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**OPTIMAL DEGREE OF FUNDING OF  
PUBLIC SECTOR PENSION PLANS**

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# Optimal degree of funding of public sector pension plans\*

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## Abstract

This paper explores the optimal degree of funding of public sector pension plans. It is assumed that a benevolent social planner decides on the contribution of current tax-payers to the funding of public sector pensions next period, weighing the interests of current and future tax payers. Two elements play a role in the optimal funding decision: the optimal-portfolio choice (i.e. the tradeoff between the expected excess return and the additional risk of funding vis-à-vis pay-as-you-go) and intergenerational redistribution (i.e. whether the current generation of tax payers is willing and capable to prefund the pension obligations of current public sector workers or shifts the burden to future generations via a pay-as-you-go scheme). The optimal degree of funding appears to vary over time, depending not only on the relative weight given to the current generation, risk aversion, and the distribution of financial risk and human capital risk, but also on the actual state of the economy, i.e. on wage income, funding in the past and the realization of the excess return on this funding.

**Keywords:** public sector pension plans, funding, implicit debt, portfolio approach

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# 1 Introduction

Most countries have separate pension plans for public sector employees. Traditionally, these specific arrangements are justified by the guarantee they provide for security, integrity and independence of the employees and by their contribution to the attractiveness of a career in the civil service. The future fiscal burden of these plans can be substantial as the government usually is the largest employer and pension promises in the public sector tend to be relatively generous compared to the private sector (Palacios and Whitehouse 2006). The generosity of pension entitlements in the public sector can be explained by the aim to offset the lower wages in the public sector compared to the private sector (Schieber 2011, Disney **et al.** 2009).

The funding of public sector pension plans has attracted much public attention since the recent fiscal crisis in western world<sup>1</sup>. There is a growing concern regarding the size of the associated fiscal burden. Recent studies report on the size of this implicit government debt (Müller **et al.** 2009, Ponds **et al.** 2012). Table 1 displays for a selection of countries the present value of net unfunded liabilities on a fair value base. The largest implicit debt positions can be found in countries with predominantly pay-as-you-go based plans. For France and Germany for example, the implicit debt of public sector pensions plans as percentage of GDP end of 2008 would amount to 90% respectively 60%. Countries with funded plans generally have much lower implicit debt positions, compare the Netherlands, Canada, and Sweden. The US have an in-between position with an implicit debt in state pension funds of around 30% of GDP end of 2008.

What may explain the cross-national diversity in funding practice of public sector pension plans and associated implicit public debt positions? Munnell **et al.** (2011) put forward that each generation of tax payers should pay the full cost of the public service it receives. If a public servant's compensation includes promised pension income in retirement, then the cost of that benefit should be recognized and funded at the time the employee is in service, not when the pension benefit actually is paid out. However full funding is rather the exception than the rule. Public sector pension schemes in many countries are financed on a pay-as-you-go basis. In case of funding, they often are deeply underfunded according to official estimates, and the underfunding is even larger when market-based accounting principles are used.

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<sup>1</sup>Compare for example Müller *et al.* (2009) for an extensive report on the EU countries in order of the European Central Bank, Ponds *et al.* (2011) for a comparing study for a number of OECD countries, the special issue of the Journal of Pension Economics and Finance 2011, vol. 7(2) devoted to the US state pension funds, and for the UK the Final Report of the Independent Public Service Pensions Commission Commission Public Sector Pension Economics of March 2011.

Table 1: Public sector pension liabilities as implicit debt compared to explicit government debt; as percentage of GDP end of 2008

Country	Implicit debt	Explicit debt	Total debt	Financing method
France	91	61	152	PAYG primarily
Finland	77	38	115	PAYG primarily
UK	63	75	138	PAYG and funded
Germany	58	44	102	PAYG primarily
US	31	53	84	Funded
Norway	19	26	45	PAYG primarily
Sweden	8	38	46	PAYG primarily
Netherlands	5	50	55	Funded
Canada	5	36	41	Funded

cf. Ponds et al. 2011, OECD

Why are public servants' pension benefits not fully funded? We discuss a number of arguments featuring in the literature.

Firstly, there always will be an incentive to shift the costs of public servants' pensions accruals to future taxpayers. Misguiding accounting principles may serve this willingness by unrealistic high discount rates for valuing future pension liabilities, thus downplaying the costs (Gold 2003, Novy-Marx and Rauh 2009, Brown and Wilcox 2009). The accountability horizon of pension fund management and politicians is much shorter than the horizon over which pension promises have to be met by adequate funding. This horizon gap may lead to underestimation of the costs and risks and to overestimation of the earning capacity of assets. The US state pension funds for example are allowed to use a discount rate of 8%, even for plans wherein future pensions can be adjusted for wage inflation or price inflation. Underfunding of local plans may also be a substitute for government debt when the local government is constrained from issuing additional debt.

