

Demographic Structure, Adjustment Costs, and Capital Price *

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Abstract

Population of advanced economies is aging rapidly while emerging countries follow closely the same transformation. The paper investigates the impact of demographic structure changes on capital prices. Demographic structure is mainly driven by three phenomena: the global fertility rate, the increase in life expectancy and the postponing of motherhood. We study the impact of these three factors on capital price in a three period overlapping generations model for a closed economy with adjustment costs. We consider two working adulthood periods during which agents may have children. The birth rates as well as the life expectancy are picked from UN's historical and projection data for France. We simulate the demographic transitions and their consequences on capital price. In order to give a more detailed analysis of demographic effects, we simulate a model of 16 generations with different behaviors for each generation. In this case, we find a price variation of around 10% due to demographic transition.

Keywords: Aging, capital price, Overlapping generations,

JEL Classification: J1, G12, E21

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1 Introduction

Over the last several decades, major advanced economies have experienced significant demographic changes. In the post second world war period, these economies exhibit important rise in the number of births. This baby-boom is a consequence of a positive transitory shock on the total fertility rate. Since the mid-sixties, however, subsequent decline in the fertility rate has occurred. Combined with this baby bust, gains in life expectancy has caused the aging of population. In addition, the postponement of motherhood is observed as a recent common trend. Longer education and higher participation rate of women to the labor market have increased the age at first birth. Aging of population is rapidly becoming a global phenomenon. For the next few decades many currently developing economies will experience similar demographic transitions.

There are many works that investigate the relationship between demographic factor and financial market (see e.g. Auerbach & Herrmann (2002)). How does demographic structure affect capital price? A common idea is that the capital price are influenced by the relative cohort size of young generation and old generation, which are considered correspondingly the buyers and the sellers of assets. The asset demands of a large working generation, such as the baby boomers, drives capital price up. When the large old generation reaches retirement, baby boomers will need to sell assets to support retirement consumption. The economy is aging and the ensuing young generation is relatively smaller. This mismatch between asset supply and demand makes the capital price decline. In this view, the demographic changes have consequences at the market-wide level.

In the literature, a great variety of economic models suggest a link between demographic structure and asset prices (see e.g Brooks (2002, 2006)). A common framework to investigate the demographic effect on asset prices is an overlapping generations (OLG) model with production and capital accumulation as in Diamond (1965) or Auerbach & Kotlikoff (1987). In standard neoclassical growth model, there are no capital adjustment costs. In this case, the price of capital is constantly equal to one unit of consumption goods. Introduction of the adjustment cost in the model admits the capital to be priced

endogenously. Abel (2003) studies the theoretical effects of a baby boom on stock prices and capital accumulation in an two overlapping generation model with adjustment cost. He finds that after an increase in stock prices during their working age, the retirement of baby-boomers leads to a meltdown in stock prices. This increase is due to the higher share of the oldest worker in the working age population who saves for his/her retirement. When the baby-boomers retire, the demographic consequences are decreases in prices as they all progressively sell their portfolio.

Empirical researches have found evidence that population age structure affects asset prices, but the consistency of this evidence is not overpowering (e.g. Poterba (2004)). Even if theoretical models suggest an association between population age structure and asset prices, it is difficult to find robust evidence of their correlation. Poterba (2001) analyzes the historical relationship between population structure and real returns from investment in Treasury bills, long-term government bonds and stocks. His estimates offered weak evidence of a demographic impact on returns for the US, Canada, and the U.K. Geanakoplos et al. (2004) suggest that demographic shocks could have impacts on asset values, but is not sufficient to explain the actual peak-to-trough movements in the stock market. Actually, the movements are two to three times greater than what can be explained by the demographic analysis. Takáts (2010) investigates the impact of aging on housing price in 22 advanced economies and find significant evidences of the effects of demographic factors on real house prices. Davis & Li (2003) also find that an increase in the fraction of middle-aged people (aged 40-64) tends to boost real asset prices. Brunetti & Torricelli (2010) support the influence of demographics on financial markets in the Italian case. Favero et al. (2015) find that the slowly evolving age structure of population fits well the persistent component in interest rate.

The complication of these empirical researches is the choice of the dependent variables that represent the asset prices and the independent variables that represent demographic factors. Besides econometric specification problems, the estimated effects are usually sensitive to the periods and to the countries included in the sample. These unfortunately unsatisfactory findings in empirical studies make the simulated models an important in-

strument for understanding and evaluating the conceivable consequences of demographic transition. This might be due to the very low frequency of demographic movements.

In this paper we study the impact of demographic structure changes on capital price. The aging of the population may result either from a decrease in the total birth rate, an increase in life expectancy or a postponing of the average age of motherhood. Baby boom has consequences on population structure transitory dynamics, but contrarily to what is usually thought, it has no particular impact on aging as the larger number of baby-boomers have a larger number of children.

We build on Abel (2003) by adding another generation to his model. We have therefore a three period OLG model with adjustment cost. Our population structure is based on Pestieau & Ponthière (2014) and Momota & Horii (2013) where workers are divided in two categories: young and old. This allows us to incorporate the postponement of motherhood in the model with two fertility rates: one for young workers and another for old workers.

The main goal of this paper is to study the impact of the postponing of parenthood and the increase in life expectancy on capital price. The rest of the paper is organized as follows. The second section gives a presentation of the model with a complete demographic structure characterized by three demographic parameters (two birth rates plus a surviving probability at 75 years old). The third section is devoted to an extension of our model to 16 generations. In the fourth section, we calibrate a more detailed model in order to calibrate the previous model in order to reproduce French data and capture the direct consequences of aging on capital price. Last section concludes.

2 The Model

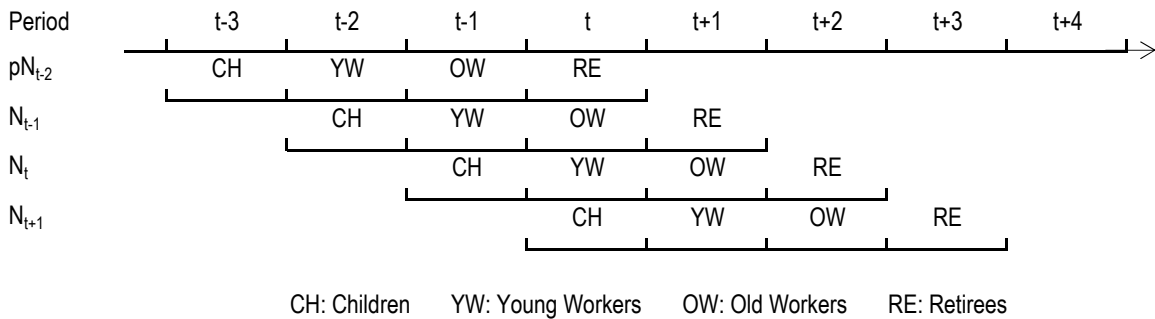
We begin with a simple model including only three overlapping generations in order to illustrate the channel of transmission of demographic shocks. In this economy, agents live for four periods but they are active decision-maker only in the last three periods of their life : young working age, old working age, and retirement. During the two

working periods, agents can have children, supply labor in-elastically, receive a wage, consume part of their incomes and save the rest. Comparing to Abel (2003), our model has a more complicated demographic structure since all workers (young and old) can have children. This additional child-bearing period allows to consider a more flexible demographic setting that includes the phenomena of delayed child-bearing. Moreover, old workers pass to retirement period with a probability p . After retirement, they consume the totality of their savings with interest.

2.1 Demography

The demographic structure is as follows, at time t , there are three groups of agents in the economy, where N_t is the number of young workers, N_{t-1} the number of old workers, and pN_{t-2} the number of retirees. N_{t+1} is the number of children born in t , in childhood, agents are not active decision-maker.

