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The optimal portfolio for the mandatory pension system – the case of Poland

Abstract: In this study we challenge two questions regarding the asset allocation between the social security (NDC) and capital (FDC) pillars assuming that the objective of the mandatory system is to provide the minimum consumption floor in the elderly age. Firstly, should we rely only on NDC or maybe on both pillars? Secondly, what is the optimal portfolio structure if we plan to diversify the pension contributions? We deliver the answers to the aforementioned questions by running a set of Monte Carlo simulations using the copula function approach to approximate the tail's probability. Our results clearly confirm that the role of capital pillar should be greater than it is nowadays and the evidence on time-varying (life-cycle) approach to assets allocation is mixed and depends on the shortfall probability.

Keywords: mandatory pension system, pension funds, dynamic portfolio optimization, Roy's Safety-First criterion

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1. Introduction

The Polish pension reform which took place in 1999 aimed to shift from the defined benefit (DB) to defined contribution (DC) system. From that moment the pension benefits became, at least in theory, tightly linked to the total of contributions accumulated on individual accounts. This fundamental reform was motivated by the society's ageing process. Otherwise the working generation would experience the growing tax burden, which could hamper further the economy's potential. Though, assuming that no budgetary pensions' subsidizing will be available, there emerges a question about the value of a replacement rate since poverty reduction in the old age is one of the main objectives of the public pension system.

In order to manage this poverty risk the government has decided to divide the mandatory contribution into two pillars (non-financial and financial one) and the risk-reduction view of this move has been contained in a meaningful statement *Security through diversity* (Warsaw: Office of the Government Plenipotentiary for Social Insurance Reform, 1997). However, the division of the contribution between the pillars and the investment policy employed in the financial pillar was driven not only by the poverty reduction motive but was also heavily influenced by the current budgetary needs[1] and the expected impact of the reform on the development of the local financial market (Holzmann, 2009). Hence, it was not surprising that some further research studies claimed for different regulations in this matter.

The objective of this work is to provide the estimate of optimal pension contribution division which would minimize the risk of not achieving the predetermined replacement rate (shortfall probability) in the decumulation (retirement) period. To achieve this goal we perform a set of Monte Carlo (MC) simulations using the copula function approach to approximate the tail's probability. The results of our study show that the existing rules are far from optimal according to the employed criterion. We hope this may shed a new light on regulatory policy.

Our study contributes to the existing literature by accounting for a few facts which have been only partly addressed or even absent in the debate. First, for our modelling task we employ the GDP projection till 2060 (European Comission, 2015) to make the conclusions really forward looking. Then, we verify if asset-mix should be time varying. Finally, imposing only a short-sale restriction, but no caps nor floors, we run an optimization procedure which enables us to provide the guidelines applicable not only to Polish conditions but also to other countries as well.

The remainder of the paper is organized as follows: Section 2. provides a brief overview of Polish pension system and recent changes in the pension landscape in CEE to

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[1] The analysis of the consequences of the introduction of financial pillar on public finance can be found *e.g.* in (Kempa, 2010) and (Égert, 2012).

highlight the relevance of the research questions investigated in this paper. Section 3. surveys the literature in the discussed area. Section 4. describes our methodological approach and data employed. Section 5. reports the empirical outcomes. Section 6. concludes the study.

2. Pension system in Poland and recent reforms in CEE region

The pension policy reform introducing a DC rule[2] has widespread across Central and Eastern European (CEE) states since the late 90' *e.g.* Latvia (1995), Hungary (1998) and Romania (2000). In Poland this fundamental reform was implemented in 1999, when two mandatory and one voluntary pillar were established. The first pillar operates on a non-financial[3] defined contribution (NDC) basis[4] *i.e.* the members' contributions are recorded on individual accounts, hence, they may be treated like claims on government. However, these receivables differ from T-bonds in terms of their legal status, which determines their default risk (probably higher comparing to financial securities). On the other hand the 1st pillar claims are not recorded, unlike T-bonds in the public debt statistics, which allows for less budgetary discipline. In the second mandatory pillar the individual accounts have also been settled, but the members collect here the financial assets (FDC). The 2nd pillar is formed by Open Pension Funds (OPF) manged by private companies. The third pillar, also of FDC type, offers supplementary occupational schemes or voluntary pension savings, which are facilitated by some tax benefits. In the decumulation phase the value of retirement from the public system (1st and 2nd pillar) is calculated as a total value of assets divided by the expected lifetime in months. Therefore, this system creates the natural incentive to stay longer in the labour market, which addresses the key economic challenge for the next decades.

