A Tradeoff between the Output and Net Foreign Asset Effects of Pension Reform

Abstract

and

I. INTRODUCTION

and

II. THE MODEL

A. Households

$$U = \int^{L} u \; c \;\, s \quad \cdot e^{-\beta s} \cdot ds \qquad L$$

$c \;\, s$ $\qquad s \qquad \beta$

$$\dot{a} \;\, s \;\, = r \cdot a \;\, s \;\, + H \;\, s \;\, - c \;\, s \qquad a \;\, s$$

s

$r \qquad H \;\, s$

$$H \;\, s \;\, = \begin{cases} w - \tau & \leq s \leq R \\ \quad b & R < s \leq L \end{cases}$$

$R \qquad < R < L \quad w \qquad \tau$

b

Assumption 1 *The net of tax wage earning is greater than the pension benefit:* $w - \tau > b$.

$$a \quad = a \;\, L \;\, =$$

$$\int^{L} c \;\; s \;\; \cdot e^{-rs} \cdot ds = \int^{R} (w-\tau) \cdot e^{-rs} \cdot ds + \int_{R}^{L} b \cdot e^{-rs} \cdot ds = \left(\frac{\;}{r} \right) \cdot \left\{ (w-\tau) \cdot \left(\;\; - e^{-rR} \right) + b \cdot e^{-rR} \cdot \left(\;\; - e^{-r \;\; L-R} \;\; \right) \right\}$$

$$u \;\; c \;\; s \;\;\; = \lambda \cdot e^{\;\beta - r \;\; \cdot s}$$

$$\beta = r$$

$$c = (w-\tau) \cdot \frac{\left(\;\; - e^{-rR} \right)}{\left(\;\; - e^{-rL} \right)} + b \cdot \left[\;\; - \left(\frac{\; - e^{-rR}}{\; - e^{-rL}} \right) \right]$$

t

L

R

$C = L \cdot c$

$$A^{h} = \;\; {}^{R} \left(\frac{w-\tau-c}{r} \right) \cdot \left(e^{rs} - \;\; \right) \cdot ds + \;\; {}_{R}^{L} \left(\frac{c-b}{r} \right) \cdot \left[\;\; - e^{-r \;\; L-s} \;\; \right] \cdot ds = \frac{(w-\tau-b)}{r} \cdot \left[L \cdot \left(\frac{\; - e^{-rR}}{e^{-rL}} \right) - R \right]$$

B. Firms

$$Y = \left(K^f\right)^{\alpha} \cdot \left(N^f\right)^{-\alpha}$$

N^f K^f ——

$$K = \alpha \cdot R \cdot \left(\frac{\alpha}{r}\right)^{\overline{\ \ -\alpha}} \qquad Y = \alpha^{\alpha} \cdot R \cdot \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{\ -\alpha}}$$

$$w = \alpha^{\alpha} \cdot (\ \ -\alpha) \cdot \left[\frac{\alpha}{r}\right]^{\frac{\alpha}{\ -\alpha}}$$

$$b = \frac{\tau \cdot R}{L - R}$$

τ

$$A^h(R\ \ \tau) = \frac{\ }{r} \cdot \left[w - \tau \cdot \left(\frac{L}{L-R}\right)\right] \cdot \left[L \cdot \left(\frac{\ -e^{-rR}}{\ -e^{-rL}}\right) - R\right]$$

τ

$R < L \cdot \ \ -\frac{\tau}{w}\ \ = \overline{R}$ R

$\overline{R}$ $A^h\ \ R\ \ \tau$

Lemma 1 *For a given tax* τ *, the aggregate household assets function* $A^h\ \ R\ \ \tau$ *is: a) positive for all the interior feasible values of* R *, when* $R \in\ \ \ \overline{R}$ *; b) equal to 0 when* R *approaches its boundary feasible values:* $\lim_{R\to}\ A^h = \lim_{R\to\overline{R}} A^h = \ $ *; and c) increasing and concave in* R *for all* $R \in\ \ R_m$ *, and decreasing (either concave or convex) in* R *for all* $R \in\ R_m\ \overline{R}$ *, where* R_m *denotes the value of* R *at which* $A^h\ \ R\ \ \tau$ *achieves its maximum.*