Figure 1: Demographic Structure



The model will thus have three demographic parameters: the number of child per young workers (n), the number of child per old workers (m), and survival probability at around 75 years old (p). This will allow us to describe all aging phenomena: *i*) baby-booms with transitory increases of both fertility rates, *ii*) the postponing of the average age of motherhood with a combination of permanent decrease in young's fertility rates and a permanent increase in old's fertility rate, *iii*) permanent decrease in the birth rate with permanent decrease in both fertility rates, and *iv*) the increase in survival probability at 75. The impact of the first phenomena on asset prices has been studied in Abel (2003).

The individual has two reproduction periods with respectively two exogenous birth rates n_t and m_t . So the number of young worker at the period t (born at $t - 1$) is:

$$N_t = n_{t-1}N_{t-1} + m_{t-1}N_{t-2} \quad (1)$$

And the cohort size growth factor is:

$$G_t \equiv \frac{N_t}{N_{t-1}} = n_{t-1} + \frac{m_{t-1}}{G_{t-1}} \quad (2)$$

Thus if n and m are constant, then

$$G = \frac{n + \sqrt{n^2 + 4m}}{2} \quad (3)$$

We define $G = 1 + g$ with $g = \frac{n + \sqrt{n^2 + 4m}}{2} - 1$ defined as the cohort size growth rate.

2.2 Consumers

An adult consumer in t born as a child at the beginning of period $t-1$ chooses consumption when young (c_t), in middle age (d_{t+1}), and when old (e_{t+2}) in order to maximize his lifetime welfare subject to the inter-temporal budget constraint. Let p be the probability of survival from old active period to retirement.¹ Assume that $u(\cdot)$ is the utility function, the agent born at the beginning of period $t - 1$ has the following expected lifetime utility in t :

$$U(c_t, d_{t+1}, e_{t+2}) = u(c_t) + \beta u(d_{t+1}) + p\beta^2 u(e_{t+2})$$

where β is a time preference factor with $0 < \beta < 1$. Assume that consumers have logarithmic utility,

$$u(c) = \ln(c)$$

¹Here, people age and have a probability p of surviving during the last period. They are not "perpetual youth" like in Yaari (1965) and Blanchard (1985).

The present value of the lifetime resources of this consumer is:

$$\Omega_t \equiv w_t + \frac{w_{t+1}}{R_{t+1}}$$

The budget constraints for the three periods are:

$$\begin{aligned} c_t &= w_t - s_t \\ d_{t+1} &= w_{t+1} + R_{t+1}s_t - z_{t+1} \\ e_{t+2} &= \frac{R_{t+2}}{p}z_{t+1} \end{aligned}$$

assuming a perfect annuity market².

Note that s_t is the saving of the young active agent and z_{t+1} is not the saving of the old active agent but his wealth. His saving is written as:

$$w_{t+1} + (R_{t+1} - 1)s_t - d_{t+1} = z_{t+1} - s_t$$

The life time budget constraint of an agent born at the beginning of period $t - 1$ is:

$$c_t + \frac{d_{t+1}}{R_{t+1}} + \frac{pe_{t+2}}{R_{t+1}R_{t+2}} = \Omega_t$$

By solving the optimization problem, we get the optimal consumption as:

$$c_t = \frac{1}{1 + \beta + p\beta^2}\Omega_t \tag{4}$$

$$d_{t+1} = \frac{\beta R_{t+1}}{1 + \beta + p\beta^2}\Omega_t \tag{5}$$

$$e_{t+2} = \frac{\beta^2 R_{t+1}R_{t+2}}{1 + \beta + p\beta^2}\Omega_t \tag{6}$$

and the optimal wealth as:

²This is a simplification assumption which will be raised in the simulation section in order to have realist consumption profiles.

$$s_t = \frac{1}{1 + \beta + p\beta^2} \left((\beta + p\beta^2)w_t - \frac{w_{t+1}}{R_{t+1}} \right) \quad (7)$$

$$z_{t+1} = \frac{p\beta^2}{1 + \beta + p\beta^2} (R_{t+1}w_t + w_{t+1}) \quad (8)$$

2.3 Production

Production technology uses both capital and labor. Capital must be installed in our model thus is subject to adjustment costs as in Abel (2003). The capital also depreciates at rate δ . Assume that the consumption good technology use a Cobb-Douglas production function:

$$F(K_t, L_t) = AK_t^\alpha L_t^{1-\alpha}$$

If we note $k_t \equiv \frac{K_t}{L_t}$, consumption good production per worker will be:

$$f(k_t) = Ak_t^\alpha \quad (9)$$

The depreciation is assumed to be linear thus:

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (10)$$

where $0 \leq \delta \leq 1$, and investment, I_t , is the aggregate amount of consumption goods diverted from the consumption goods technology and used as an input in the capital accumulation.

The adjustment cost function is assumed to be quadratic in investment as in Caballero et al. (2006) :

$$C(I_t, K_t) = \left(\frac{a}{2} \right) \frac{I_t^2}{K_t} \quad (11)$$

where the parameter $a \geq 0$ measures the magnitude of quadratic adjustment cost.

Let q_t be the price of capital at the end of period t, which is the quantity of consumption goods in the period t that needed to acquire an additional unit of capital for use in

period $t + 1$, w_t the wage rate and r_t , the interest rate from period $t - 1$ to t . $q_{t-1}K_t$ is the value of the capital used in t and the firm has to borrow to acquire capital. We choose to modelize only one firm that produces both the consumption and the capital goods. It maximizes its profits for one period.³ The profit of the firm is thus composed of the cash-flow corresponding to the sells of the consumption goods (Y_t) and the sells of the quantity of capital at price q_t that is going to be used in $t + 1$ minus the costs. The costs come from the production of consumption goods and the capital goods (which is different from the consumption good only by the adjustment cost). These costs are wages ($w_t L_t$), the price of installed capital needed to produce in t ($q_{t-1}K_t$), plus the interest rate service on the corresponding amount ($r_t q_{t-1}K_t$ as the capital is assumed to be financed by borrowing), and the cost of investment I_t and its corresponding installation cost $C(I_t, K_t)$ that are needed in order to get K_{t+1} . Therefore, the producer maximizes its profit through the following program:

$$\begin{aligned} \text{Max}_{\{K_t, L_t, I_t\}} \quad & \pi = F(K_t, L_t) + q_t K_{t+1} - w_t L_t - (1 + r_t)q_{t-1}K_t - I_t - C(I_t, K_t) \\ \text{s.t.} \quad & K_{t+1} = (1 - \delta)K_t + I_t. \\ \text{with} \quad & F(K_t, L_t) = AK_t^\alpha L_t^{1-\alpha} \\ & C(I_t, K_t) = \left(\frac{a}{2}\right) \frac{I_t^2}{K_t} \end{aligned}$$

Assuming that the economy is perfectly competitive, the firm takes q_t , w_t , and r_t as given to maximize its profit. First order condition are given by:

$$\begin{aligned} \frac{\partial \pi_t}{\partial L_t} = 0 & \Leftrightarrow \frac{\partial Y_t}{\partial L_t} - w_t = 0 \\ \frac{\partial \pi_t}{\partial K_t} = 0 & \Leftrightarrow \frac{\partial Y_t}{\partial K_t} + q_t \frac{\partial K_{t+1}}{\partial K_t} - (1 + r_t)q_{t-1} - \frac{\partial C_t}{\partial K_t} = 0 \\ \frac{\partial \pi_t}{\partial I_t} = 0 & \Leftrightarrow q_t - 1 - \frac{\partial C_t}{\partial I_t} = 0 \end{aligned}$$

³We could have chosen to have a two sector model: one for consumption good and another for capital good. We also could have an infinity life firm.

and one gets

$$w_t = f(k_t) - f'(k_t)k_t \quad (12)$$

$$1 + r_t = \frac{f'(k_t) + q_t(1 - \delta) + \frac{a}{2} \left(\frac{I_t}{K_t}\right)^2}{q_{t-1}} \quad (13)$$

$$q_t = 1 + a \left(\frac{I_t}{K_t}\right) \quad (14)$$

From the equation (14), we can find that the investment to capital stock ratio, $\frac{I_t}{K_t}$, is important in the determination of the capital price. By dividing both side of the equation (10) by K_t , one gets:

$$\frac{K_{t+1}}{K_t} = (1 - \delta) + \frac{I_t}{K_t} \quad (15)$$

thus, the investment to capital stock ratio, $\frac{I_t}{K_t}$, affects also the growth rate of the capital stock.

