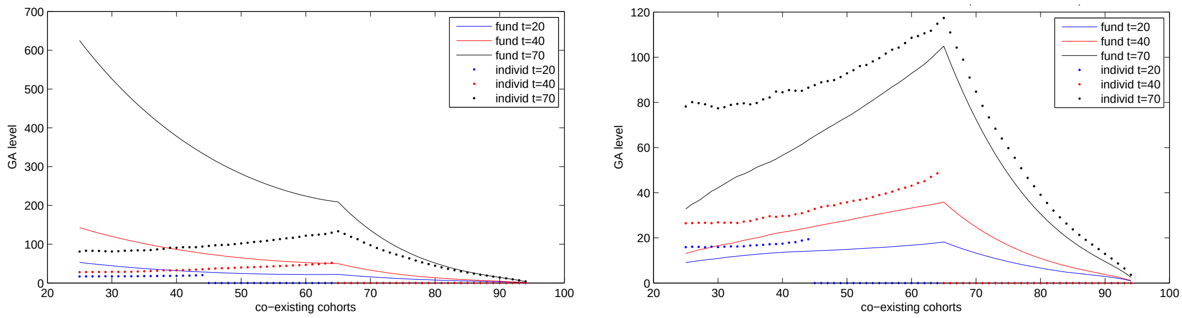
A : GA in Baseline Model

Figure 20: Generation Account Value in Time for DB-“hard” Plan



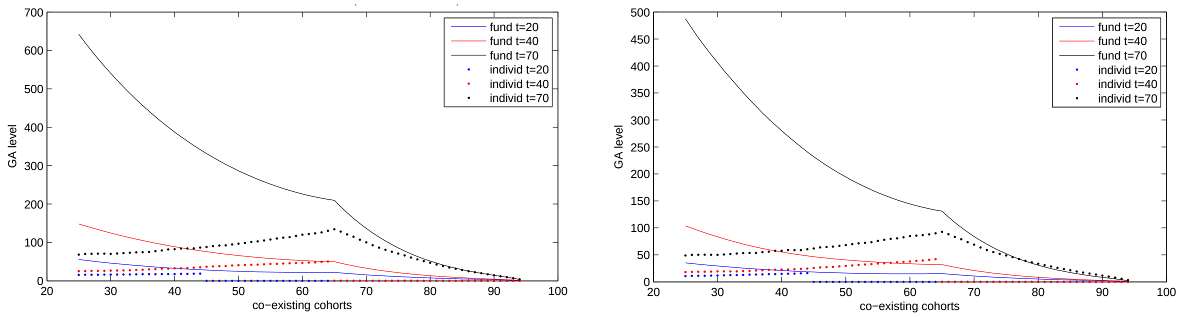
Source: Model: baseline; Plan: DB-“hard”; Framework: no PSC (on the left), yes PSC (on the right); time: year 20, 40 and 70.

Figure 21: Generation Account Value in Time for DB-“soft” Plan



Source: Model: baseline; Plan: DB-“soft”; Framework: no PSC (on the left), yes PSC (on the right); time: year 20, 40 and 70.

Figure 22: Generation Account Value in Time for CDC Plan



Source: Model: baseline; Plan: CDC; Framework: no PSC (on the left), yes PSC (on the right); time: year 20, 40 and 70.