R

$L - R$ R

τ K Y A^h τ b

E R

b

A^h

K A

A τ

A^h τ K A τ

$R \in$ R_m $R \in$ R_m $\overline{R}$

R_m R A^h R τ K

A^h R b

$$R > \frac{b \cdot L}{w} = \underline{R}$$

A^h R b

Lemma 2 *For a given pension benefit* b *the aggregate household assets function* A^h R b *is convex in* R, $\frac{\partial\ A^h\ R\ b}{\partial R} <$, *and satisfies* $_{R \to L} \frac{\partial A^h\ R\ b}{\partial R} <$ *If* R *is the value of* R *such that* $\frac{\partial A^h\ R\ b}{\partial R} = \alpha \cdot \left(\frac{\alpha}{r}\right)^{\overline{\ \ }-\alpha}$, *then* $R > R_m$

Assumption 2 *At the initial equilibrium, the retirement age satisfies* $R > R > R_m$

A^h R b A^h R τ

$A = A^h - K$

$A \quad R \quad \tau \qquad A \quad R \quad b$

III. Effects of Pension Reforms

τ

$R \qquad R$

$b \qquad E \qquad E$

$\tau \qquad \tau$

$A^h \;\; \tau \qquad A^h \;\; \tau$

$\frac{\partial A^h \;\; R \;\; b}{\partial R} < \alpha \cdot \left(\frac{\alpha}{r} \right)^{\overline{\;\;} - \alpha} \qquad E \qquad P$