Note that the equation (13) might be written:

$$r_t + \delta \frac{q_t}{q_{t-1}} = \frac{f'(k_t)}{q_{t-1}} + \frac{\frac{a}{2} \left(\frac{I_t}{K_t}\right)^2}{q_{t-1}} + \left(\frac{q_t}{q_{t-1}} - 1\right)$$

So the gross rate of interest expressed in consumption goods is equal to the marginal productivity of capital in the consumption sector plus the marginal productivity of capital in the capital sector plus the capital gains. This is a generalization of the usual first order condition when there is no capital cost adjustment (where $a = 0$ and $q = 1$).

We can check that the profit in t of the firm $F(K_t, L_t) + q_t K_{t+1} - w_t L_t - R_t q_{t-1} K_t - I_t - C(I_t, K_t) = 0$.

2.4 Market equilibria

The labor market is at its equilibrium when the labor demand L_t is equal to the working-age population:

$$L_t = N_{t-1} + N_{t-2} \quad (16)$$

The output of the good technology is used as consumption and investment. So the equilibrium of the good market is given by:

$$F(K_t, L_t) = N_{t-1}c_t + N_{t-2}d_t + pN_{t-3}e_t + I_t + C(I_t, K_t) \quad (17)$$

The wealth market is written:

$$N_{t-1}s_t + N_{t-2}z_t = q_t K_{t+1}$$

per head of workers, one has:

$$\frac{N_{t-1}}{L_t}s_t + \frac{N_{t-2}}{L_t}z_t = q_t \frac{L_{t+1}}{L_t}k_{t+1} \quad (18)$$

2.5 Intertemporal equilibrium

The dynamics of the economy is thus defined by a system of 9 equations with 9 variables $s_t, z_t, k_t, w_t, R_t, q_t, \frac{K_{t+1}}{K_t}, \frac{I_t}{K_t}$ and L_t

$$s_t = \frac{1}{1 + \beta + p\beta^2}((\beta + p\beta^2)w_t - \frac{w_{t+1}}{R_{t+1}}) \quad (19)$$

$$z_{t+1} = \frac{p\beta^2}{1 + \beta + p\beta^2}(R_{t+1}w_t + w_{t+1}) \quad (20)$$

$$f(k_t) = Ak_t^\alpha \quad (21)$$

$$\frac{K_{t+1}}{K_t} = (1 - \delta) + \frac{I_t}{K_t} \quad (22)$$

$$w_t = f(k_t) - f'(k_t)k_t \quad (23)$$

$$R_t = \frac{f'(k_t) + q_t(1 - \delta) + \frac{a}{2} \left(\frac{I_t}{K_t}\right)^2}{q_{t-1}} \quad (24)$$

$$q_t = 1 + a \left(\frac{I_t}{K_t}\right) \quad (25)$$

$$L_t = N_{t-1} + N_{t-2} \quad (26)$$

$$\frac{N_{t-1}}{L_t}s_t + \frac{N_{t-2}}{L_t}z_t = q_t \frac{L_{t+1}}{L_t}k_{t+1} \quad (27)$$

This could be reduced to a two equation system.

$$\begin{aligned}k_{t+1} &= g(k_t, k_{t-1}, k_{t+2}) \\ q_t &= g(k_t, k_{t+1})\end{aligned}$$

2.6 Price of capital at steady state

We have:

$$\frac{k_{t+1}}{k_t} G_{t+1} = (1 - \delta) + \frac{I_t}{K_t} \quad (28)$$

with $k_t = \frac{K_t}{L_t}$ the capital labor ratio. Therefore, the investment to capital stock ratio at steady-state depends on the cohort size growth rate g and the depreciation ratio δ :

$$\frac{I}{K} = g + \delta$$

The price of capital q at steady state depends also on these two elements. From the equation (25), we have

$$q = 1 + a(g + \delta)$$

since $\frac{\partial g}{\partial n} > 0$ and $\frac{\partial g}{\partial m} > 0$, we also have

$$\frac{\partial q}{\partial n} > 0; \quad \frac{\partial q}{\partial m} > 0.$$

Thus the price of capital q increases with the two birth rates n and m at the steady state. This means that an increase of any of the birth rates leads to a larger young cohort. The subsequent growth of asset demand will raise asset prices. Oppositely, a decrease of birth rate and thus a smaller young cohort will cause a decrease of asset prices.

If we consider a particular case with $n + m = 1$, so $g = 0$, the number of births will always be equal to N for each period. At the steady state, the price of capital will not be affected by the change in birth rates as long as their sum is still equal to 1.

3 The computable general equilibrium model

We keep the spirit of the model of section 2 with constant elasticities of substitution for both utility and production, but now we allow them to be different of one.

3.1 Demographic evolution

In the following, we want to compare the relative impact on capital price of the demographic parameters. With five years for each period, we therefore have a 16 generation model (20 with the children). The behaviors of individual in each period of life are summarized in the following table.

The cohort size of generation t , born at the period $t - 4$, is determined by:

$$N_t = n_{0,t-4}N_{t-4} + n_{1,t-4}N_{t-5} + n_{2,t-4}N_{t-6} + n_{3,t-4}N_{t-7} + n_{4,t-4}N_{t-8} + n_{5,t-4}N_{t-9} \quad (29)$$