In Poland, the contribution to the public pension system has been settled at the level of 19,52% of gross salary, where the 12,22% was initially transferred to the 1st and the remaining 7,3% to the 2nd pillar. Therefore, the emergence of the financial pillar reduced the budgetary revenues. Initially, this loss in public finance were to be covered by the increased revenues from privatization and the funds moved to the 2nd pillar were to be partly used to buy the equities of the privatized companies (Góra, 2003, pp. 195-196). There was also a common belief among economists that the need to balance the budget would lead to the structural reform of public finance and finally the reduction of government expenditures. Actually, it happened only partially. The economic crisis that started about 2007 adversely hit the public finance around the globe. Declining tax revenues and automatically rising social transfers led to a sharp hike in public debt/GDP ratio of many states. Therefore, it was not surprising that governments started to look

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[2] However we should note, that in some of the countries the DC rule applied only to financial pillars and the reforms were implemented gradually.

[3] Also called 'notional' instead of 'non-financial'.

[4] Sometimes it is called PAYG, however in our opinion this term is not very precise cause it hides the DC or DB nature of the employed solutions

for a fast track to provide additional budgetary financing. In some of the countries the authorities decided to temporarily suspend or reduce the contribution to the 2nd pillar *e.g.* in Estonia, Hungary, Latvia, Lithuania and Romania. In Poland since 2011 this contribution has been reduced from 7.3% to 2.3%, with a possible increase to 3.5% in 2017 and beyond. However, the Polish government did not stop at this point and since 2014 the 2nd pillar has become solely voluntary. If individual wished still to contribute to this pillar she or he had to submit the statement, otherwise the whole contribution started to be transferred to the 1st pillar solely. It is not surprising that after employing the NDC as a default option, only about 15% of the individuals decided to stay in FDC.

Together with the shifts in the contribution division between the pillars, there have been implemented the regulatory changes regarding asset allocations in FDC. Analysing investment protocols across the CEE states we observe large diversity of the employed policies. One issue is the share of local T-bonds in pension fund portfolios. In Poland at the beginning the floor was established for government securities share at 40% level. The objective of this solution was to guarantee the adequate demand for the securities and to facilitate the development of this market segment. Currently, the OPF cannot buy T-bonds any more, and the previously purchased bonds have been converted into the NDC records since 2014. At a first glance it may look like pension contributions has started to be invested very aggressively. However, we should not forget that currently the contribution to the 2nd pillar is very low (2,92%) and the remaining part (16,6%) is allocated into NDC records, which resembles the risk of T-bonds. The other question is the opportunity for pension funds to invest abroad. In Poland till 2011 there was a cap on foreign holdings at a very low 5% level[5]. The regulator intention was again to facilitate the development of local equity market. However, in some other countries like Estonia and Latvia pension funds have been allowed to allocate even 100% of their portfolio into foreign securities as the supply of securities on the local markets has seemed to be too low, hence, the risk of speculative bubble has considered to be significant. Finally, in some of the states, *e.g.* in Slovakia, pension funds can offer various portfolios in terms of the expected risk and return, while in the others, like Poland, this type of market segmentation is unavailable. Therefore, the observed divergence of regulatory solutions tends to ask the question about the main motive of the employed solutions and we cannot be sure that the dominant one was targeting the retirement wealth.

3. Literature review

There has been an intensive academic debate about the optimal time diversification of an investment portfolio, but only a few of these studies refer to the particular questions

[5] In the April 2009 European Commission accused Poland of breaking the rule of free movement of capital by imposing the excessively restrictive limits on foreign investments made by Open Pension Funds (OPF). Finally, according to the sentence of the European Court of Justice from 21st Dec 2011, Polish government was forced to remove these constraints increasing them gradually to 30% cap.

we address. In the literature review provided below we highlight these differences to clarify our research objective and indicate the areas that deserve further attention.

The studies that advocate for time-varying portfolio at first need to challenge "no time diversification" theorem developed by (Samuelson, 1969) and (Merton, 1969), (Merton, 1971). According to their works in an efficient market, an expected utility optimizer with constant relative risk aversion (CRRA) does not change the portfolio weights over time. This statement contains a few strong assumptions, however. Additionally, the theorem assumes that an investor does not possess any human capital (net present value of an individual's future labour income).

Let's challenge the postulate of market efficiency first. According to this statement the asset returns follow a random-walk process. The empirical research is mixed in supporting this view. In fact, numerous studies *e.g.* (Poterba & Summers, 1988), (Barberis, 2000), (Spierdijk, Bikker, & van den Hoek, 2012) found returns to be mean reverting, which means that volatility of equities is expected to diminish over time.

It is also quite debatable, if relative risk aversion is actually constant. While some of the works confirm this view (Chiappori & Paiella, 2011) the other seriously question this argument *e.g.* (Brunnermeier & Nagel, 2008), (Liu, Yang, & Cai, 2016).

