

Shadow Prices, Constraints and Intergenerational Transfers through Pay-As-You-Go Social Security

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Should Social Security's benefits in the United States be raised because they are relatively low compared to some other countries or should they be lowered as part of a reform to restore solvency? While these questions are ultimately political, since either approach is economically feasible, this paper argues that economics has something to say about the likely outcome. This paper presents a model based on shadow prices and constraints that permits an analysis of intergenerational transfers through Social Security using the tools of standard price theory. This paper also addresses the question as to what would be a sustainable benefit formula in the long term for Social Security, given the assumption that (at some point) further increases in the payroll tax rate are unacceptable. It then considers issues of political sustainability. A social security benefit formula or automatic adjustment formula that is financially sustainable may not be politically sustainable. That situation could occur if it leads to a decline in the benefit replacement ratio that is politically unsustainable, necessitating further changes in the social security program.

Introduction

Should Social Security's benefits be raised because they are relatively ungenerous, as advocated by Democratic Presidential candidate Bernie Sanders? Or should they be lowered as part of a reform package to restore solvency, as advocated by Republican Presidential candidate Ted Cruz (Center for Retirement Research at Boston College 2015)? Social Security is projected to have insufficient funds to pay promised benefits in 2034 (Social Security Trustees 2015). While these questions are ultimately political, since either approach is economically feasible, this paper argues that economics has something to say about the likely outcome. This paper presents a model based on shadow prices and constraints that permits an analysis of Social Security reform using standard price theory.

While a number of countries have converted their social security systems to include defined contribution plans, many countries retain traditional pay-as-you-go systems. Because of population aging, which raises the old-age dependency ratio, these systems are under stress. Countries are questioning whether they can afford to maintain the generosity of their social security old-age benefits. Advocates of defined contribution plans have criticized traditional pay-as-you-go systems as being financially unsustainable. The second part of the paper analyzes the Social Security benefit formula to explain why the benefit formula is unsustainable and what a sustainable benefit formula would look like.

Since the late 1990s, starting with an innovative reform in Sweden, a growing number of countries have reformed their traditional social security systems to include automatic adjustment mechanisms. A number of different adjustment mechanisms have been developed. At least twelve countries have adopted life-expectancy indexing of benefits or automatic adjustments tied to social security insolvency. Both types of reforms provide automatic adjustment mechanisms

for sustaining the solvency of social security systems, but most of them do not assure solvency over the long run, requiring additional adjustments. The paper analyzes the features of a sustainable automatic adjustment mechanism, which are similar to the features of a sustainable benefit formula.

A benefit formula or automatic adjustment mechanism that is financially sustainable may not be politically sustainable if it leads to reductions in the benefit replacement rate that are politically unacceptable. In the final section, the paper considers further adjustments to the Social Security program that may be required beyond those required for financial sustainability.

The paper begins with a discussion of the role of shadow prices and constraints in affecting the outcome of Social Security reform.

Shadow Prices and Constraints

With pay-as-you-go financing, Social Security faces the pay-as-you-go budget constraint that the annual inflow of revenue must equal the outflow of benefits

$$BN = twL \quad (1)$$

where B is the average annual benefit, N is the total number of Social Security beneficiaries, BN is total annual benefit payments, t is the payroll tax rate, w is average annual wage income, L is the total number of Social Security covered workers and twL is total annual Social Security contributions. This constraint is a hard constraint in that it is determined by the financing requirements of a pay-as-you-go system.

In addition to this constraint, two soft constraints are defined by the differing views of the population as to the acceptable role of government in providing retirement benefits. The first soft constraint is that there is a maximum acceptable level for the Social Security payroll tax rate t_{\max} , so that the actual payroll tax rate must be less than or equal to t_{\max} .

$$t \leq t_{\max} \quad (2)$$

People differ as to what that rate is, and its level thus depends on the political strength of the different viewpoints. Sweden, for example, has adopted the policy that all future adjustments to its social security system will be made through changes in benefit levels, with the social security payroll tax rate having reached its maximum acceptable level (Turner 2004).