Table 1: The periods of life

Age	Period	Fertility	Survival Probability	Participation Rate	Consumption	Saving
0-4	children	-	-	-	$\mu * c$	-
5-9	children	-	-	-	$\mu * c$	-
10-14	children	-	-	-	$\mu * c$	-
15-19	children	-	-	-	$\mu * c$	-
20-24	worker and parent	$n_{0,t}$	$p_{0,t}$	$\varphi_{0,t}$	$c_{0,t}$	$s_{0,t}$
25-29	worker and parent	$n_{1,t+1}$	$p_{1,t+1}$	$\varphi_{1,t+1}$	$c_{1,t+1}$	$s_{1,t+1}$
30-34	worker and parent	$n_{2,t+2}$	$p_{2,t+2}$	$\varphi_{2,t+2}$	$c_{2,t+2}$	$s_{2,t+2}$
35-39	worker and parent	$n_{3,t+3}$	$p_{3,t+3}$	$\varphi_{3,t+3}$	$c_{3,t+3}$	$s_{3,t+3}$
40-44	worker and parent	$n_{4,t+4}$	$p_{4,t+4}$	$\varphi_{4,t+4}$	$c_{4,t+4}$	$s_{4,t+4}$
45-49	worker and parent	$n_{5,t+5}$	$p_{5,t+5}$	$\varphi_{5,t+5}$	$c_{5,t+5}$	$s_{5,t+5}$
50-54	worker	$n_{6,t+6}$	$p_{6,t+6}$	$\varphi_{6,t+6}$	$c_{6,t+6}$	$s_{6,t+6}$
55-59	worker	$n_{7,t+7}$	$p_{7,t+7}$	$\varphi_{7,t+7}$	$c_{7,t+7}$	$s_{7,t+7}$
60-64	worker	$n_{8,t+8}$	$p_{8,t+8}$	$\varphi_{8,t+8}$	$c_{8,t+8}$	$s_{8,t+8}$
65-69	worker	$n_{9,t+9}$	$p_{9,t+9}$	$\varphi_{9,t+9}$	$c_{9,t+9}$	$s_{9,t+9}$
70-74	retiree	$n_{10,t+10}$	$p_{10,t+10}$	$\varphi_{10,t+10}$	$c_{10,t+10}$	$s_{10,t+10}$
75-79	retiree	$n_{11,t+11}$	$p_{11,t+11}$	$\varphi_{11,t+11}$	$c_{11,t+11}$	$s_{11,t+11}$
80-84	retiree	$n_{12,t+12}$	$p_{12,t+12}$	$\varphi_{12,t+12}$	$c_{12,t+12}$	$s_{12,t+12}$
85-89	retiree	$n_{13,t+13}$	$p_{13,t+13}$	$\varphi_{13,t+13}$	$c_{13,t+13}$	$s_{13,t+13}$
90-94	retiree	$n_{14,t+14}$	$p_{14,t+14}$	$\varphi_{14,t+14}$	$c_{14,t+14}$	$s_{14,t+14}$
95+	retiree	$n_{15,t+15}$	$p_{15,t+15}$	$\varphi_{15,t+15}$	$c_{15,t+15}$	$s_{15,t+15}$

3.2 Consumers

We use here a constant inter-temporal elasticity of substitution (CIES) utility function,

$$u(c) = \frac{c^{(1-\frac{1}{\sigma})}}{1 - \frac{1}{\sigma}}$$

where $\sigma > 0$, $\sigma \neq 1$, σ measures the elasticity of inter-temporal substitution. For all $c > 0$, $u'(c) > 0$ and $u''(c) < 0$, so u is strictly increasing and concave. The expected lifetime utility in t is:

$$U(c_{0,t}, c_{1,t+1}, c_{2,t+2}, \dots, c_{15,t+15}) = \sum_{i=0}^{15} \left(\beta^i \cdot u(c_{i,t+i}) \cdot \prod_{j=0}^i p_{j,t+j} \right) \quad (30)$$

We now take into account the cost of children consumption in the budget constraints.

In period 0, the consumer has no saving or income, so

$$(1 + \mu\nu_{0,t})c_{0,t} + s_{0,t} = \varphi_{0,t}w_t$$

We do not assume a perfect annuity market in order to get reasonable consumption profiles. Indeed, perfect annuity market leads to very huge yields of savings at very old age. Accidental bequest is taxed by the government to finance a public good. The budget constraints for each period $i \in (1, 2, \dots, 15)$ are

$$(1 + \mu\nu_{i,t+i})c_{i,t+i} + s_{i,t+i} = \varphi_{i,t+i}w_{t+i} + R_{t+i} \cdot s_{i-1,t+i-1}$$

with

$$\nu_{i,t+i} = n_{i-3,t+i-3} + n_{i-2,t+i-2} + n_{i-1,t+i-1} + n_{i,t+i}$$

note that for $i < 0$, $n_i = 0$.

The present value of the lifetime resource is Ω_t :

$$\Omega_t \equiv \varphi_{0,t}w_t + \sum_{i=1}^9 \left(\frac{\varphi_{i,t+i}w_{t+i}}{\prod_{j=1}^i R_{t+j}} \right) \quad (31)$$

The life time budget constraint is:

$$(1 + \mu\nu_{0,t})c_{0,t} + \sum_{i=1}^{15} \left(\frac{(1 + \mu\nu_{i,t+i})c_{i,t+i}}{\prod_{j=1}^i R_{t+j}} \right) = \Omega_t \quad (32)$$

According to the first order conditions, the optimal consumption for the generation 0 at period t is:

$$c_{0,t} = \frac{\Omega_t}{(1 + \mu\nu_{0,t}) + \sum_{i=1}^{15} \beta^{i\sigma} (1 + \mu\nu_{0,t})^\sigma (1 + \mu\nu_{i,t+i})^{1-\sigma} \left(\prod_{j=1}^i p_{j,t+j} \right)^\sigma \left(\prod_{j=1}^i R_{t+j} \right)^{\sigma-1}} \quad (33)$$

and the consumptions at others periods $i \in (1, 2, \dots, 15)$ of life are:

$$c_{i,t+i} = \left(\beta^i \prod_{j=1}^i p_{j,t+j} \prod_{j=1}^i R_{t+j} \right)^\sigma \left(\frac{1 + \mu\nu_{0,t}}{1 + \mu\nu_{i,t+i}} \right)^\sigma c_{0,t} \quad (34)$$

so, the optimal individual wealth are given by:

$$s_{0,t} = \varphi_{0,t} w_t - (1 + \mu\nu_{0,t}) c_{0,t} \quad (35)$$

for $i = 0$ and the wealths at others periods $i \in (1, 2, \dots, 14)$ of life are:

$$s_{i,t+i} = \varphi_{i,t+i} w_{t+i} + R_{t+i} \cdot s_{i-1,t+i-1} - (1 + \mu\nu_{i,t+i}) c_{i,t+i} \quad (36)$$

3.3 Production

We use here a constant elasticity of substitution (CES) production function,

$$Y_t = F(K_t, L_t) = A \left(\alpha K_t^{1-\frac{1}{\rho}} + (1 - \alpha) (\Gamma_t L_t)^{1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \quad (37)$$

where $0 < \alpha < 1$, $A > 0$, $\rho > -1$, $\rho \neq 0$. Y_t is the aggregate output of consumption goods, K_t is the aggregate capital stock at the beginning of period t and L_t is the aggregate labor in period t . The elasticity of substitution between the factors of production, capital and labor, is given by ρ . $\Gamma_t = \prod_{i=1}^t (1 + \gamma_t)$ where γ_t is the rate of labor productivity growth in period t .

By maximizing the firm's profit, we have the income per effective worker,

$$\tilde{w}_t = (1 - \alpha) A \left((1 - \alpha) + \alpha \left(\frac{K_t}{\Gamma_t L_t} \right)^{1-\frac{1}{\rho}} \right)^{-\frac{1}{1-\frac{1}{\rho}}} \quad (38)$$

the rate of return,

$$1 + r_t = \frac{1}{q_{t-1}} \left(\alpha A \left(\alpha + (1 - \alpha) \left(\frac{K_t}{\Gamma_t L_t} \right)^{\frac{1}{\rho} - 1} \right)^{-\frac{1}{1-\rho}} + q_t(1 - \delta) - \left(-\frac{a}{2} \left(\frac{I_t}{K_t} \right)^2 \right) \right) \quad (39)$$

and capital price

$$q_t = 1 + a \left(\frac{I_t}{K_t} \right). \quad (40)$$

Note that the income per worker is given by:

$$w_t = \Gamma_t \tilde{w}_t \quad (41)$$

3.4 Market equilibria

The labor market is at its equilibrium when the labor demand L_t is equal to the working-age population:

$$L_t = \sum_{i=0}^9 \left(\left(\prod_{j=0}^i p_{j,t-i+j} \right) \varphi_{i,t} N_{t-i} \right) \quad (42)$$

The wealth market is written:

$$\left(\sum_{i=0}^{14} N_{t-i} s_{i,t} \left(\prod_{j=0}^i p_{j,t-i+j} \right) \right) = q_t K_{t+1} \quad (43)$$