Secondly, underfunding may be the outcome of a rational strategy of government agencies to control the claims of representatives of public sector employees. In many countries labor unions hold a strong position in the public sector. This may lead to asymmetric risk sharing. Strong unions may claim pension fund surpluses for improvement of the remuneration, whereas they will force tax payers to bail out pension fund deficits (Immergut **et al.** 2006, Tsebelis 1995). In such a configuration, the government can have a strong incentive to underfund the public sector plans. A public sector pension fund in a political environment with more veto points and veto players is likely to have higher funding ratios as tax payers may have more opportunities to block legislative claims for benefits increases and pension

bailouts in the public sector.

Thirdly, the cross country variation in pension system design can also be explained by the political preferences prevailing in the late 1930 and early 1950s when most of universal mandatory pension in the developed countries were established (Perotti and Schwiabacher 2009). The severe economic shocks in the interwar period might have had impact on the then prevailing political preferences. Large inflationary shocks devastated middle class savings in a number of countries, among them typically the countries in Continental Europe with nowadays a large call on paygo financing (Germany, France, Italy, Belgium). The political majority shifted support away from pension savings and free markets to social insurance and a strong role for state intervention. Countries without severe war destruction and price shocks tended to rely more on savings, like the Scandinavia, Switzerland and the Netherlands in Continental Europe as well as the UK and its allies Canada, the US, and Australia. Once pension system parameters are set, reversing these choices will involve major legislative changes that may be difficult to realize because the transformation process may be long-lasting and costly and hampered by vested interests, such as those of the elderly.

These arguments may explain why underfunding or no funding is practice in many countries and they also might help to explain the variety in the degree of underfunding across countries.

The economic literature may provide guidance regarding the question what the optimal degree of funding might be. Remarkably, there is only a limited number of studies related to this issue. First we have the study of d'Arcy **et al.** (1999) using the tax smoothing argument, by which the preferred tax policy for pensions is to be a constant percentage of taxable income over time. A range of funding paths over time can be optimal, depending on the relationship between the deterministic growth rates of pension costs and taxable income. The tax rate has to be chosen such that at the end of the horizon the position of full funding will be realized. In the context of the US state pension funds system, Bohn (2011) relates optimal funding to the comparison of taxpayers' cost of funds with the return on pension assets. He finds zero funding is optimal as taxpayers' borrowing costs are larger than the pension funds' return on their assets. The presence of legal ambiguities and defaults risks may warrant some funding.

Lucas and Zeldes (2009) put forward that funding and investment policy considerations would be irrelevant in a completely frictionless market setting. Individuals then are able to offset any effect of tax policy on the timing of consumption (Ricardian neutrality), and, in case of funding, taxpayers will undo the impact of risk taking in public sector pension funds by adjusting the asset allocation of their private savings (Modigliani-Miller neutrality). Important conditions for these neutralities are that the public is fully informed and aware of the level of the implicit government debt in public sector plans and can undertake any action

without costs and frictions.

Also the portfolio choice approach (cf. Dutta **et al.** 2000, Matsen and Thøgersen 2004) to pension plans may provide useful insights in the optimal funding of public sector pension plans. The portfolio approach is based on the welfare aspects of pay-as-you-go (paygo) pension plans in dynamic efficient economies (Merton 1983, Gordon & Varian 1988). A paygo program effectively is a government created asset that permits one generation to trade in the human capital returns of the next. The portfolio-choice approach considers the low-yielding but also low-risky paygo pension as a quasi financial asset. In the literature the portfolio-choice framework is used to explore the optimal mix of an unfunded social security plan (with a rate of the return equal to the growth rate of the gross wage bill) and a funded individual pension plan (providing the capital market return). As wages and capital market returns are less than perfectly correlated, a mix of an unfunded and a funded plan benefits from risk diversification. The optimal size of paygo social security programs appears to depend critically on the settings of the plan (fixing either the contribution rate or the replacement rate, see e.g. Wagner 2003); the type of the individual welfare function (relative standing, constant relative risk aversion Knell 2010); social welfare evaluation (Rawlsian *ex ante* or traditional interim, see e.g. Matsen and Thøgersen 2004); the characteristics of the economy, such as open or closed economies (Krueger and Kubler 2006); wage process dynamics with either temporarily or permanent wage shocks (Thøgersen and Bøhlerengen 2010), cross-country differences in the dynamics of wage and investment income and their covariance (Borgmann 2005, de Menil **et al.** 2012).