$E \qquad P \qquad K$

$K \qquad K \qquad Y \qquad Y$

$A \qquad A$

$b \qquad b \qquad R$

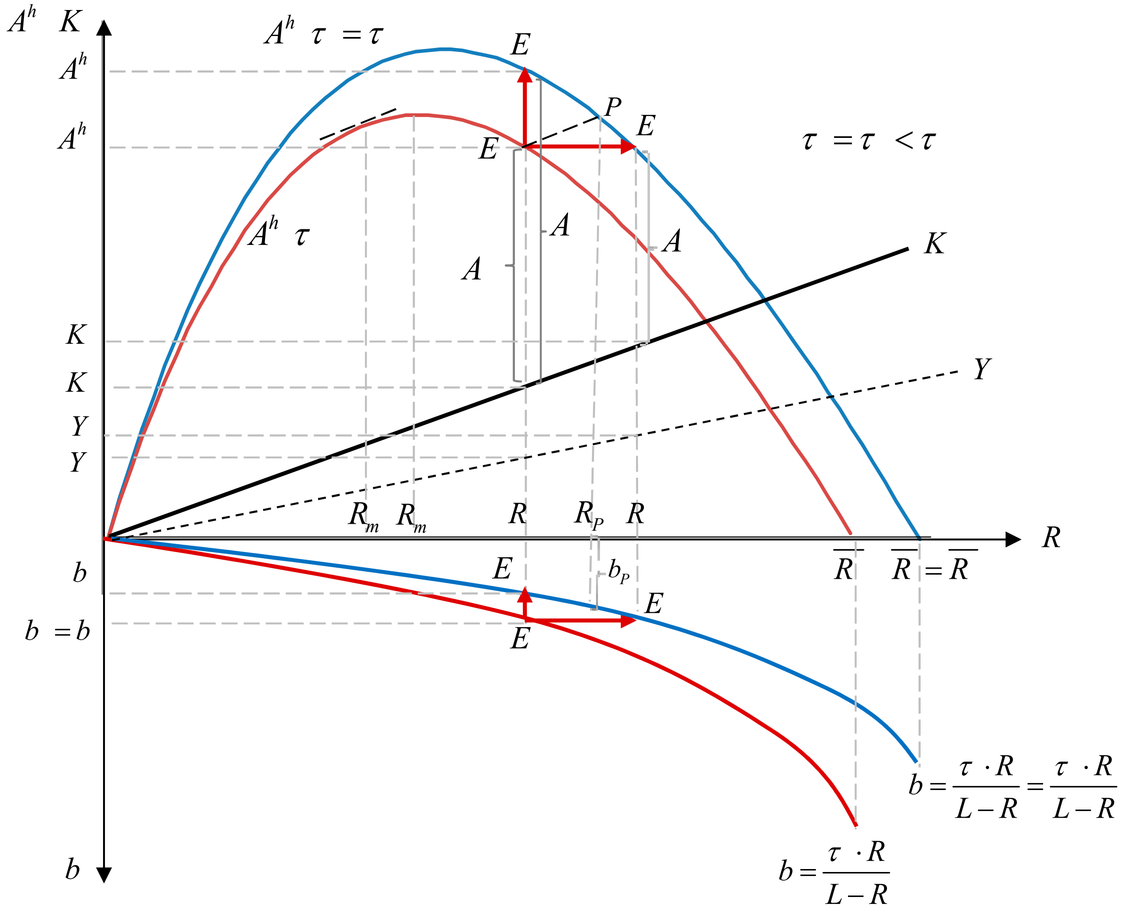
$E \qquad E \qquad \tau$

$\tau \qquad \tau \;\; = \tau \;\; < \tau \qquad A^h \;\; \tau \qquad A^h \;\; \tau$