The tax is:

$$T_t = (1 - p_{0,t}) N_t s_{0,t} (1 + r_t) + \left(\sum_{i=1}^{14} \left(\prod_{j=0}^{i-1} p_{j,t-i+j} \right) (1 - p_{i,t}) N_{t-i} s_{i,t} (1 + r_t) \right) \quad (44)$$

and the government budget balance is in equilibrium:

$$G_t = T_t \quad (45)$$

The consumption good market is:

$$F(K_t, L_t) = \left(\sum_{i=0}^{15} \left(\prod_{j=0}^i p_{j,t-i+j} \right) N_{t-i} (1 + \mu\nu_{i,t}) c_{i,t} \right) + I_t + C(I_t, K_t) + G_t \quad (46)$$

3.5 General equilibrium

$$N_t = \sum_{i=0}^5 n_{i,t-4} N_{t-i-4} \quad (47)$$

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (48)$$

$$L_t = \sum_{i=0}^9 \left(\left(\prod_{j=0}^i p_{j,t-i+j} \right) \varphi_{i,t} N_{t-i} \right) \quad (49)$$

$$w_t = (1 - \alpha) A \Gamma_t \left((1 - \alpha) + \alpha \left(\frac{K_t}{\Gamma_t L_t} \right)^{1 - \frac{1}{\rho}} \right)^{-\frac{1}{1-\rho}} \quad (50)$$

$$R_t = \frac{1}{q_{t-1}} \left(\alpha A \left(\alpha + (1 - \alpha) \left(\frac{K_t}{\Gamma_t L_t} \right)^{\frac{1}{\rho} - 1} \right)^{-\frac{1}{1-\rho}} + q_t (1 - \delta) - \left(-\frac{a}{2} \left(\frac{I_t}{K_t} \right)^2 \right) \right) \quad (51)$$

$$q_t = 1 + a \left(\frac{I_t}{K_t} \right) \quad (52)$$

$$c_{0,t} = \frac{\Omega_t}{(1 + \mu\nu_{0,t}) + \sum_{i=1}^{15} \beta^{i\sigma} (1 + \mu\nu_{0,t})^\sigma (1 + \mu\nu_{i,t+i})^{1-\sigma} \left(\prod_{j=1}^i p_{j,t+j} \right)^\sigma \left(\prod_{j=1}^i R_{t+j} \right)^{\sigma-1}} \quad (53)$$

with

$$\Omega_t \equiv \varphi_{0,t} w_t + \sum_{i=1}^9 \left(\frac{\varphi_{i,t+i} w_{t+i}}{\prod_{j=1}^i R_{t+j}} \right)$$

$$c_{i,t+i} = \left(\beta^i \prod_{j=1}^i p_{j,t+j} \prod_{j=1}^i R_{t+j} \right)^\sigma \left(\frac{1 + \mu\nu_{0,t}}{1 + \mu\nu_{i,t+i}} \right)^\sigma c_{0,t} \quad i \in (1, 2, \dots, 14)$$

$$s_{0,t} = \varphi_{0,t} w_t - (1 + \mu\nu_{0,t}) c_{0,t} \quad (54)$$

$$s_{i,t+i} = \varphi_{i,t+i} w_{t+i} + R_{t+i} \cdot s_{i-1,t+i-1} - (1 + \mu\nu_{i,t+i}) c_{i,t+i} \quad i \in (1, 2, \dots, 14)$$

$$\left(\sum_{i=0}^{14} N_{t-i} s_{i,t} \left(\prod_{j=0}^i p_{j,t-i+j} \right) \right) = q_t K_{t+1} \quad (55)$$

We simulate the stationarized equations.

3.6 Deflated General equilibrium

To stationarize the model, we deflate $Y_t, K_t, I_t, w_t, \Omega_t, c_t, sc_t, sd_t, se_t, sf_t, sg_t$ by $\Gamma_t = \prod_{i=1}^t (1 + \gamma_i)$. The stationarized variable X_t is noted \tilde{X}_t

Thus the general equilibrium of the economy is characterized by: $N_t, \tilde{K}_t, L_t, \tilde{I}_t, \tilde{w}_t, r_t, q_t, \tilde{c}_{i,t+i}, \tilde{s}_{i,t+i}$ for $i \in (0, 2, \dots, 14)$. (37 variables, 37 equations)

$$N_t = \sum_{i=0}^5 n_{i,t-4} N_{t-i-4} \quad (56)$$

$$\tilde{K}_{t+1} = \frac{(1 - \delta)\tilde{K}_t + \tilde{I}_t}{1 + \gamma_{t+1}} \quad (57)$$

$$L_t = \sum_{i=0}^9 \left(\left(\prod_{j=0}^i p_{j,t-i+j} \right) \varphi_{i,t} N_{t-i} \right) \quad (58)$$

$$\tilde{w}_t = (1 - \alpha) A \left((1 - \alpha) + \alpha \left(\frac{\tilde{K}_t}{L_t} \right)^{1-\frac{1}{\rho}} \right)^{-\frac{1}{1-\rho}} \quad (59)$$

$$R_t = \frac{1}{q_{t-1}} \left(\alpha A \left(\alpha + (1 - \alpha) \left(\frac{\tilde{K}_t}{L_t} \right)^{\frac{1}{\rho}-1} \right)^{-\frac{1}{1-\rho}} + q_t(1 - \delta) - \left(-\frac{a}{2} \left(\frac{\tilde{I}_t}{\tilde{K}_t} \right)^2 \right) \right) \quad (60)$$

$$q_t = 1 + a \left(\frac{\tilde{I}_t}{\tilde{K}_t} \right) \quad (61)$$

$$\tilde{c}_{0,t} = \frac{\tilde{\Omega}_t}{(1 + \mu\nu_{0,t}) + \sum_{i=1}^{15} \beta^{i\sigma} (1 + \mu\nu_{0,t})^\sigma (1 + \mu\nu_{i,t+i})^{1-\sigma} \left(\prod_{j=1}^i p_{j,t+j} \right)^\sigma \left(\prod_{j=1}^i R_{t+j} \right)^{\sigma-1}} \quad (62)$$

with

$$\tilde{\Omega}_t \equiv \varphi_{0,t} \tilde{w}_t + \sum_{i=1}^9 \left(\varphi_{i,t+i} \tilde{w}_{t+i} \prod_{j=1}^i \frac{1 + \gamma_{t+j}}{R_{t+j}} \right)$$

$$\tilde{c}_{i,t+i} = \left(\beta^i \prod_{j=1}^i p_{j,t+j} \prod_{j=1}^i R_{t+j} \right)^\sigma \left(\frac{1 + \mu\nu_{0,t}}{1 + \mu\nu_{i,t+i}} \right)^\sigma \frac{\tilde{c}_{0,t}}{\prod_{j=1}^i (1 + \gamma_{t+j})} \quad (63)$$

$$i \in (1, 2, \dots, 14)$$

$$\tilde{s}_{0,t} = \varphi_{0,t} \tilde{w}_t - (1 + \mu\nu_{0,t}) \tilde{c}_{0,t} \quad (64)$$

$$\begin{aligned} \tilde{s}_{i,t+i} &= \varphi_{i,t+i} \tilde{w}_{t+i} + R_{t+i} \cdot \frac{\tilde{s}_{i-1,t+i-1}}{1 + \gamma_{t+i}} - (1 + \mu\nu_{i,t+i}) \tilde{c}_{i,t+i} \\ & \quad i \in (1, 2, \dots, 14) \end{aligned} \tag{65}$$

$$\left(\sum_{i=0}^{14} N_{t-i} \tilde{s}_{i,t} \left(\prod_{j=0}^i p_{j,t-i+j} \right) \right) = q_t \tilde{K}_{t+1} (1 + \gamma_{t+1}) \tag{66}$$