In addition, a politically determined minimum acceptable level for the generosity of Social Security benefits also acts as a soft constraint. The generosity of Social Security benefits is typically measured by the Social Security replacement rate, which is the ratio of Social Security benefits to wages $\frac{B}{w}$. Thus, there is a minimum acceptable replacement rate $\left(\frac{B}{w}\right)_{\min}$, with the actual replacement rate being greater or equal to that rate.

$$\frac{B}{w} \geq \left(\frac{B}{w}\right)_{\min} \quad (3)$$

In Sweden, because all adjustments to Social Security are made by cutting the generosity of benefits, the replacement rate has declined over time. At some point, it can be predicted that the replacement rate will have fallen to a level that the political consensus is that further cuts are not politically acceptable.

The old-age dependency ratio for Social Security is the ratio of beneficiaries to covered workers $\frac{N}{L}$. It is widely recognized that an increasing old-age dependency ratio, which is caused by population aging, increases the difficulty of financing Social Security benefits. Turner (1984) shows that the old-age dependency ratio acts as a shadow price p for Social Security benefits in the context of a pay-as-you-go system.

$$p = \frac{N}{L} \quad (4)$$

To demonstrate this concept, when the ratio of beneficiaries to workers is 1 to 10, it costs each worker \$.1 to raise the average benefit level by \$1. By comparison, when the ratio is 1 to 2, it costs each worker \$0.5 to raise the average benefit level by \$1. A similar shadow price can be calculated for federal spending on the young, but spending on the young is roughly 20 percent as large as spending on those age 65 and older (Burtless 2015), and is not considered here.

These constraints and the shadow price can be used to form the basis of a price-theoretic analysis of the level of Social Security benefits in Social Security reforms. The demand for the level of Social Security benefits can be written as a function of earnings and the shadow price of Social Security benefits.

$$B^d = B(w, p) \quad (5)$$

The demand function can be written in percentage change form as

$$E(B^d) = a_1 E(w) + a_2 E(p) \quad (6)$$

where E is the percentage change operator (the derivative of the natural logarithm), a_1 is the income elasticity of demand for the level of Social Security benefits, which is positive, and a_2 is the price elasticity, which is negative.

The pay-as-you-go constraint can also be written in percentage change terms as

$$E(BN) = E(twL) \quad (7)$$

Equation 2 is a dynamic budget constraint. It indicates that for social security to maintain financial balance over time, the growth rate in total real benefit payments must equal the growth rate in total real payroll tax payments.

Splitting the dynamic budget constraint into its component parts, equation 7 becomes

$$E(B) + E(N) = E(t) + E(w) + E(L) \quad (8)$$

The growth rate in total social security contributions equals the sum of the growth rates of the payroll tax rate, average real wages, and the labor force. The growth rate in total benefits equals the sum of the growth rate of benefits per beneficiary and the growth rate of beneficiaries.

Expressing the equation in terms of the percentage change in benefits gives

$$E(B) = E(t) + E(w) + E(L) - E(N) \quad (9)$$

Because the policy interest concerning benefit levels relates to the replacement rate $\frac{B}{w}$, equation

9 can be rewritten as

$$E\left(\frac{B}{w}\right) = E(t) - E\left(\frac{N}{L}\right) \quad (10)$$

Thus, if the tax rate has reached its maximum acceptable level ($E(t) = 0$), the change in the replacement rate is not determined by the income and price elasticities but by the requirements of the pay-as-you-go budget constraint.

$$E\left(\frac{B}{w}\right) = -E\left(\frac{N}{L}\right) \quad (11)$$

Since with population aging, the rate of growth of the old-age dependency ratio is positive, the policy outcome must be that the replacement rate will decline at the same rate that the old-age dependency ratio is increasing.