In this paper, we apply the portfolio-choice framework to the optimal degree of funding of public sector pension plans. We analyse the optimal combination of funding and pay-as-you-go in the financing of public sector pensions as a trade-off between the excess return and the additional risk of funding vis-à-vis paygo. Taxable income primarily consists of wages and wage-linked income. So taxable income has a good match with total remuneration of public sector workers as public sector wages follow private sector wage formation and public sector pension plans often are structured as wage-indexed defined-benefit plans. Full paygo financing therefore implies low tax rate volatility, but in a dynamic efficient economy it also leads to a higher tax rate compared to funding. Funding of public sector pensions gives prospect on earning the capital market rate of return and thus on a lower, but more volatile tax rate.

We do not only take this optimal portfolio perspective into account, however. In addition, we also include an intergenerational redistribution perspective. Prefunding of a public sector pension plan implies that the current generation of tax payers is willing and is capable to prefund the pensions of the current generation of public sector workers. With defined-benefit

pensions, the next generation of tax payers then still has to warrant the pensions claims of the retired public sector workers, but these claims are backed by the contributions plus the investment proceeds, and only the difference between the pension claims and accumulated capital in the fund is for account of the tax payers at that moment. With a paygo pension scheme for public sector workers, the bill of the future pension claims of current workers in that sector is completely shifted to the next generation of tax payers. Likewise, current tax payers have to pay for the paygo financed pensions of previous generations of workers in the public sector.

We assume that a benevolent planner takes both the optimal portfolio perspective and the intergenerational redistribution perspective into account and weights the interests of current and next-period tax payers when deciding on the contribution of the current generation of tax-payers in the funding of public sector pensions next period. The willingness to prefund is then determined by the wage income of the current generation of workers and the residue of the public sector pension scheme fund which has to be absorbed by the taxpayers. The social planner preferences as described by the social welfare function capture the individual preferences, so that tax payers have no incentive to undo the impact of the decisions taken by the planner. However, as we abstract from wage-indexed bonds, the social planner can raise social welfare by enabling the intergenerational trade of risks that is not possible in the market.

We find (for plausible assumptions for the relevant variables) a large preference for paygo financing of public sector pension funds<sup>2</sup>. When the social planner only takes the utility of future generations of tax payers into account (i.e., when only the optimal portfolio perspective is taken into account)<sup>3</sup>, the optimal degree of funding is completely determined by the expected excess return of funding (which raises the optimal funding rate), the additional risk of funding compared to paygo financing (which lowers the optimal funding rate) and the degree of risk aversion (more risk aversion lower the optimal funding rate). The more weight is attached to the utility of current tax payers, i.e. the more important the intergenerational redistribution perspective, the lower (ceteris paribus) the optimal degree of funding. Another important result is that, if the utility of current tax payers is taken into account, the optimal funding rate is not constant but depends on the funding in the previous period and on the actual state of the economy, i.e. on wage income and the excess return on the investments in the pension fund.

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<sup>2</sup>it should be noted that we abstract from a social security plan, so all risk diversification between savings wage-related taxation has to come from the financing of public sector pensions.

<sup>3</sup>This is comparable to the Rawlsian ex ante evaluation of social welfare in Matsen and Thogerson, 2004).

## 2 The model

As tax payers ultimately have to bear the costs and risks of public sector pension plans, we relate the optimal degree of funding to the risk profile of tax payers. A simple two-period stochastic OLG model is used to analyze the optimal degree of funding of a public sector pension plan. The setting is a two-sector small open economy. Population growth is zero and labour supply is exogenous. The allocation of the working generation over the private and public sector is stable over time.

### 2.1 Taxation and public sector pensions

The wage income of the private sector workers ( $X_t$ ) is taxed to pay for the remuneration of the public sector employees. The latter consists of two parts: the wage income in the first period of life ( $W_t$ ) and the pension benefit received when old ( $B_{t+1}$ ) in the second period. The pensions of the public sector employees are of the defined-benefit type and indexed to wages, so we can write  $B_{t+i} = aX_{t+i}$ . To keep the model tractable, it is assumed that there are no other government outlays besides the remuneration of the public sector employees. So disposable income of the private sector workers  $Y$  is what remains from private sector gross wage income  $X$  after paying the public sector wages  $W$  and the tax to back the pension obligations  $T$ :

$$Y_t = X_t - W_t - T_t \quad (1)$$

The tax rate in period  $t$  is the sum residue in the pension scheme (i.e. the difference between the pension benefits to be paid and the funding in the previous period plus rate of return,  $B_t - F_{t-1}(1 + R_t)$ ) and funding for public sector pensions next period ( $F_t$ ):

$$T_t = B_t - F_{t-1}(1 + R_t) + F_t \quad (2)$$

So the tax in period  $t$  depends on the choices made regarding the degree of funding in the past and in the current period. In the case of pure paygo-financing, i.e. no funding in the past nor in the current period, the tax equals the current pension obligations:  $T_t = B_t$ .