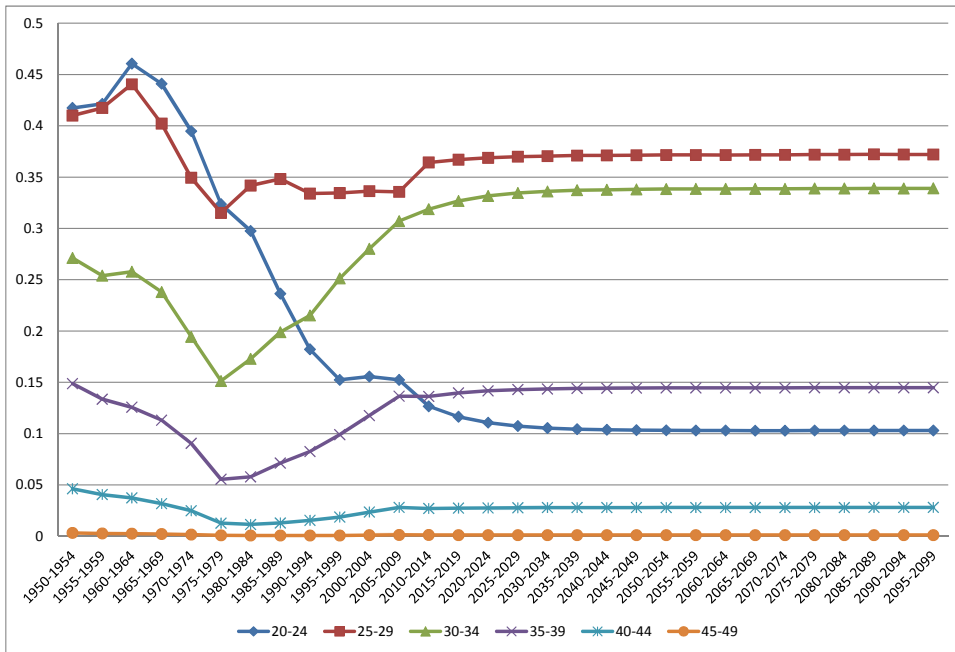
4 The French case

In this section, we calibrate the previous model on French data. We study the impact of aging on capital price. We also give some robustness checks.

4.1 Exogenous variables

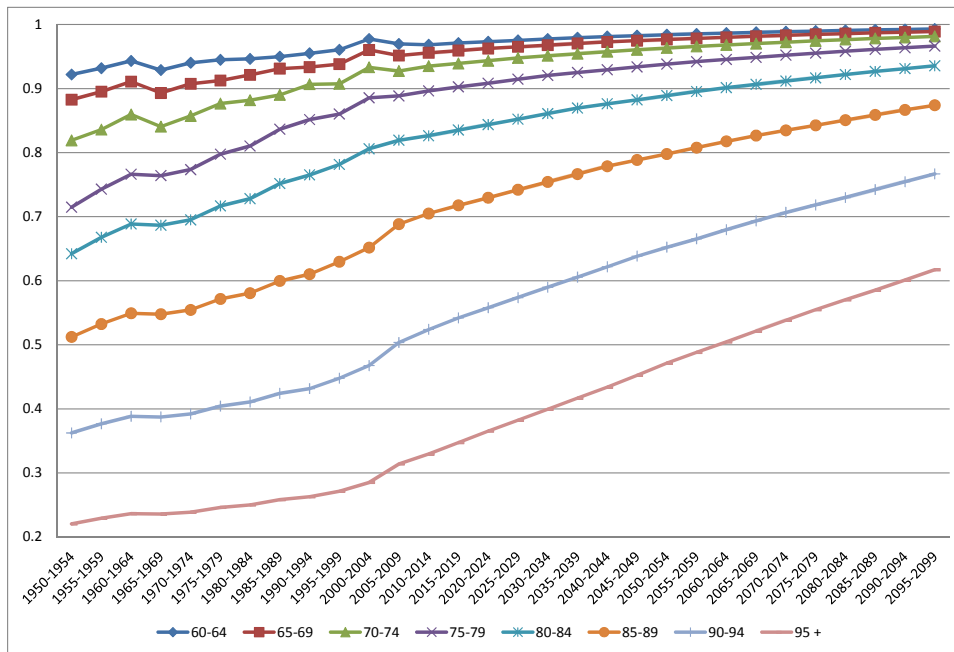
In this section, we give the tendency of exogenous variables: fertility rates and survival probabilities for different age groups (see Figure 2 and 3). In accordance with the model's definition, the birth rate is defined as the number of child per person (not just woman) in five years. The French birth rate for people aged 20-24 continue to decline after 1960-1964. This could be explained by the increase of female labor force participation. The birth rates of other middle age groups (25-29,30-34,35-39) decrease until 1975-1979 and improve after this period. This suggests the postponement of motherhood. Also, the improvement of medical care contribute to the gains of survival probability of old people over all age groups.

Figure 2: Fertility rates



The Figure gives the evolution of French fertility rates of different age groups.

Figure 3: Survival probabilities



The Figure gives the evolution of French survival probabilities of different age groups.

4.2 Calibration

For the production function, we choose an allocation rule between Capital and Labor as 0.4 and 0.6. For a period of five years, we choose a depreciation rate at 25%. We use a annual time preference factor at 0.99 so $0.99^5 = 0.951$ for five years. This parameter has been chosen in order to reproduce the capital output ratio observed for France. We try to calibrate our model in the French case for the period of 2010-2015. Recent estimation works (e.g. Antràs (2004)) suggest that the elasticity of substitution should be below one. We use a relatively low elasticity of substitution at 0.9 for production function. Parameters are shown in Table 2 and the targeted variables in Table 3.

Table 2: Parameters for simulations

Parameters		Values
Consumer parameters		
Elasticity of inter-temporal substitution	σ	0.5
Time preference factor	β	0.951 ¹
Technology parameters		
Scale parameter	A	10
Relative efficiency of capital	α	0.4
Relative efficiency of labor	$1 - \alpha$	0.6
Elasticity of substitution	ρ	0.9
Rate of depreciation	δ	0.25 ²
Adjustment cost parameter	a	3
Exogenous variables		
Number of child per worker at age i	n_i	cf. Section 4.1 ³
Survival probability at age i	p_i	cf. Section 4.1 ³
Labor productivity growth rate	γ	3% ⁴
Labor-force participation rate	φ	cf. Table (1)

¹Annualized factor: 0.99

²Annualized rate: about 5%

³UN, World Population Prospects: The 2012 Revision

⁴Annualized rate: 0.6%

Table 3: Targeted variables

Variable (Annualized)	Simulations	Observations	Period
Capital return	3.6%	4.4%	2010-2015 ¹
Growth rate	1.0%	1.2%	2010-2013 ²
Investment Output Ratio	17%	22%	2010-2013 ²
Labor Income Ratio	66%	58%	2010-2013 ²
Capital Output Ratio	2.55	2.05	2010-2013 ³

¹Average of interest rate (Indice Taux de l'Echéance Constante (TEC) 5 ans) and capital price variation (Annualized CAC 40 variation between May 31st 2010 and May 31st 2015).

²INSEE, GDP in volume.

³INSEE, Household wealth accounts.