Assuming that the payroll tax rate has not reached its acceptable maximum level, the effect of policy changes on the benefit replacement rate can be analyzed in terms of the levels of the wage and price elasticities. The benefit demand equation 6 can be rewritten in terms of replacement rates by subtracting the percentage change in wages $E(w)$ from both sides of the equation

$$E\left(\frac{B}{w}\right) = (a_1 - 1)E(w) + a_2E(p) \quad (12)$$

where the superscript D indicating demand has been suppressed for simplicity. If the income price elasticity equals 1, the outcome of policy reform on the Social Security replacement rate will depend entirely on the price elasticity. With a negative price elasticity, because of the increase in the shadow price (increasing old-age dependency ratio), the policy reform will result in a decreasing Social Security replacement rate. Thus, under conditions of population aging, a necessary condition for policy reform to result in an increase in the Social Security replacement rate is for the income elasticity of demand for Social Security benefits to be greater than one. For example, depending on the percentage changes in income and in the old-age dependency ratio, the policy reform outcome could be an increase in the generosity of Social Security benefits, as measured by the replacement rate, if the income elasticity were sufficiently high and the price elasticity sufficiently low (in absolute value).

According to the intermediate estimates of the Social Security actuaries, real covered wages will grow by 1.17 percent per year between 2015 and 2035. Over that period, the number of OASDI beneficiaries per 100 covered workers will rise from 36 to 44, or by about 1.0 percent a year (Social Security Trustees 2015). With these growth rates, if, for example, the income elasticity was 1.2 and the price elasticity was less than 0.2 in absolute value, policy reform would result in an increase in the benefit replacement rate. The greater the degree that people consider that there are no good substitutes for Social Security benefits and that Social Security benefits are a necessity, the lower would be the price elasticity in absolute value. Because the employer half of the payroll tax payment is not salient, that may lower the price elasticity. However, the more that people consider private savings and pensions to be a substitute for Social Security, the higher would be the price elasticity.

In sum, this model analyzes reform affecting the generosity of Social Security benefits in a price theoretic framework, with the outcome of reform depending on the income and price elasticities, along with the associated changes in income and the shadow price. With population aging, reform will result in an increase in the generosity of Social Security benefits only if the price elasticity is sufficiently small in absolute value and the income elasticity exceeds one.

Requirements for Financial Sustainability in Social Security Financing

The basic conditions concerning financing pay-as-you-go Social Security from the perspective of optimality of resource use were developed by Samuelson (1958) and extended by Aaron (1966). Pay-as-you-go Social Security can improve welfare if the sum of the rate of growth of population plus real wages exceeds the real interest rate. This section addresses a different and more pragmatic issue. It addresses the issue of the financial sustainability of Social Security. Are the benefit formula and the financing mechanism together sustainable over the long term?

The mathematics of pay-as-you-go systems clarifies the role of indexing implicit in Social Security benefit formulas with respect to both economic and demographic changes. It indicates what type of benefit formula or automatic adjustment mechanism is needed to maintain sustainability of social security financing.

We begin the analysis of this section by returning to the dynamic budget constraint as expressed in equation 9

$$E(B) = E(t) + E(w) + E(L) - E(N) \quad (9)$$

For countries where the payroll tax rate t is fixed ($E(t)=0$), having reached the maximum level considered politically acceptable, the dynamic constraint for a sustainable benefit formula can be seen in equation 13

$$E(B) = E(w) - E\left(\frac{N}{L}\right) \quad (13)$$

Equation 13 can be interpreted as a dynamic benefit formula that is consistent with sustainable pay-as-you-go financing when the payroll tax rate is fixed. It indicates that a sustainable social security program with pay-as-you-go financing would have benefits growing at less than the real wage earnings growth rate. They would grow at the rate of real wage earnings growth less an adjustment for the rate of growth in the old-age dependency ratio. Adjustment mechanisms or benefit formulas that are not consistent with equation 13 will not be sustainable over the long run. The U.S. Social Security benefit formula has benefits growing at the rate of the real wage growth rate over the long term, and is thus not sustainable with population aging and a fixed payroll tax rate.