### 2.2 Households

In the first period of life, a tax payer of generation  $t$  has a net income of  $Y_t$ . This income can be used for consumption when young ( $C_t^y$ ) or saved ( $S_t$ ) for consumption during old age. So  $C_t^y = Y_t - S_t$ . Savings can be invested in risk free bonds with a return  $R^f$  or in stocks with a stochastic return  $R_t$ . Let  $\theta_t$  be the share of the portfolio invested in stocks, then the portfolio return next period is given by  $R_{t+1}^p = \theta_t R_{t+1} + (1 - \theta_t) R^f$  and the consumption of



a member of generation  $t$  when old equals:  $C_{t+1}^o = (1 + R_{t+1}^p)S_t$ . We assume that individuals choose  $\theta_t$  and  $S_t$  so as to maximize expected lifetime utility, which is described by a standard CRRA function:

$$E\{U(C_t^y, C_{t+1}^o)\} = \frac{(C_t^y)^{1-\gamma}}{1-\gamma} + \frac{1}{1+\rho} E \left\{ \frac{(C_{t+1}^o)^{1-\gamma}}{1-\gamma} \right\} \quad (3)$$

where  $\rho$  is the rate of time preference and  $\gamma > 1$  is a parameter determining the aversion. In Appendix 1 we show that these optimal choices result in an indirect utility function  $\tilde{U}(Y_t) = \Psi \frac{Y_t^{1-\gamma}}{1-\gamma}$  where  $\Psi$  is a constant determined by the risk free interest rate and the mean and the variance of the return on stocks.

### 2.3 Government

We assume that the government acts on behalf of the private sector workers in defining the finance structure of the public sector pensions. This may either be pure pay-as-you-go, partial funding, or (more than full) funding. The government in period  $t$  chooses the rate of funding as a fraction  $\phi_t$  of net labour income  $(1-a)X_t$ . So  $F_t = \phi_t(1-a)X_t$ . This funding rate is chosen so as to maximize expected social welfare which is described as:

$$E\{V_t\} \equiv E\{\tilde{U}(Y_t)^\alpha \tilde{U}(Y_{t+1})\} \quad (4)$$

It is assumed that the social planner expects that the funding rate chosen today will also be applied next period. That is:

$$Y_{t+1} = (1-a)X_{t+1} + F_t(1 + R_{t+1}) - F_{t+1} \quad (5)$$

$$= (1-a)X_{t+1}(1 - \phi_t) + \phi_t(1-a)X_t(1 + R_{t+1}) \quad (6)$$

$$= (1-a)X_t[1 + G_{t+1} + \phi_t(R_{t+1} - G_{t+1})] \quad (7)$$

where  $G$  is the growth rate of labour income, i.e., the return on human capital<sup>4</sup>.

Note that if  $\alpha = 0$  the government maximizes the utility of the next generation. Given the assumption  $\phi_t = \phi_{t+1}$  this implies that the government maximizes ex ante utility of a steady-state generation<sup>5</sup>. In this situation, only the optimal portfolio perspective plays a role. If  $\alpha > 0$  the utility of the current generation is taken into account as well and so the intergenerational redistribution perspective comes into play. In this case, also the return on

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<sup>4</sup>We leave unspecified the stochastic nature of the gross wage growth, i.e. the implicit return on the paygo program. Compare Thogersen and Bohlerengen (2010) for an analysis of the impact of wage shocks as being either temporarily or permanent on the optimal size of paygo.

<sup>5</sup>This is comparable to the Rawlsian ex ante evaluation of social welfare in Matsen and Thogerson, (2004).

funding of public sector pensions by the previous generation (if any) is considered:

$$Y_t = (1 - a)X_t + F_{t-1}(1 + R_t) - F_t \quad (8)$$

$$= (1 - a)X_t[1 - \phi_t] + (1 + R_t)\phi_{t-1}(1 - a)X_{t-1} \quad (9)$$

$$= (1 - a)X_t[1 - \phi_t + Q_t] \quad (10)$$

where  $Q_t \equiv \phi_{t-1} \frac{1+R_t}{1+G_t}$ . This variable  $Q$  can be interpreted as the effective funding of public sector pensions by the previous generation. This effective funding is larger if the previous generation invested more in funding (larger  $\phi_{t-1}$ ). If there was funding by the previous generation ( $\phi_{t-1} > 0$ ), the size of  $Q_t$  depends also positively on the excess return on funding, i.e., on the return on stocks compared to the return on human capital. It should also be noted that the government at time  $t$  can observe the value of  $Q_t$  before deciding on  $\phi_t$ .