4.3 Simulation strategy

The simulation is computed with the corresponding parameter values from an initial steady state computed with demographic values for 1950-1954 to a final steady state where the demographic parameter values are corresponding to the central scenario of United Nations for France. For the initial steady state, we use the actual probabilities of survival after 60 and the actual distribution for birth rates at different age at motherhood (data from UN). This is not exactly the real distribution of birth by age of motherhood as to compute the steady state we have to set the sum to 1. For the final steady state, in the 2100 UN projections the sum is almost already equal to one due to their convergence assumptions.

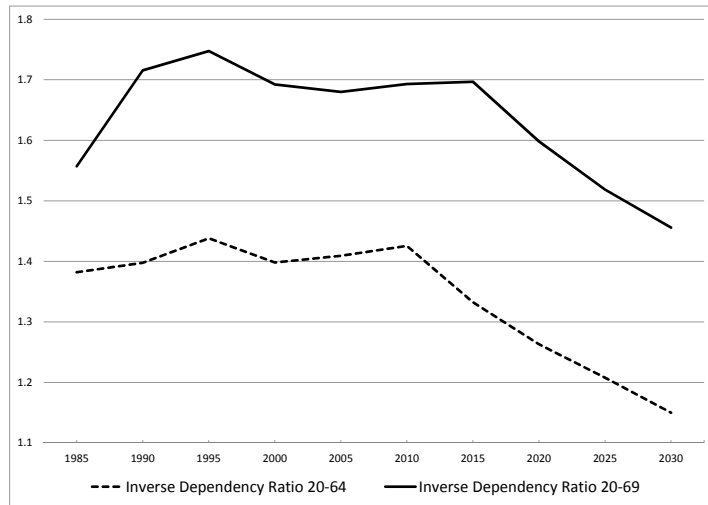
4.4 Results

In this section, we compute the capital price variations due to the French demographic evolution. The calibration, as mentioned above, has been done for France over the period 2010-2015.

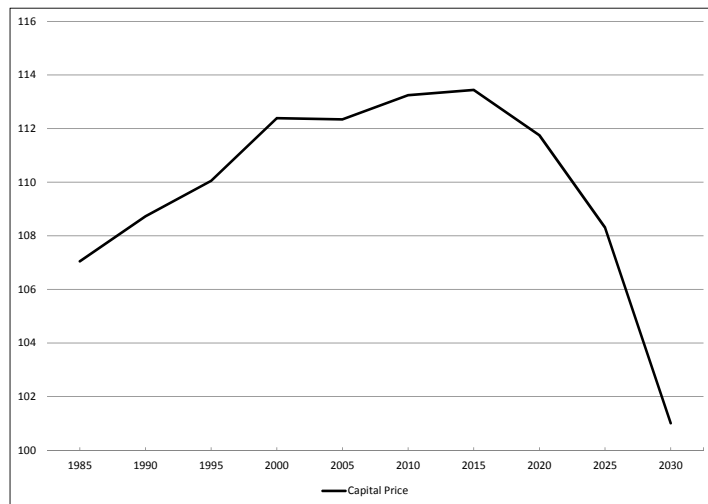
We report the simulation results below.

The inverse dependency ratio is defined as the ratio of working-age population to the rest of the population. The Figure 4a gives the evolution of the inverse dependency ratio in two ways: one with age 20-64 people as working age population which is common used

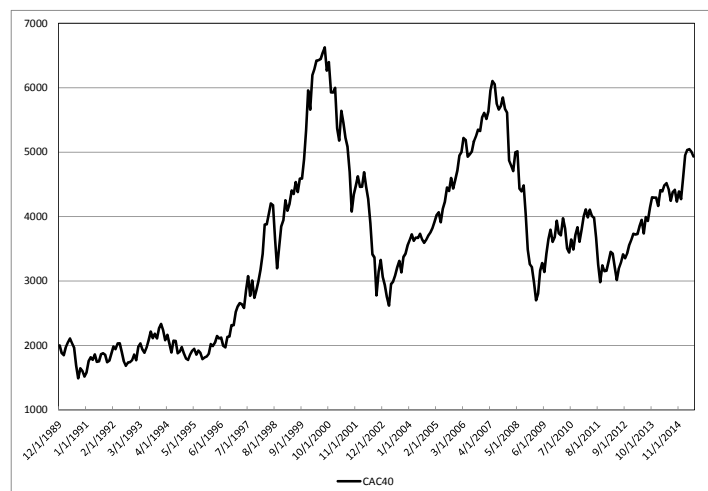
Figure 4: Pure demographic impact



(a)



(b)



(c)

and another with age 20-69 people as working age population which is the definition of our model. If we compare their evolutions with the capital price change in Figure 4b, we can find approximately the same trend.

The global impact on capital prices is roughly of 6% between 1989 and 2014 (see Figure 4b). This is far less than the magnitude of the CAC40 index variation between 1989 and 2014 which is approximately of 150%. If the first peak is roughly at the same date (2000 for the CAC40 and 2000-2004 for the simulations), it is not the case for the second peak (2007 for the CAC40 and 2010-2014 or even further for the simulations).

It seems that the demographic has a role to play in capital price. In our simulation with all demographic changes (birth rates and survival probabilities), the capital prices expressed in terms of the consumption good increase and then decrease according to the demographic cycles. But the magnitude of the phenomena under rational expectations in a closed economy is very small compared to the stylized facts. Part of the missing effect is probably a consequence of the five year period model that flatten phenomena but it's far from explaining the whole huge gap between the model and the data. Other parts of the missing effect probably come from international capital flow and bubbles. Our model does not take into account either transitory financial bubbles (Caballero et al. (2006)) nor fundamental rational bubbles as in Tirole (1985).

4.5 Robustness checks

In this section we present robustness checks that address three parameters: the elasticity of substitution in production, the cost of children in percentage of parents' consumption and the adjustment cost parameter.

The targeted variables are mostly sensitive to the elasticity of substitution in production, when changing to a higher elasticity boost the capital return but weaken the labor income. The child cost parameter does not have dramatic effects on targeted variables. The adjustment cost parameter has an effect on the level of investment.

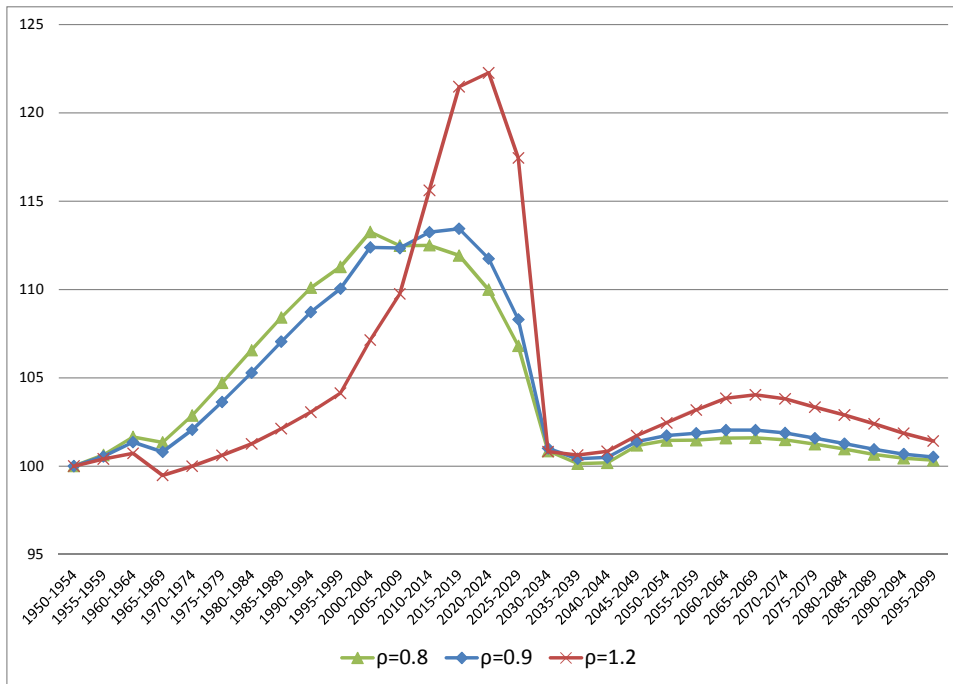
Table 4: Robustness checks

Changed parameters	Capital return	Growth rate	Investment Output Ratio	Labor Income Ratio	Capital Output Ratio
$\rho = 1.2$	7.69%	0.54%	17.36%	48.38%	2.53
$\rho = 0.8$	2.41%	0.33%	16.54%	72.74%	2.49
$\mu = 0.5$	4.10%	0.45%	16.68%	66.15%	2.45
$\mu = 0$	2.95%	0.37%	17.89%	66.65%	2.74
$a = 4$	3.87%	0.42%	14.89%	65.83%	2.23
$a = 2$	3.29%	0.43%	20.13%	66.95%	3.00

In the following, we show the evolution of capital prices when the parameters change. In order to better examine the shifts of the capital price evolution curve, all the values are initialized at 100.