In sum, the Social Security budget constraint limits countries' Social Security options. If countries have decided that they will not raise the Social Security payroll tax rate, their choices are further limited. Because of falling birth rates and increasing life expectancy at older ages, the number of beneficiaries is growing faster than the number of workers. In this situation, the Social Security budget constraint indicates that countries must reduce the generosity of Social Security benefits relative to wages. With a fixed early retirement age, this means that the replacement rate must fall. Benefit formulas and automatic adjustment mechanisms that are not consistent with this constraint will ultimately fail to be sustainable.

The assumption of a fixed payroll tax rate appears to apply for some countries, and may eventually apply for most countries after future increases have caused the rate to reach the highest level that is politically feasible. Even in those situations, however, there may be people who disagree with the political consensus and favor instead maintaining the replacement rate ($E(B/w)=0$) so as to preserve the level of generosity of the Social Security program. In that case, equation 10, with rearrangement of terms, becomes

$$E(t) = E(N/L) \quad (14)$$

Thus, if the replacement rate is fixed so as to maintain the generosity of the social security program, the payroll tax rate must increase at the same rate as the old-age dependency ratio.

This analysis thus far has taken the old-age dependency rate as being determined by demographics, given a fixed social security benefit claiming age (retirement age). However, an alternative approach is to raise the eligibility age for social security benefits. From equation 10, if both the replacement rate and the payroll tax rate are considered fixed, social security solvency can still be maintained by raising the eligibility age over time so as to keep the old-age dependency rate constant

$$E(N/L) = 0 \quad (15)$$

Equation 15 assumes that raising the eligibility age is done in such a way that benefits received at the new age are the same as those received at the previous age.

Application to the U.S. Social Security System

The basic U.S. Social Security benefit formula maintains a constant replacement ratio over time and thus can be represented in percentage change terms as the following

$$E(B/w) = 0 \quad (16)$$

This benefit formula is sustainable without increases in the payroll tax rate, so long as the old-age dependency ratio is stable or declining. Thus, the Social Security benefit formula was stable for years while the Baby Boom generation was swelling the ranks of the workforce and the old-age dependency ratio was declining.

The current Social Security benefit formula is no longer sustainable with a fixed payroll tax rate because the old-age dependency ratio is declining, which implies a replacement rate that declines over time at the same rate as the increase in the old-age dependency ratio, as shown in equation 11. The increase in the Normal Retirement Age to age 67, which is currently being

phased in, provides a temporary period when the replacement rate is declining, but that is not an inherent aspect of the Social Security benefit formula.

As discussed earlier, calculations using the Intermediate Assumptions for the 2015 Trustees Report indicate that between 2015 and 2035 the old-age dependency ratio is projected to increase at roughly 1 percent per year (Social Security Board of Trustees 2015). Using equation 11, this implies that the replacement rate for financial sustainability must decrease at 1 percent per year. This conflict between the actual Social Security benefit formula and a sustainable formula is one way of viewing the inherent problem in financing under the current Social Security benefit formula with the constraint of a fixed payroll tax rate.

Thus the financial unsustainability of the U.S. Social Security program is due to a flaw in its benefit formula that does not adjust to an increasing old-age dependency ratio. The current demographics of an increasing old-age dependency ratio plus the political economics of a seemingly fixed payroll tax rate dictate that the Social Security replacement rate must fall. It is not possible to maintain the current generosity of Social Security with an increasing old-age dependency ratio and a fixed payroll tax rate.