We show in Appendix 2 that the maximization of social welfare leads to a simple first-order condition for the optimal funding rate  $\phi_t$ :

$$\phi_t = \frac{[\mu_{r-g} + \frac{1}{2}\sigma_{r-g}^2] - \frac{\alpha}{(1-\phi_t+Q_t)}}{\gamma\sigma_{r-g}^2} \quad (11)$$

where  $\mu_{r-g}$  and  $\sigma_{r-g}^2$  denote the mean and the variance of the excess return of stocks over human capital. In general, equation (11) is a quadratic equation in  $\phi_t$ . This equation can easily be solved, but the resulting expression is complex and does not provide much insight. For the special case  $\alpha = 0$  (when the government only looks at the utility of the next generation, i.e., only the optimal portfolio perspective is relevant) the expression for the optimal funding rate reduces to:

$$\phi_t = \frac{\mu_{r-g} + \frac{1}{2}\sigma_{r-g}^2}{\gamma\sigma_{r-g}^2} \quad (12)$$

From this equation it immediately follows that funding is always positive if  $\alpha = 0$ . That is, from an optimal portfolio perspective it is always optimal to invest in stocks and trade capital market risk and human capital risk via the public sector pension scheme. Note further, that it follows from (11) that  $\frac{\partial\phi_t}{\partial\alpha} < 0$  (as long as  $\phi_t < 1 + Q_t$ ). So the larger the weight of the current generation in the funding decision, i.e. the more important the intergenerational redistribution perspective, the lower the degree of funding. Numerical simulation indeed shows that the optimal funding rate rapidly decreases if  $\alpha$  increases. In fact,  $\phi$  may easily become negative, that is, if intergenerational redistribution toward the current generation is very important for the government, then it might be optimal to borrow on the capital market instead of investing in the capital market by funding future public sector pensions. So funding will not be introduced (or will be abolished) if the current generation is important in government decision making. Numerical simulation also shows that (if  $\alpha > 0$ ) the optimal

funding ratio rises if  $Q_t$  increases. So if the utility of the current generation is taken into account, the rate of funding will be higher if effective funding by the previous generation is larger.

### Introducing a minimum subsistence level

The model can easily be extended by allowing for a minimum subsistence level by assuming that utility is only determined by lifetime income in so far as this exceeds a minimum level  $\bar{Y}$ , i.e.  $\tilde{U}(Y_t) = \Psi \frac{(Y_t - \bar{Y})^{1-\gamma}}{1-\gamma}$ . In Appendix 3 it is shown that in that case the first-order condition for the optimal funding rate is:

$$\phi_t = \frac{[\mu_{r-g} + \frac{1}{2}\sigma_{r-g}^2] - \frac{\alpha}{(1-\phi_t+Q_t)(1+m)}}{[1 + (\gamma - 1)(1 + m)]\sigma_{r-g}^2} \quad (13)$$

where  $m$  is a parameter that measures how important the minimum subsistence level is (see Appendix 3). As it is assumed that  $\gamma > 1$ , it is evident that the larger the value of  $m$  (i.e. the more important the minimum subsistence level), the smaller the amount of funding. Note, however, that it still holds that funding is always positive if  $\alpha = 0$ .

### Summarizing our results:

- funding will only be introduced if the government does not give a too large weight to the initial generation;
- it is more likely that funding is introduced if the current generation is rich because of a high net wage income relative to the minimum subsistence level (low  $m$ ) or due to large effective funding by the previous generation (high  $Q$ );
- if funding exist it may well be abolished by future generations;
- it is more likely that funding will be abolished in a period where the return on previous funding is low and/or the wage income is low.

## 3 Simulation results

We have derived that the optimal degree of funding depends on a number of factors. A crucial factor is related to the economic characteristics as realized in the current state and the expectations regarding those characteristics in the next period. Also an important factor are the decisions taken in the previous state regarding the degree of funding of pensions to be paid out in the current period. The higher the degree of funding, the more the disposable

income of the current tax payers will be modified by the pension fund result. Finally the optimal degree of funding next period will also be determined crucially by the relative weights attached to the current and next-period tax payers.