What seems to be a really crucial argument for the time-varying lifetime portfolio choice is the inclusion of human capital into the analysis. The main idea of the theoretical works (Bodie, Merton, & Samuelson, 1992), (Campbell & Viceira, 2002) underlines the fact that human capital is a low risky asset. Additionally, the covariance between labour income and equity volatility seems to be rather limited. Consequently, as (Blake, Wright, & Zhang, 2014) state, human capital may be treated similarly to the bond-like asset, because future labour income is fairly stable over time and can be interpreted as the 'dividend' on the individual's implicit holding of human capital. In the younger age, the stock of human capital is high relative to the stock of risky equities. Therefore, to diversify the risk of large human capital stock younger investors should overweight equities in the securities portfolio. Over time, the proportion of labour asset stock to financial assets reverses and the demand for risky asset decays.

Nevertheless, two fundamental questions need to be raised there if we focus our attention on public pension system. First, what should be the portfolio optimization criterion? Second, should we manage within-investment horizon risk, end-horizon risk or both of them?

In the discussed studies the portfolio choice should maximize the utility function which describes the individually perceived trade-off between risk and return. Moreover, the most commonly employed measure of risk is standard deviation which reflects the risk definition according to its neutral concept *i.e.* both negative and positive deviations. This approach does not seem to fit the case of public pension system. The government which objective is to guarantee the consumption floor in the elderly age does not need to follow individuals' risk attitude, rather it should focus on minimizing the risk of not achieving the target, hence, should employ **the Roy's safety first criterion** (Roy, 1952). Investment performance better than expected will not induce any political tensions, but the reported shortfall will undoubtedly create the need of budgetary subsidizing increasing the tax burden on working age generation.

Secondly, in case of public pension system we should think what really matters: the end-horizon or within-horizon performance. (Chhabra, 2005, p. 4) puts this dilemma in simple words: "A stream may have an average depth of five feet, but a traveller wading through it will not make it to the other side if its midpoint is 10 feet deep". Again, in our story there is no need to focus the attention on the within-horizon risk, as **what matters only is the terminal value of wealth**. This statement is true as long as the investment horizon is known and there is no option for the contributors to withdraw the funds before the end date (Trainor, 2005). Both of these conditions seem to be satisfied for public pension system[6].

(Booth, 2004) analysed the probability of achieving the standard retirement targets such as "70@65", which means 70% replacement rate at 65, and found the asset mix to be a function of an investment horizon, where the bonds' share should grow with an investor's age. (Schleef & Eisinger, 2007) run the analysis of various asset allocation strategies rather than finding the optimal asset mix, as they considered this type of an optimization procedure as theoretically robust but computationally intractable. Surprisingly, (Schleef & Eisinger, 2007) have concluded that falling equity allocation over time fail to increase the likelihood of reaching a targeted portfolio value compared with fixed asset allocation models. (Ervin, Faulk, & Smolira, 2009) included into the analysis also social security payments which constitute a part of retirement income. Their results revealed that after accounting for social security benefits, the required share of equities in the securities portfolio is reduced. Nevertheless, in this study the social security benefits are not linked with social security contributions, which is a crucial point for any DC case, like Polish one.

In fact we find only one study that mimics closely the Polish conditions and addresses the problem of the overall pension from two pillars. (Mielczarek, 2013) has simulated the probability distributions of the retirement wealth of Polish pensioner according to two different scenarios. These scenarios reflected the regulatory shift regarding the division of pension contribution between the two pillars, which happened in Poland in 2011, as described in Section 2. Using MC simulation (Mielczarek, 2013) revealed that the regulation establishing a greater contribution to the 2nd pillar resulted in a higher terminal value of wealth and lower shortfall probability. Nevertheless we still notice a potential to develop this research further. First of all, (Mielczarek, 2013) noticed that one limitation of the performed study may be the use of historical data and trends and

[6] The-end date can always be at some risk due to political decisions, but the possible deviations will not be greater than a few years, hence, it is not substantial.

there was no attempt to account for drastic changes in GDP due to, for example, financial crisis. In our study we challenge this issue by including the GDP projection into the analysis which accounts for society's ageing process. Secondly, there is also an opportunity to model the interdependence of the returns generated between the two pillars. The theory suggests that the labour and financial market cannot be absolutely unrelated, as both are driven by GDP. In fact, the empirical studies confirm this prediction indicating a limited positive relationship (Żuk P. , 2012). Third, the minimization of a shortfall risk requires a deep investigation of the tail's probability; hence, we employ the copula function approach for this purpose. Last not least, our study differs from (Mielczarek, 2013) simulation in its general idea: we do not analyse any particular asset-mix, rather we approximate the optimal one to indicate the directions of eventual regulatory changes. Therefore, we consider the aforementioned points as the existing research gap we would like to bridge.

4. Methodology and data

4.1. Copula functions

The definition of the copula function is as follows: 2-dimensional function C: $[0, 1]^2 \to [0, 1]$ is referred to as the copula function if it meets the following conditions (Joe, 1997); (Nelsen, 2006):

a) $C(u, v)$ is an increasing function regarding u and v,
b) $C(u, 0) = 0$, $C(0, v) = 0$,
c) $C(u, 1) = u$, $C(1, v) = v$.