An Alternative Model

Voting models provide an alternative approach to analyzing the issue of what will happen to the future level of Social Security benefits. A simple model would indicate that the greater the number of beneficiaries and people near retirement age relative to younger workers, the greater the likelihood that benefits will be increased because that is in their own narrow self interest. An implication of this model is that the large Baby Boom generation would force high Social Security payroll tax rates on their children to finance increased Social Security benefits for themselves. Given the interconnectedness of different generations through families, it seems

implausible that the Baby Boom would want to do this. Also, given a median voter model, it seems implausible that they would have sufficient voting power to achieve that outcome if they desired it.

Political Sustainability

A Social Security benefit formula that is financially sustainable may not be politically sustainable. In particular, the requirement that the replacement ratio or benefit generosity level decline over time may eventually cause benefits to fall to a level of generosity that is not politically acceptable to a majority of voters. This outcome suggests policy adjustments based on the assumption that a declining replacement rate at some point is not politically sustainable.

While the dynamic benefit formula of equation 13 is sustainable in a budgetary sense, it implies a declining replacement ratio over time, and thus may not be sustainable in a political sense over long periods. Further adjustments may be needed to maintain the generosity of Social Security benefits, such as gradually increasing the eligibility age for Social Security benefits over time. Raising the eligibility age may be justified as life expectancy and health at older ages continue to improve, while the percentage of the workforce with physically demanding jobs is declining. Such an adjustment, however, would need to take into consideration the needs of workers unable to continue working due to unemployment, the physical difficulty of their work, or their own health. A number of countries have made this change (Turner 2007). However, this change can penalize workers who are no longer able to work – often those at the lower end of the income scale whose jobs are low skilled or have involved physical labor. Thus, other changes in programs may be needed as well, such as, perhaps, changes in disability insurance programs.

Conclusions

The paper has analyzed Social Security reform using a simplified form of a pay-as-you-go Social Security program. It shows that increasing Social Security benefit generosity, even with population aging, would be feasible, if the wage elasticity of demand is sufficiently high and the price elasticity is sufficiently low. In practice, the elasticity parameter values required appear to be unlikely.

The paper goes on to consider the characteristics of a sustainable Social Security benefit formula, and why the current formula is not sustainable. While it considers several financing options, others could be adopted. For example, other sources of financing besides the payroll tax rate could be used. Some countries rely to some extent on general revenue financing.

The Social Security pay-as-you-go budget constraint can be analyzed to determine the properties of sustainability for social security programs, either through the structure of their benefit formulas or through automatic adjustment mechanisms. When countries have reached the point where further increases in the payroll tax rate are no longer politically feasible, the implications for the generosity of Social Security benefits are clear. With increasing old-age dependency ratios, the generosity of benefits, as measured by the replacement ratio, must decline. This decline can be offset by increasing the age of eligibility for benefits, when that is done so that the benefits received the higher age are the same as those received at the previous eligibility age.

This paper develops a benefit formula for pay-as-you-go social security programs that will assure solvency over the long run. The proposed benefit formula automatically adjusts to economic and demographic changes in a way that is stable and sustainable. The paper demonstrates that for a country that has reached its maximum acceptable social security payroll tax rate, a social security system with a benefit formula that sets the growth in average real

benefits over time equal to the growth in the real wage minus the growth in the old-age dependency ratio will be sustainable with respect to demographic and economic fluctuations. Social Security programs, such as that in the United States, which set the rate of growth of real benefits per beneficiary equal to the rate of growth of real wages, which maintains a constant replacement rate over time, are not sustainable over the long run.

While the proposed benefit formula is sustainable in a budgetary sense, it implies a declining replacement ratio over time, and thus may not be sustainable in a political sense over long periods. Further adjustments may be needed to maintain the generosity of social security benefits, such as gradually increasing the early retirement age over time as life expectancy and health at older ages continue to improve, while the percentage of the workforce with physically demanding jobs is declining. Such an adjustment, however, would need to take into consideration the needs of workers unable to continue working due to unemployment, the physical difficulty of their work, or their own health.

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