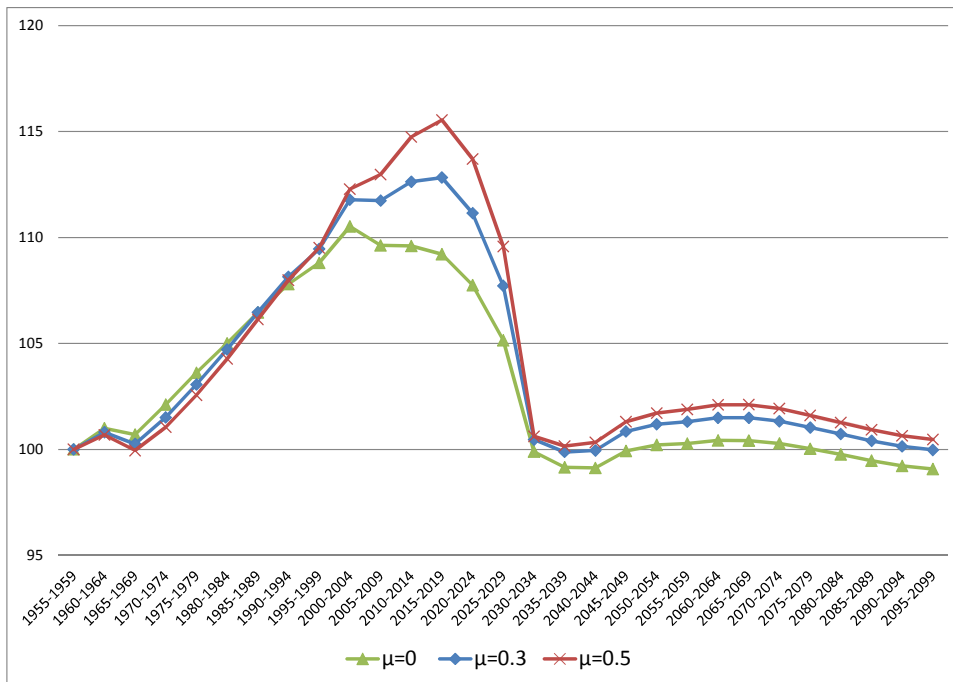
A higher elasticity of substitution leads to a more volatile capital price, thus to a higher maximum prices. But the maximum is later. A higher (receptively lower) children cost parameter seems to delay (receptively advance) the date at which the maximum price is reached. In addition, the different adjustment cost does not change the curve form of the capital price but the magnitude of the price level variation. A larger parameter causes a higher price level.

Figure 5: Capital price variations for different elasticity of substitution



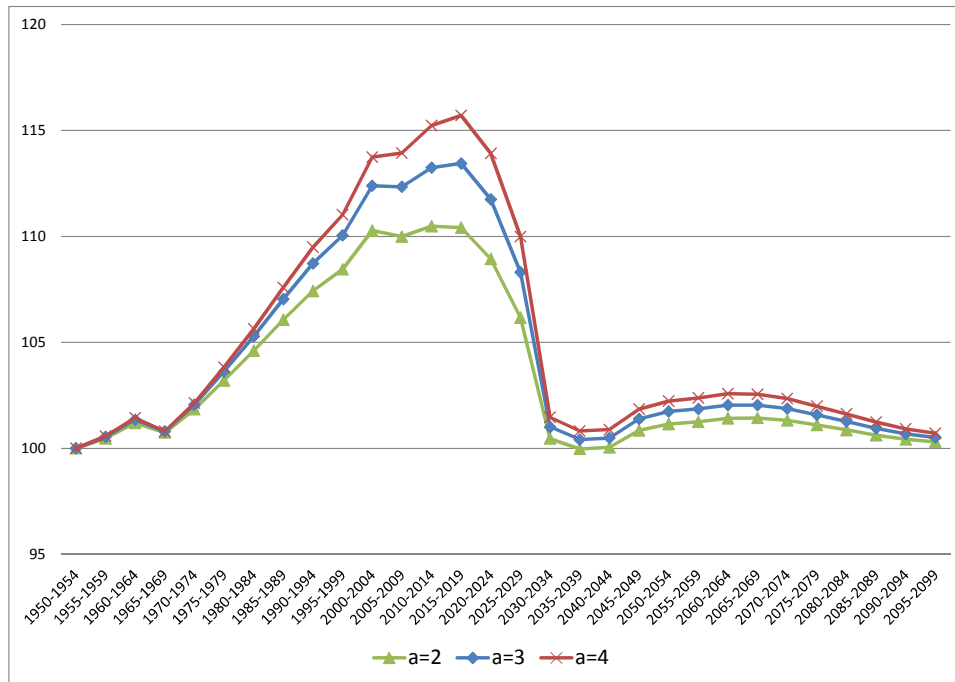
The Figure gives the evolution of capital price over the simulated period by changing the elasticity of substitution parameter. In the benchmark case, $\rho = 0.9$. Two other alternative parameters are chosen to test the form of price curve: 0.8 and 1.2.

Figure 6: Capital price variations for different adjustment cost parameters



The Figure gives the evolution of capital price over the simulated period by changing the Cost of children in % of parents' consumption parameter. In the benchmark case, $\mu = 0.3$. Two other alternative parameters are chosen to test the form of price curve: 0 and 0.5

Figure 7: Capital price variations for different adjustment cost parameters



The Figure gives the evolution of capital price over the simulated period by changing the adjustment cost parameter. In the benchmark case, $a = 3$. Two other alternative parameters are chosen to test the form of price curve: 2 and 4

5 Conclusions

The role of demographic variables in explaining asset prices has been largely studied in theoretical and empirical researches. This paper constructs a three period OLG model with two working and reproductive periods as well as adjustment costs to study the impact of demographic variables on capital prices.

The theoretical exercise shows that the effects of the decrease of young parents' birth rates could be compensated by the increase of older parents' birth rates if the variations of the two birth rates happen in the same time and with the same magnitude.

We also build a more sophisticated model with shorter periods and 16 generations, in order to be more realistic. It is calibrated on France. We find an impact of demographic variables of 13% on capital prices. This price variation is very far to be enough to explain the observed magnitude of the phenomena. First, we could investigate the role of bubbles to complete our model. We could secondly study other ways to build expectations and

not only rational ones. In particular, the assumption of myopic expectations could be investigated. Thirdly, independently that we live in an open world, it seems that some amplification mechanisms of demographic phenomena must be at work and we have to identify them. In particular, the model studied here is real and the role of monetary policy stance has not been studied. This is left for future research.

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