The importance of the copula function in the multivariate analysis stems from Sklar's theorem (Sklar, 1959); (Schweizer & Sklar, 1974):

Let H be the two-dimensional cumulative distribution function whose marginal distributions are respectively denoted by F and G. Then there is a relationship C where

$$H(u, v) = C(F(u), F(v)). \qquad (1)$$

If the F and G are continuous, then C is uniquely determined. In addition, if F and G constitute cumulative distribution functions, the function H defined by the above equation is the two-dimensional cumulative distribution.

In other words, this function gives the full dependence structure between the marginal cumulative distributions, creating along with these distributions a multidimensional one, of course subject to the assumptions set forth above.

4.2. Data

For the estimation of the copula model we used historical yearly logarithmic returns for the period 1995-2015.

To convert data for cumulative distributions needed for copula estimation we use t-Student distribution which relatively best reflects the financial properties of the time series.

Based on this historical data, we estimate the parameters of the copula model.

Forecast of returns on social security (ZUS) records collected in the 1st pillar is based on the following assumptions:

a) The forecasted returns consist of deterministic (trend) and stochastic (volatility) component;
b) The returns on ZUS records guarantee that ZUS is actuarially balanced. Therefore our rate of return in every period is equal to the real wage bill growth in the economy;
c) We estimate the trend of a real wage bill growth as a product of the forecasted labour productivity growth[7] and employment for Poland for the years 2015-2060 (European Comission, 2015);
d) The volatility component is based on historical volatility of Polish GDP growth rate for the years 1995-2015;
e) The interdependence between the volatility component of ZUS returns and the volatility component of equity returns has been based on the estimates provided by (Żuk P. , 2013).

Similarly to ZUS returns the forecast of Polish stock market consists of trend and volatility components. This latter one has been based on historical volatility of Polish equity index (WIG). On the other hand, the trend component is equal to the ZUS trend plus 2 or 3% risk premium according to the results provided by (Dimson, Marsh, & Staunton, 2008). Analysing the equity premia of seventeen countries, and a World index over a 106-year sample (Dimson, Marsh, & Staunton, 2008) found that on average the investors expected a premium on the World index of around 3-3.5% on a geometric mean basis[8]. What is also quite appealing is that the variation of the estimates through the decades was extremely high. It was possible to find decades with positive two digit excess returns, as well as prolonged periods of negative equity market premia. Consequently, (Dimson, Marsh, & Staunton, 2008) concluded that it would be misleading to project the future equity premium from data for the previous decade. This statement seems to be even more justified if we take into account the extremely long perspective of a future pensioner.

To sum up:

$$R_{ZUS_i} = r_{ZUS_i} + \epsilon_{ZUS_i} \quad (2)$$

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[7] Hence, we also assume that the marginal product of labour is fully paid.

[8] Therefore, our assumption of equity premium is a conservative one in order to avoid a potential overestimating of the equities share in the pension portfolio.

$$R_{EQUITY_i} = r_{EQUITY_i} + \epsilon_{EQUITY} \ (3)$$

$$r_{EQUITY_i} = r_{ZUS_i} + RP \ (4)$$

$$\epsilon_{ZUS_i} \sim T(0, \sigma_{ZUS}, \nu_{ZUS}) \ (5)$$

$$\epsilon_{EQUITY_i} \sim T\left(0, \sigma_{EQUITY}, \nu_{EQUITY}\right) \ (6)$$

where: R_{ZUS_i}- forecast of ZUS returns in period *i*,

R_{EQUITY_i}- forecast of Polish stock market returns in period *i*,

r_{ZUS_i}- forecast of ZUS trend component,

r_{EQUITY_i}- forecast of Polish stock market trend component,

RP – risk premium,

ϵ_{ZUS_i} - forecast of ZUS stochastic component,

ϵ_{EQUITY_i}- forecast of Polish stock market stochastic component,

$T(\mu, \sigma, \nu)$ – t-Student distribution with μ mean , σ standard deviation and ν degrees of freedom.

Pseudoobservations from both T distributions are generated using estimated t-Student copula functions with parameter $\theta = 0.3$ which is consistent with estimates provided by (Żuk P. , 2013).

We use estimated copula model parameters to simulate 30 or 35[9] two-dimensional observations in a form of cumulative distributions values. Next, those values were converted into annual deviations from a trend established before by using the parameters of the one-dimensional t-Student distributions.

We optimize the weights of the portfolio, with optimization goal to minimize the shortfall probability. The portfolio weights are optimized every year.

We repeat the process of simulating data and optimizing the weights of the portfolio 100 times using Monte-Carlo simulation of two-dimensional distributions modelled by copula function. We average the weight of individual portfolio items for each year and that is the result of our estimations.