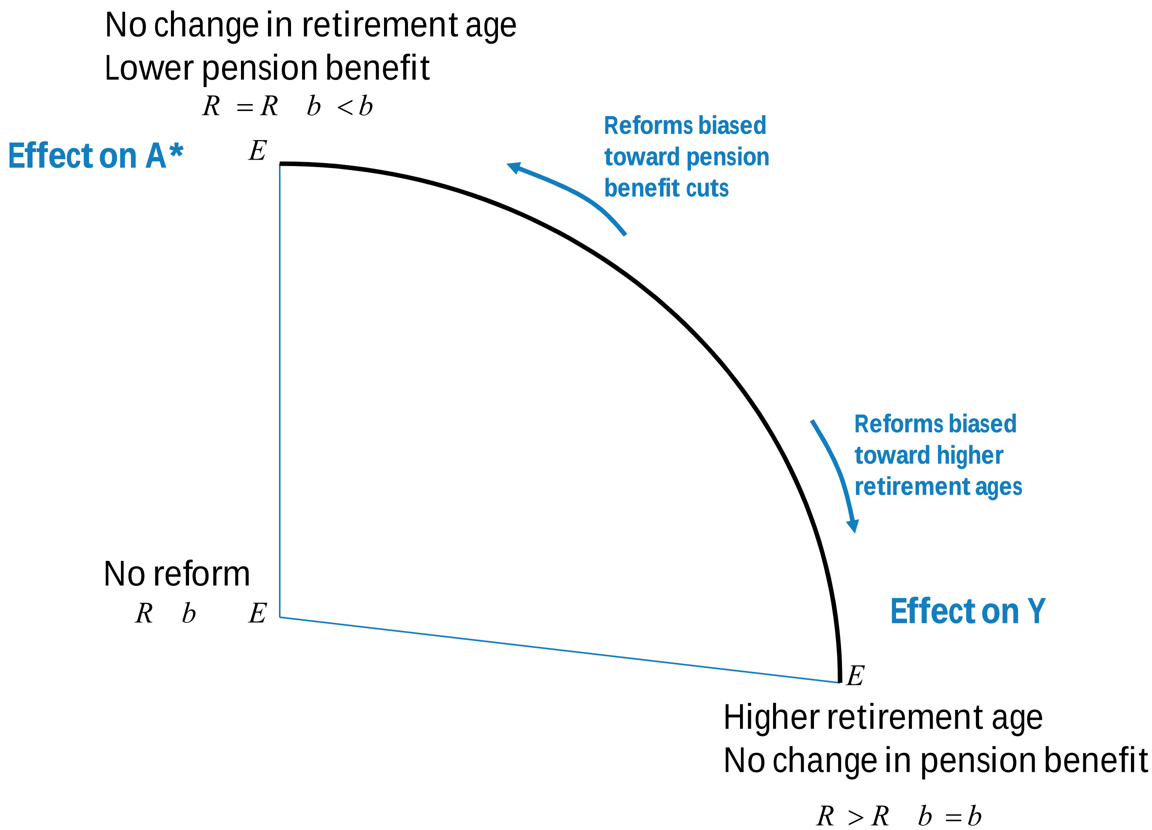
$A^h \qquad A^h \qquad A$

Proposition 1 *Under assumptions 1 and 2, there is a tradeoff between the long-term output and net foreign asset effects of pension reforms that achieve similar fiscal targets: a) reforms that increase the retirement age have an expansionary effect on output, but a negative effect on net foreign assets; b) reforms that cut pension benefits improve the net foreign asset position but have no output effect.*

$E \quad E \qquad E$



A^h K
A^h $\tau = \tau$
E
A^h
P
E
A^h
E
$\tau = \tau < \tau$
A^h τ
A
A
A
K
K
K
Y
Y
Y
R_m R_m R R_P R
R
b
E
b_P
$\overline{R}$ $\overline{R} = \overline{R}$
E
$b = b$
E
$b = \frac{\tau \cdot R}{L - R} = \frac{\tau \cdot R}{L - R}$
$b = \frac{\tau \cdot R}{L - R}$
b



No change in retirement age
Lower pension benefit
R = R b < b
Reforms biased toward pension benefit cuts
Effect on A*
E
No reform
R b E
Reforms biased toward higher retirement ages
Effect on Y
E
Higher retirement age
No change in pension benefit
R > R b = b

APPENDIX. PROOFS

Proof of Lemma 1

$A^h \; R \; \tau$ $\quad w-\tau\cdot\left(\frac{L}{L-R}\right)>$ $\quad R\in \quad \overline{R}$

$R<L\cdot \quad -\frac{\tau}{w}$

$L\cdot\left(\frac{-e^{-rR}}{-e^{-rL}}\right)-R>$ $\quad R<\overline{R}<L$ $\quad A^h \; R \; \tau \; >$ $\quad R\in \quad \overline{R}$

$A^h \; R \; \tau$ $\quad R$

$$\frac{\partial A^h}{\partial R}=-\frac{\tau\cdot L}{r\cdot(L-R)}\cdot\left[\frac{L\cdot\left(\; -e^{-rR}\right)}{\left(\; -e^{-rL}\right)}-R\right]+\left(\frac{}{r}\right)\cdot\left(w-\frac{\tau\cdot L}{L-R}\right)\cdot\left[\frac{r\cdot L\cdot e^{-rR}}{\left(\; -e^{-rL}\right)}- \;\right]$$

$$_{R\to}\left(\frac{\partial A^h}{\partial R}\right)=\frac{(w-\tau)}{r}\cdot\left[\frac{r\cdot L}{-e^{-rL}}- \;\right]>$$ $\quad \frac{r\cdot x}{-e^{-rx}}- \; >$ $\quad x>$ $\quad A^h$

R $\quad R$

$$_{R\to\overline{R}}\left(\frac{\partial A^h}{\partial R}\right)=-\frac{\tau\cdot L}{r\cdot\left(L-\overline{R}\right)}\cdot\left[\frac{L}{-e^{-rL}}-\frac{\overline{R}}{-e^{-r\overline{R}}}\right]\cdot\left(\; -e^{-r\overline{R}}\right)<$$ $\quad A^h$

R $\quad R$ $\quad \overline{R}$ $\quad \frac{\partial A^h}{\partial R}$

R $\quad \frac{r\cdot L\cdot e^{-rR}}{-e^{-rL}}-$ $\quad R$

L

$A^h \; R \; \tau$ $\quad R$

$$\frac{\partial \;\; A^h}{(\partial R)}=-\frac{\cdot\tau\cdot L}{r\cdot(L-R)}\cdot\left[\frac{L\cdot\left(\; -e^{-rR}\right)}{\left(\; -e^{-rL}\right)}-R\right]-\frac{\cdot\tau\cdot L}{r\cdot(L-R)}\cdot\left[\frac{r\cdot L\cdot e^{-rR}}{\left(\; -e^{-rL}\right)}- \;\right]-\left(w-\frac{\tau\cdot L}{L-R}\right)\cdot\frac{r\cdot L\cdot e^{-rR}}{\left(\; -e^{-rL}\right)}$$