The table below reports the optimal degree of funding next period for a specific set of the parameters. For this set  $\phi_t$  equals 0.1335. Then we have indicated how  $\phi_t$  will change for a higher value of each of the parameters (a relative increase with 10%). The realization of the capital market return  $R$  and growth rate wages  $G$  are 5% and 3% respectively. So the pension fund realizes in this state a funding surplus, where the surplus is larger the higher the prevailing degree of funding  $\phi_{t-1}$  in period  $t - 1$  has been set. The pension fund surplus raises the net wage income of tax payers and so the room to prefund next period pension payments increases. The better the prospect regarding the excess return of savings compared to paygo  $\mu_{r-g}$  is, the higher the funding rate  $\phi_t$ . A higher risk regarding capital market return  $\sigma_r$  leads to lower funding, i.e. a lower  $\phi_t$ . Also higher wage growth risk  $\sigma_g$  will lead to a lower  $\phi_t$ . This can be explained by the increase in the mismatch risk between the pension fund result, driven by the capital market return, and the tax base<sup>6</sup>. When the subsistence level is higher, there is less room to prefund next period's pensions. Finally prefunding will be lower when in the social welfare function a higher weight is given to the current generation of tax payers,  $\alpha$ . In figure 1 we have displayed the relationship between the preferred degree of funding next period and the relative weight of the current generation of tax payers in the social welfare function for the whole range for  $\alpha$  from 0 tot 1.

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<sup>6</sup>When the correlation coefficient  $\rho$  is sufficiently low, an increase in wage growth risk will lead to a higher preferred degree of funding next period because of risk diversification. Given the expression for the variance of the excess return of funding over paygo  $\sigma_g^2 + 2\rho\sigma_g\sigma_r + \sigma_r^2$ , it will hold that  $\partial\phi_t/\partial\sigma_g > 0$  when  $2\rho\sigma_g\sigma_r + \sigma_g^2 < 0$ , so when  $\rho < -\sigma_g/2\sigma_r$ .

**Table 2: The optimal degree of funding  $\phi_t$**

$\phi_t$	$\phi_{t-1}$	$\mu_{r-g}$	$\sigma_r$	$\sigma_g$	$\rho$	$R$	$G$	$\gamma$	$m$	$\alpha$
<b>0.1335</b>	0.3	0.04	0.2	0.04	0	0.05	0.03	6	0.4	0.5
<b>0.1351</b>	0.33									
<b>0.1449</b>		0.44								
<b>0.1221</b>			0.22							
<b>0.1329</b>				0.044						
<b>0.1310</b>					0.1					
<b>0.1336</b>						0.055				
<b>0.1334</b>							0.033			
<b>0.1214</b>								6.6		
<b>0.1317</b>									0.44	
<b>0.1288</b>										0.55
<i>R, G, <math>\mu_{r-g}, \sigma_r, \sigma_g</math> are reported on an annual base. One period is 20 years.</i>										

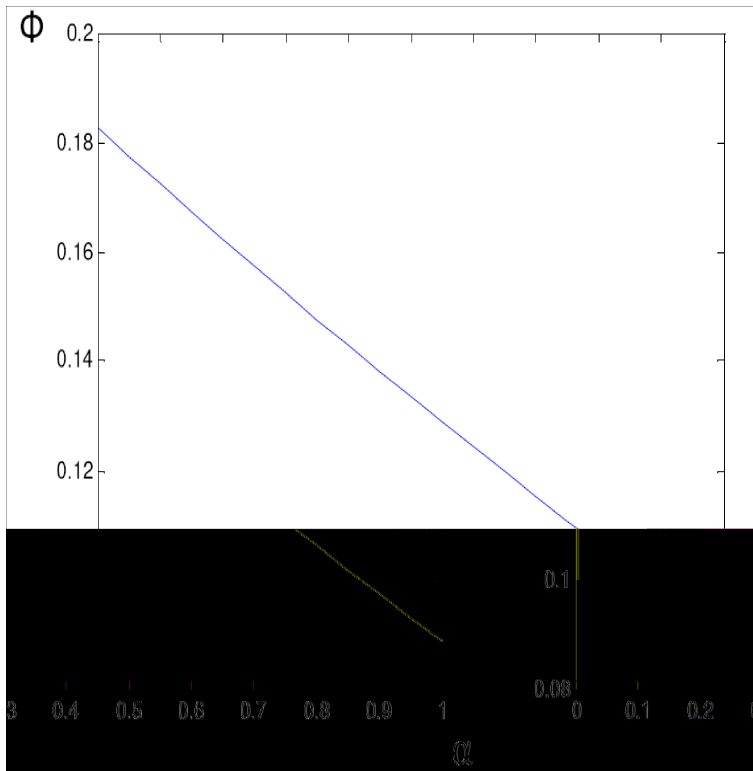


Figure 1: Optimal degree of funding  $\phi$  for  $\alpha$  ranging from 0 to 1.