Because there is a net positive cash flow from premia each year, we use the true time-weighted rate of return (TWROR).

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[9] Depending on the working life duration scenario.

TWROR is a measure of the historical performance of an investment portfolio, which compensates for external flows. We define the external flows as net movements in portfolio's value that result from transfers of cash into or out of the portfolio, with no equal and opposite movement of value and that are not income from the investments in the portfolio, such as interest, coupons or dividends. Hence, in our case, the external flows simply refer to pension contributions. To compensate for these flows, the overall time interval under analysis is divided into contiguous sub-periods at each point in time within the overall time period whenever there is a flow of contributions. The returns over the sub-periods between external flows are linked geometrically (compounded), *i.e.,* by multiplying the growth factors in all sub-periods – the growth factor in each sub-period is equal to 1 plus the return over the sub-period (Bacon, 2003) (Feibel, 2003). Therefore, the TWROR is:

$$1 + TWROR = \frac{W_1 - CF_1}{W_0} \times \frac{W_2 - CF_2}{W_1} \times \frac{W_3 - CF_3}{W_2} \times \ldots \times \frac{W_n - CF_n}{W_{n-1}}, \quad (7)$$

where W_0 is the initial portfolio value, W_t is the portfolio value at the end of sub-period *t*, immediately after external flow CF_t, W_n is the final portfolio value, CF_t is the net external flow into the portfolio that occurs just before the end of sub-period *t*, and *n* is the number of sub-periods.

The assumed income profile during the life cycle has been based on the estimates provided by (Kolasa, 2012) and the contribution rate to the mandatory pension system is equal to the true value, *i.e.* 19.52% of gross salary. We assume two scenarios of the working life duration (WLD), *i.e.* 30 and 35 years[10]. The expected lifetime after being retried is equal to 20 years.

The targeted aspiration level means the value of the collected capital which is enough to receive the monthly benefit equal to the targeted replacement rate relative to the last salary.

5. Empirical results

We start our verification procedure from analysing the scenarios where the whole pension contribution (19.52%) is invested in social security records. The results have been presented in Table 1.

[10] According to Eurostat data from 2015, the duration of working life for Poland was 32.6, while the European Union (28 countries) average was 35.4.

Table 1. The portfolio performance - no diversification between the two pillars, social security records only.

Aspiration Level (Targeted replacement rate)	**40%**	**50%**	**60%**
Shortfall probability (30 yrs WLD)	0,62	0,84	1
Shortfall probability (35 yrs WLD)	0,56	0,67	0,93

Source: own study.

We also performed the similar analysis for the scenarios where the whole pension contribution is invested in local equities (Table 2.).

Table 2. The portfolio performance - no diversification between the two pillars, local equities only.

Aspiration Level (Targeted replacement rate)	**40%**	**50%**	**60%**
Shortfall probability (30 yrs WLD, 2% risk premium)	0.0993	0.1423	0.5629
Shortfall probability (35 yrs WLD, 2% risk premium)	0.0001	0.0002	0.0717
Shortfall probability (30 yrs WLD, 3% risk premium)	0.0788	0.1129	0.4576
Shortfall probability (35 yrs WLD, 3% risk premium)	0.0001	0.0001	0.0469

Source: own study.

We notice that relying solely on the social security assets results in the substantial shortfall probability. Additionally, both for social security or equity scenarios the factor that mostly determines the reduction of this probability is the increase in the WLD.

In the next step we run the optimization procedure with nonnegative constraint on the portfolio weights assuming two-asset universe: social security records and local equities.

Table 3. Two-asset portfolio performance.
The assumed equity risk premium equal to 3%.

Aspiration Level (Targeted replacement rate)	**40%**	**50%**	**60%**
Shortfall probability (30 yrs WLD)	≈0	0.0076	0.1072
Shortfall probability (35 yrs WLD)	≈0	≈0	0.0019
The average weight of ZUS records during the investment period (30 yrs WLD)	65.13%	37.06%	28.28%
The average weight of ZUS records during the investment period (35 yrs WLD)	70.45%	62.62%	41.53%

Source: own study.

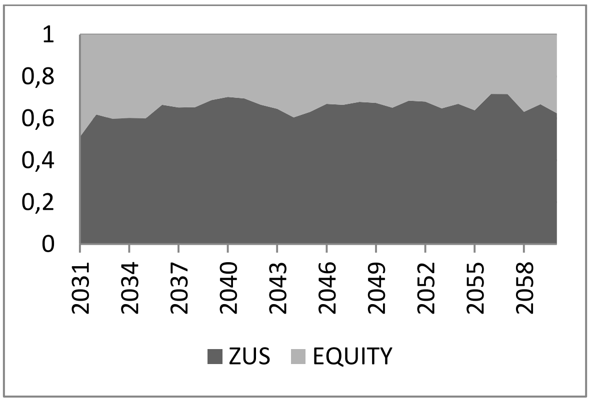

Figure 1. Asp. level 40%, 30 yrs WLD, 3% equity prem.