$R<R_m$ $\quad R$ $\quad L$

A^h R

$\overline{R}$ $\overline{R}$ L $_{R\to\overline{R}}\left(w-\frac{\tau\cdot L}{L-R}\right)=$

$\frac{\partial\ A^h}{(\partial R)}$ $\frac{\partial\ A^h}{(\partial R)}>$

Proof of Lemma 2

$A^h\ b\ R$ R

$$A^h\ b\ R\ =-\frac{}{r}\cdot\left[w-b\cdot\frac{L}{R}\right]\cdot\left[L\cdot\left(\frac{-e^{-rR}}{-e^{-rL}}\right)-R\right]$$

$$\frac{\partial A^h\left(b\ R\right)}{\partial R}=\frac{b\cdot L}{r\cdot R}\cdot\frac{\left[\ -e^{-rR}\cdot\left(\ +r\cdot R\right)\right]}{\left(\ -e^{-rL}\right)}+\frac{w}{r}\cdot\left[\frac{r\cdot L\cdot e^{-rR}}{-e^{-rL}}-\ \right]$$

$$\frac{\partial\ A^h\left(b\ R\right)}{(\partial R)}=-\left(w-\frac{b\cdot L}{R}\right)\cdot\left(\frac{L\cdot r\cdot e^{-rR}}{-e^{-rL}}\right)-\frac{\cdot b\cdot L}{r\cdot R}\cdot\frac{\left[\ -e^{-rR}\cdot\left(\ +r\cdot R\right)\right]}{\left(\ -e^{-rL}\right)}<$$

R

$$\frac{\partial A^h\left(b\ R\right)}{\partial R}=\left(\frac{}{r}\right)\cdot\left(w-\frac{b\cdot L}{R}\right)\cdot\left[\frac{r\cdot L\cdot e^{-rR}}{-e^{-rL}}-\ \right]+\frac{b\cdot L}{r\cdot R}\cdot\left[\frac{L\cdot\left(\ -e^{-rR}\right)}{R\cdot\left(\ -e^{-rL}\right)}-\ \right]$$

R L $_{R\to L}\left(w-\frac{b\cdot L}{R}\right)\cdot\left[\frac{r\cdot L\cdot e^{-rR}}{-e^{-rL}}-\ \right]<$

$_{R\to L}\left[\frac{L\cdot\left(\ -e^{-rR}\right)}{R\cdot\left(\ -e^{-rL}\right)}-\ \right]=$ $_{R\to L}\frac{\partial A^h\left(b\ R\right)}{\partial R}<$

$A^h\left(R\ b\right)$ $A^h\left(R\ \tau\right)=A^h\ \ R\ \tau\ b\ R$

$$\frac{\partial A^h\left(R\ b\right)}{\partial R}=\frac{\partial A^h\left(R\ \tau\right)}{\partial \tau}\cdot\frac{\partial \tau\ \ R\ b}{\partial R}+\frac{\partial A^h\left(R\ \tau\right)}{\partial R}$$

$$\frac{\partial A^h\left(R\ b\right)}{\partial R}-\frac{\partial A^h\left(R\ \tau\right)}{\partial R}=\frac{b\cdot L}{r\cdot\left(L-R\right)}\cdot\left[\frac{L\cdot\left(\ \ -e^{-rR}\right)}{R\cdot\left(\ \ -e^{-rL}\right)}-\ \ \right]>$$

$$R=R_m \quad \frac{\partial A^h\left(R_m\ b\right)}{\partial R}>\frac{\partial A^h\left(R_m\ \tau\right)}{\partial R}=\alpha\cdot\left(\frac{\alpha}{r}\right)^{\overline{\ \ -\alpha}}$$

$A^h\ \ R\ b$ R R

$$\frac{\partial A^h\ \ R\ \ \ b}{\partial R}=\alpha\cdot\left(\frac{\alpha}{r}\right)^{\overline{\ \ -\alpha}} \qquad R\ \ >R_m$$

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