## 4 Conclusion

Worldwide one can observe a large variety in finance practices of public sector pension plans, ranging from full funding to full pay-as-you-go. This paper provides an analytical model to explain this variety. Using a portfolio-choice framework and assuming realistically that tax-payers ultimately have to bear all risks and costs of the pensions of public sector workers, we find that the optimal degree of funding is endogenous over time and is determined by a combination of distinctive drivers: the actual disposable income of tax payers including the absorption of the pension fund result, the expectations as to next-period returns and risks on funding and paygo, risk aversion and the relative weights of the current and next-period generations of tax payers. Our findings fits well with a strand in the literature that the across-country variety in pension plan design reflects differences in political preferences in the thirties and fifties during which most current plans were established (Perotti and Schwienbacher 2008). In turn these preferences can be seen as a reflection of the economic experiences in the interbellum period.

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## Appendix 1: Derivation of the indirect utility function

Assume that the distribution of the return on stocks is lognormal. We can then derive an explicit solution for optimal savings  $S_t$  and the optimal portfolio choice  $\theta_t$  following Campbell and Viceira (2002):

$$S_t = \Omega Y_t \tag{14}$$

$$\theta_t = \frac{\mu - r^f}{\gamma \sigma^2} \tag{15}$$

where



$$\Omega \equiv \frac{\exp[0.5(\gamma - 1)\sigma_p^2][E\{1 + R^p\}]^{\frac{1}{\gamma}-1}}{\exp[0.5(\gamma - 1)\sigma_p^2][E\{1 + R^p\}]^{\frac{1}{\gamma}-1} + (1 + \rho)^{\frac{1}{\rho}}} \quad (16)$$

$$r^f \equiv \log(1 + R^f) \quad (17)$$

$$\mu \equiv \log E\{1 + R_{t+1}\} \quad (18)$$

and  $\sigma^2$  is the variance of the stock market return.

From these equations it follows that the optimal consumption when young and old can be written as:

$$C_t^y = (1 - \Omega)Y_t \quad (19)$$

$$C_{t+1}^o = (1 + R_{t+1}^p)\Omega Y_t \quad (20)$$

Substituting these equations in the utility function (3) gives the indirect utility function:

$$\tilde{U}(Y_t) \equiv \Psi \frac{Y_t^{1-\gamma}}{1-\gamma} \quad (21)$$

where  $\Psi \equiv \left[ (1 - \Omega)^{1-\gamma} + \frac{1}{1+\rho} \Omega^{1-\gamma} E\{(1 + R_{t+1}^p)^{1-\gamma}\} \right]$

## Appendix 2

The government chooses the funding rate  $\phi_t$  so as to maximize the expected social welfare as described in equation (4) subject to

$$Y_t = (1 - a)X_t[1 - \phi_t + Q_t] \quad (22)$$

$$Y_{t+1} = (1 - a)X_t(1 + Z_{t+1}) \quad (23)$$

where  $Z_{t+1} \equiv G_{t+1} + \phi_t(R_{t+1} - G_{t+1})$ .

Maximizing  $E\{V_t\}$  is equivalent to maximizing  $\log(E\{V_t\}) = \alpha \log(\tilde{U}(Y_t)) + E\{\log(\tilde{U}(Y_{t+1}))\}$ . Using (22) and leaving out constants  $\log(\tilde{U}(Y_t))$  can be rewritten as:

$$\log(\tilde{U}(Y_t)) = (1 - \gamma)[\log((1 - a)X_t) + \log(1 - \phi_t + Q_t)] \quad (24)$$

Also:

$$\log(E\{\tilde{U}_{t+1}\}) = \log(E\{Y_{t+1}^{1-\gamma}\}) \quad (25)$$

$$= (1 - \gamma)[E\{y_{t+1}\} + 0.5(1 - \gamma)\sigma_y^2] \quad (26)$$

where:

$$y_{t+1} = \log(Y_{t+1}) \quad (27)$$

$$= \log((1 + Z_{t+1})[(1 - a)X_t]) \quad (28)$$

$$= z_{t+1} + v_t \quad (29)$$

and

$$z_{t+1} \equiv \log(1 + Z_{t+1}) \quad (30)$$

$$v_t \equiv \log((1 - a)X_t) \quad (31)$$

so that it follows that

$$E\{y_{t+1}\} = v_t + E\{z_{t+1}\} \quad (32)$$

$$\sigma_y^2 = \sigma_z^2 \quad (33)$$

It follows from the definitions of  $Z$  and  $z$  that:

$$z_{t+1} - g_{t+1} = \log[1 + \phi_t(\exp(r_{t+1} - g_{t+1}) - 1)] \quad (34)$$

where  $g_{t+1} \equiv \log(1 + G_{t+1})$  and  $r_{t+1} \equiv \log(1 + R_{t+1})$ . A Taylor approximation of this function around  $r_{t+1} - g_{t+1} = 0$  gives:

$$z_{t+1} - g_{t+1} \approx \frac{f}{1-a}(r_{t+1} - g_{t+1}) + \frac{1}{2} \frac{f}{1-a} \left(1 - \frac{f}{1-a}\right) \sigma_{r-g}^2 \quad (35)$$

where  $\sigma_{r-g}^2 = \sigma_r^2 + \sigma_g^2 + 2\rho\sigma_r\sigma_g$  and  $\rho$  denotes the covariance between  $R$  and  $G$ .