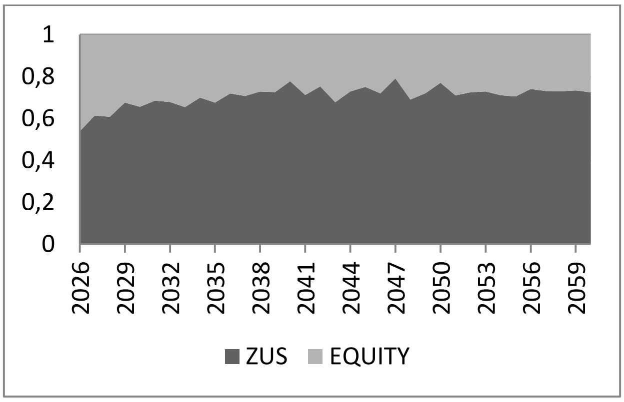

Figure 2. Asp. level 40%, 35 yrs WLD, 3% equity prem.

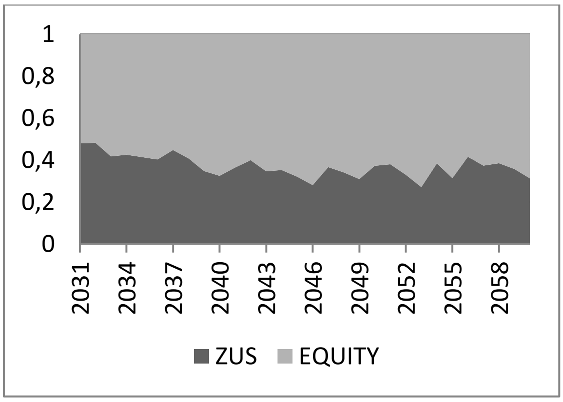

Figure 3. Asp. level 50%, 30 yrs WLD, 3% equity prem.

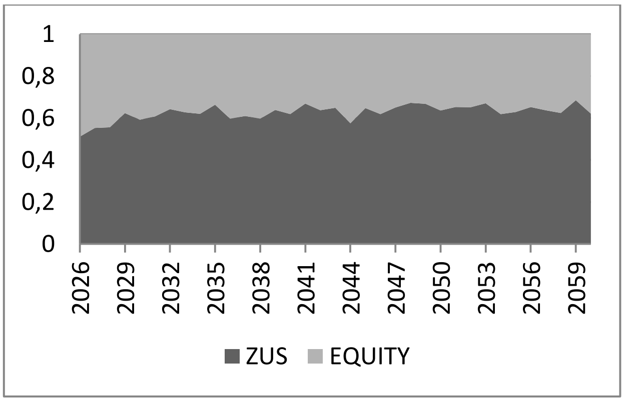

Figure 4. Asp. level 50%, 35 yrs WLD, 3% equity prem.

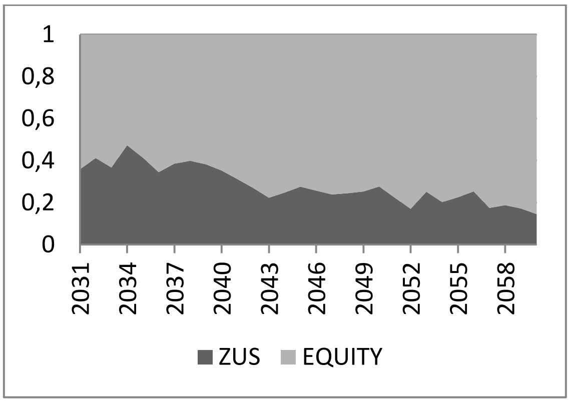

Figure 5. Asp. level 60%, 30 yrs WLD, 3% equity prem.

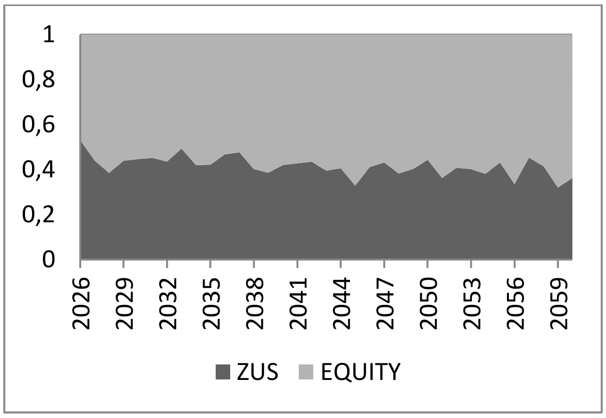

Figure 6. Asp. level 60%, 35 yrs WLD, 3% equity prem.

We observe (Table 3.) that for every targeted level of replacement rate the shortfall probability is lower respective to the scenarios based only on social security records (Table 1.) and equity holdings (Table 2.). This is definitely beneficial for the participants of the system.

To make our results more robust we run a more conservative scenario respective to the risk premium of equities (2% instead of 3%).

Table 4. Two-asset portfolio performance.
The assumed equity risk premium equal to 2%.

Aspiration Level (Targeted replacement rate)	**40%**	**50%**	**60%**
Shortfall probability (30 yrs WLD)	≈0	0.0604	0.2385
Shortfall probability (35 yrs WLD)	≈0	≈0	0.0304
The average weight of ZUS records during the investment period (30 yrs WLD)	66.59%	34.88%	33,77%
The average weight of ZUS records during the investment period (35 yrs WLD)	71.01%	61.18%	36.53%

Source: own study.

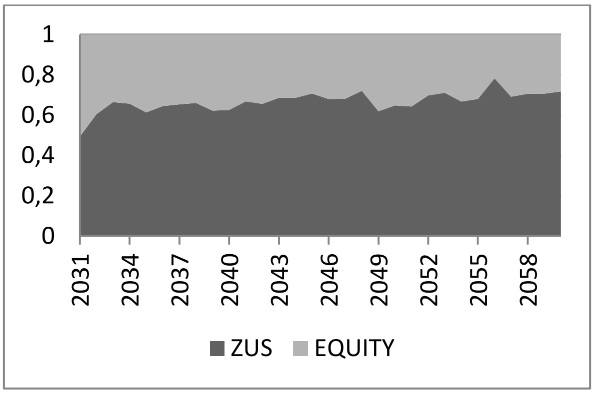

Figure 7. Asp. level 40%, 30 yrs WLD, 2% equity prem.

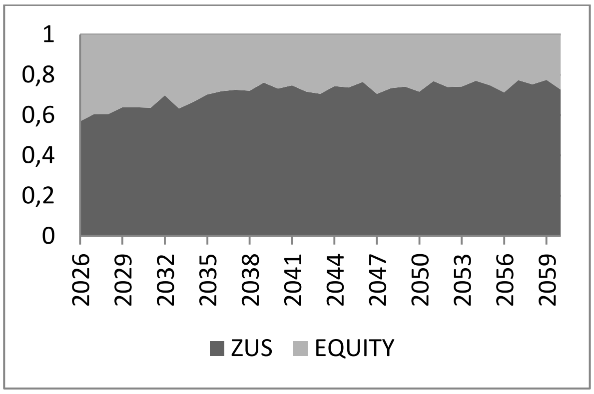

Figure 8. Asp. level 40%, 35 yrs WLD, 2% equity prem.

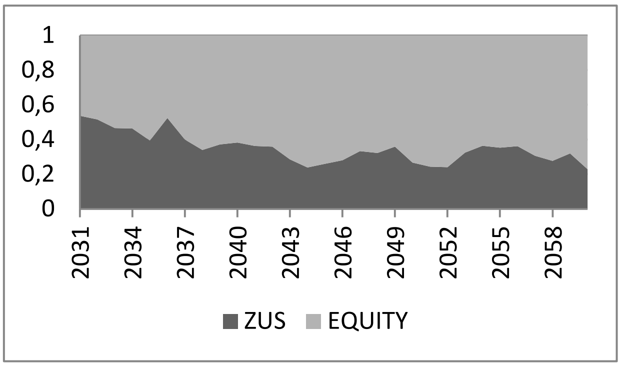

Figure 9. Asp. level 50%, 30 yrs WLD, 2% equity prem.

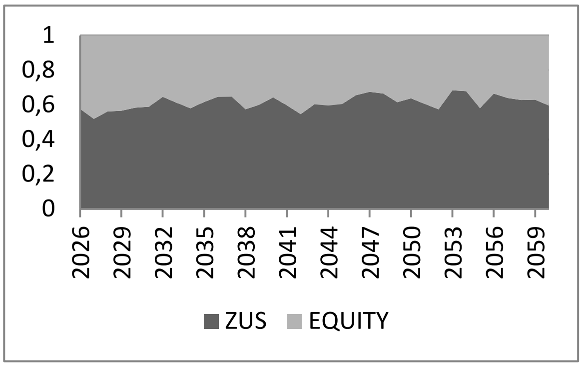

Figure 10. Asp. level 50%, 35 yrs WLD, 2% equity prem.