It follows that

$$E\{z_{t+1}\} = E\{g_{t+1}\} + \phi_t E\{r_{t+1} - g_{t+1}\} + \frac{1}{2} \phi_t (1 - \phi_t) \frac{f}{1-a} \sigma_{r-g}^2 \quad (36)$$

$$\sigma_z^2 = \phi_t^2 \sigma_{r-g}^2 \quad (37)$$

Substituting this all in (42) gives:

$$\log(E\{\tilde{U}(Y_{t+1})\}) = (1 - \gamma)[v_t + [\mu_g + \phi_t \mu_{r-g} + \frac{1}{2} \phi_y]]$$

### Appendix 3: Introducing a minimum subsistence level

The indirect utility function is now assumed to be:

$$\tilde{U}(Y_t) = \left[ (1 - \Omega)^{1-\gamma} + \frac{1}{1+\rho} \Omega^{1-\gamma} E\{(1 + R^p)^{1-\gamma}\} \frac{(Y_t - \bar{Y})^{1-\gamma}}{1-\gamma} \right] \quad (40)$$

As before, the government is assumed to chose the funding rate  $\phi_t$  so as to maximize the expected social welfare  $E\{V_t\} \equiv E\{\tilde{U}(Y_t)^\alpha \tilde{U}(Y_{t+1})\}$  subject to  $Y_t = (1 - a)X_t[1 - \phi_t + Q_t]$  and  $Y_{t+1} = (1 - a)X_t(1 + Z_{t+1})$ . It follows from (40) that:

$$\log(E\{\tilde{U}(Y_{t+1})\}) = \log(E\{\tilde{Y}_{t+1}^{1-\gamma}\}) \quad (41)$$

$$= (1 - \gamma)[E\{\tilde{y}_{t+1}\} + 0.5(1 - \gamma)\sigma_{\tilde{y}}^2] \quad (42)$$

where:

$$\tilde{y}_{t+1} = \log(\tilde{Y}_{t+1}) \quad (43)$$

$$= \log\left((1 + Z_{t+1})\left[(1 - a)X_t - \frac{\bar{Y}}{1 + Z_{t+1}}\right]\right) \quad (44)$$

$$= z_{t+1} + \log\left((1 - a)X_t - \frac{\bar{Y}}{1 + Z_{t+1}}\right) \quad (45)$$

$$= z_{t+1} + \log[\exp(\log(1 - a) + \log(X_t)) - \exp(\log(\bar{Y}) - z_{t+1})] \quad (46)$$

$$\approx \log(l_t) + m_t(z_{t+1} - E\{z_{t+1}\}) \quad (47)$$

$$= v_t + (1 + m_t)z_{t+1} \quad (48)$$

and:

$$z_{t+1} \equiv \log(1 + Z_{t+1}) \quad (49)$$

$$l_t \equiv \exp(\log(1 - a) + \log(X_t)) - \exp(\log(\bar{Y}) - E\{z_{t+1}\}) \quad (50)$$

$$m_t \equiv \frac{\log(\bar{Y} - E\{z_{t+1}\})}{l_t} \quad (51)$$

$$v_t \equiv \log(l_t) - m_t E\{z_{t+1}\} \quad (52)$$

So that it follows that

$$E\{\tilde{y}_{t+1}\} = v_t + (1 + m_t)E\{z_{t+1}\} \quad (53)$$

$$\sigma_{\tilde{y}}^2 = (1 + m_t)^2 \sigma_z^2 \quad (54)$$

Note that  $m$  can be interpreted as a parameter that measures how important the minimum subsistence level is, i.e.  $m \approx \frac{\bar{Y}}{(1-a)X_t(1+Z_{t+1})-\bar{Y}}$ .

Following the same procedure as in Appendix 2 we find:

$$\log(E\{\tilde{U}(Y_{t+1})\}) = (1 - \gamma)[v_t + (1 + m_t)[\mu_g + \phi_t\mu_{r-g} + \frac{1}{2}\phi_t(1 - \phi_t)\sigma_{r-g}^2] \quad (55)$$

$$+ \frac{1}{2}(1 - \gamma)(1 + m_t)^2\phi_t^2\sigma_{r-g}^2] \quad (56)$$

Using this equation and (24) we can optimize  $E\{V_t\}$  with respect to  $\phi_t$ . This gives first-order condition (13).