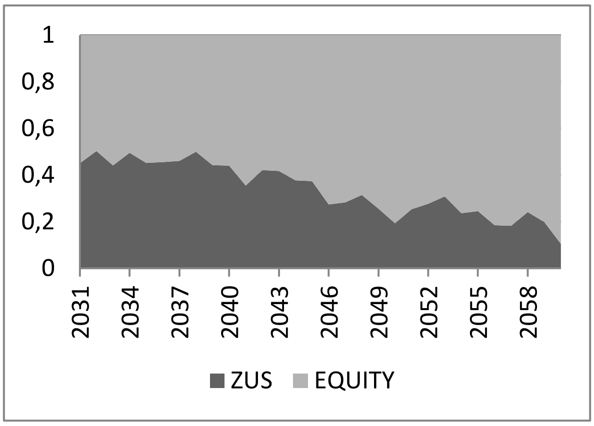

Figure 11. Asp. level 60%, 30 yrs WLD, 2% equity prem.

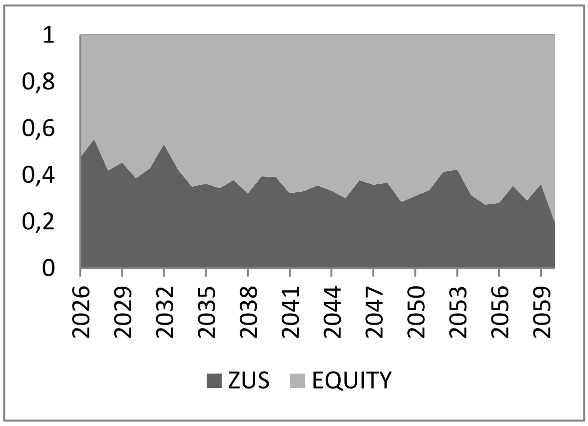

Figure 12. Asp. level 60%, 35 yrs WLD, 2% equity prem.

Even in this case the results are strongly supportive for the inclusion of equities (Table 4.). Moreover, the results show that the share of equities is greater than the available allocation of pension contribution in the second pillar. We conclude that, the existing regulations are far from optimal ones that minimize the shortfall probability.

Last but not least, we verify if the optimal share of equities in the portfolio is time varying. Consequently, we estimate for each analysed scenario the following regression:

$$w_{EQUITY_i} = \alpha + \beta t_i + \varepsilon \qquad (8),$$

where w_{EQUITY_i} is a share of equities in moment *i* and t_i is a time trend.

Table 5. Regression results of time-varying portfolio weights

Scenario	**β**	***p-value***
Asp. level 40%, 30 yrs WLD, 3% equity prem.	-0.00222	0.0096
Asp. level 50%, 30 yrs WLD, 3% equity prem.	0.0034	0.0011
Asp. level 60%, 30 yrs WLD, 3% equity prem.	0.088	$1.98*10^{-11}$
Asp. level 40%, 35 yrs WLD, 3% equity prem.	-0.0031	$3.29*10^{-5}$
Asp. Level 50%, 35 yrs WLD, 3% equity prem.	-0.0021	0.0002
Asp. level 60%, 35 yrs WLD, 3% equity prem.	0.0024	0.0004
Asp. level 40%, 30 yrs WLD, 2% equity prem.	-0.00382	$5.01*10^{-5}$
Asp. level 50%, 30 yrs WLD, 2% equity prem.	0.00689	$9.97*10^{-6}$
Asp. level 60%, 30 yrs WLD, 2% equity prem.	0.0121	$4.72*10^{-14}$
Asp. level 40%, 35 yrs WLD, 2% equity prem.	-0.0042	$9.82*10^{-6}$
Asp. Level 50%, 35 yrs WLD, 2% equity prem.	-0.0018	0.0039
Asp. level 60%, 35 yrs WLD, 2% equity prem.	0.0048	$4.87*10^{-6}$

Source: own study.

In fact the obtained results are mixed (Table 5.). We notice the time-decreasing share of equities for the scenarios where the shortfall probability was found to be low, and time-increasing for the risky scenarios.

6. Conclusions

Our findings enable us to formulate two general conclusions. Firstly, the division of the pension contribution between the two pillars is definitely beneficial for pension participants and the share of capital pillar should be greater than available. This confirms the findings obtained by (Mielczarek, 2013). Even for the lowest analysed replacement rate (40%) the average optimal weight of equity is far greater than the one established by the regulator. At the same time we notice that the shortfall probability for this investment target is nearly zero, hence; the reduction of the contribution to the capital pillar should be recognized as unfavourable for the pension system participants both in terms of risk and return. Secondly, by using the optimization approach we confirm the need of life-cycle portfolio management; however the results heavily depend on particular scenario.

Nevertheless, we see the potential to develop this study. It is intriguing to verify if the time-varying asset weights would hold under optimizing the 'within-horizon' performance not the 'end-horizon' one, like in our scenarios. However, this approach would be more appropriate for voluntary pension savings. Next, the inclusion of foreign assets may change the optimal weights. This issue is gaining the importance as in the last two years the OPFs foreign holdings were steadily growing. We hope to challenge these questions in further research.

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